Optoelectronic Materials and Devices
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Lecture - 23
R-G statistics via R-G centres

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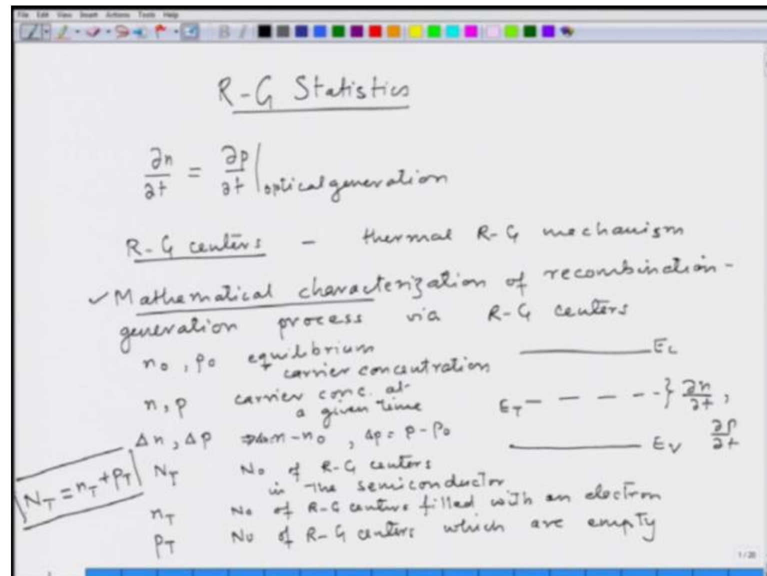

Today we will be talking about R G statistics. R G statistics, specific to when we have R G R G centres as the mechanism for recombination and generation. Before this, we have talked about recombination and generation in two cases; basically we talked about all mechanisms which would lead to the generation of electron hole pairs. We gave for change in the carrier concentration. Especially if it is an optical generation, we gave the expression when you have optical generation that is the dominant mechanism for generation.

We also looked at when recombination from band to band transition is the dominant mechanism; we looked at the recombination rate of change in the carrier concentration in that case. But in addition to this, there is a very important recombination generation process, which occurs in most devices and that is recombination and generation which is R G centre mechanism. So, R G centre centres leading to the thermal R G mechanism.

Since, this is very a important mechanism for R G processes or recombination generation processes, we devote today's lecture to understand how would we mathematically look at

the recombination generation using through the R G processes? So, affectively what we are saying is that, we have looked at different individual processes individually. We are still looking at individual processes. When you are working in a device probably more than one will be working.

Hence, you will have to incorporate all the mathematical expressions that you need in order to understand how your device will work. So, today's lecture we will be focusing on the R G statistics or how R G centres lead to recombination or generation of carriers? So, basically what we are looking at is the mathematical characterization of recombination generation process, that is via R G centres which we have discussed in the earlier lectures. So, basically we are looking at, you will like to get a handle on how to deal with them in mathematical terms.

Now, in the band diagram we had you know that our semiconductor you have conduction band, valence band edge. Then you either you have donors or acceptors; then you mentioned, that if you have centres in the middle, these act as straps or R G centres, this is the affect of this centres. How does it affect change in the carrier concentration delta n over delta t or delta p over delta t is what we would like to do now. So, to simplify things first, let us assume not a large number of mid gap state, but may be one single type of trap at a given level e t. Let us see how we can incorporate the behaviour of this trap in the R G statistics that involves these two straps.

So, with with in this semiconductor then we know few things, what is the equilibrium carrier concentration. So, we know n naught p naught we know how to get that, n naught p naught which is the equilibrium carrier consecration. We find also if carrier concentration through these R G processes is changing, so n p is the carrier concentration at any time. At a given time and then we also define access carrier concentration del n del p, which is nothing but the carrier concentration at a given time minus the equilibrium, which is del n or del p, which is p minus p naught. We further define because we have put these R G centres, which are at mid gap states. Let us say n capital N T is the total number of R G centres in the semiconductor.

In this total number of n t centres, we have a small n t which are number of R G centre levels or R G centres filled with an electron and small p t is number of R G centres, which are empty or you can they are occupied by the hole. So, you can see the

relationship is at the total number of R G centre is will be equal to the number which is occupied by electrons plus the number, which is not occupied by the electron. So, this would be the relationship in these parameters. Having defined this, let us look at how we are going to look at these processes.

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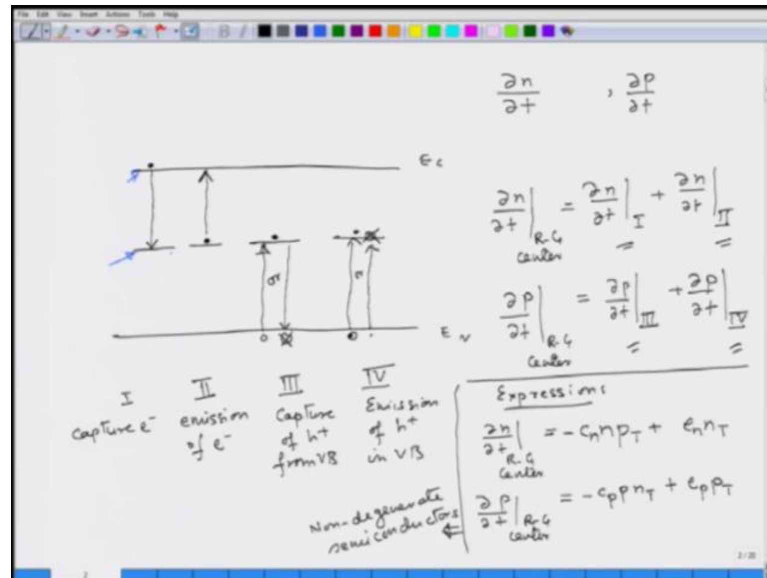

So, we are interested in finding out what would be the change because of the R G processes? What would be the change in delta n del t with time and in hole concentration with time? So, what are the processes that are happening because of these R G centres? So, let us look at it in detail. So, if this is conduction band, this is valence band and there is R G centre here. Then one thing it can do is that it can capture an electron. So, electron can be captured by this R G centre. So, this is one process that it can do which is captured electron.

And other thing it can do is, once if it has the captured electron, already it can re-emit that electron that is the second process that is emission of electron. It can also do some other things like it can capture a hole. So, if there is the electron here it can capture a hole and that is one process. This can also be shown as if the hole is getting annihilated by taking a hole from the valence band. Electron is coming down taking a hole in the valence band and this is basically captured of hole.

The third process that it can contribute towards its capture of hole from the valence band. It can emit hole as well so one can think of a situation where a electron from the valence

band goes to the trap and leaves a hole. So, this would be the process by which a electron goes to the R G centre and thereby leaving a hole here. So, that is a generation or emission of a hole this is also sometimes seen as electron annihilating a hole. So, these are different ways of looking at the same thing. Basically, this is a same process by which you are emitting.

This is the emission of a hole in valence band; this is capture of hole from valence band. So, these are the basic four processes that R G centre can get involved in. We are interested in looking at the overall effect of both recombination and generation together, what would be the net effect on the change in the carrier concentration. So, if I am trying to mathematically represented, I can represent that as change in the because of this R G centres this one particular one which we have we are considering right now.

The change in the electron concentration is going to be change due to this capture process, which is going to be a del n del t change due to the capital process first process plus change emission because of the emission process that is the second process. Similarly, I can write for holes, that is a mechanism of R G centre overall effect of recombination generation would be change due to the third process plus a change due to the fourth process. So, what are these what what what are these rates for first one, two, three and four processes? There we make certain approximations here and certain postulates on how we are going to model it.

So, if we look at the process number one. So, if I want to now give some expressions for processes one, two, three, four. Now, if I look at the process number one, what is that process doing? It is changing increasing by capturing it is decreasing the concentration of electrons in the conduction band. How it will be decreasing that? It depends on how many electrons I have; it would depend on how many electrons I have in the conduction band. What will it depend? It also needs empty trapped states in order for the electron to to combine with the trapped states, which means the rate for the process one has to be proportional to the number of electrons in the conduction band multiplied by the number of empty trap states, which was given by p t as we defined that and a proportionality constant for capturing of electrons c n.

This would reduce the electron concentration so with the negative sign. So, that is the rate for the first process. What would be for the second process? What does the second

process require? It requires such state which already has electron. So, you need it has to be then proportional to the number of states which are occupied by electron, which we have defined as n of t. That electron when it goes to the conduction band, it is basically a thermal process. It is decided only by the temperature of at that point. So, we will define that as the emission rate for electron and by this process you are adding electron in the conduction band and hence this is a positive.

So, we have taking care of the first and the second mechanism which changes the electron concentration in the conduction band due to the R G process. Similarly, I can write the change in the hole concentration. The only difference here is being that my subscripts will change. So, here it will be the capture coefficient for holes multiplied by the p n of t need empty occupied states for that and the emission of holes n p of t. So, these are the expressions general expressions, which take care of the net affect of all recombination generation processes due to the R G centres, which lead to a change in the carrier concentration.

Now, this is a general expression, which can be used in any device the calculation or characterization facility. The only only requirement here is this is for non-degenerate semiconductors. We are using this expression deriving these expressions here for non-degenerate semiconductors. So, now let us look at if this is the case, if we have a semiconductor and we have a R G centre in that semiconductor what will happen in the case of the when we have an equilibrium.

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So, if we simplify this R G statistics for equilibrium case, equilibrium what happens to those equations? Now, in equilibrium what is happening is that the number of electrons which are being captured is exactly being balanced by the number of holes which are been captured. The number of electrons which are being captured is being balanced by the number of electrons which are being emitted. Similarly, for the valence band the number of holes which are being captured is being balanced by the number of holes which are being emitted to the valence band.

Basically, equilibrium here means that each fundamental process is a detailed equilibrium; each fundamental process is balanced by an inverse process. So, basically what we are saying is that the each fundamental process being balanced by inverse process means if the electron, this is a fundamental process as emission of the electron, then it is being balanced by capture of the electron that is the inverse of that process. So, each of them will be balanced which means this balance is going to be true for electrons and for holes separately, in other words in equilibrium we are saying there is no change occurring due to R G centre processes.

So, in equilibrium due to R G centres, there is no change in the carrier concentration this is zero and the same thing can be written also for holes. So, let us look at the earlier equation that we developed in the case of the equilibrium. What will happen to it? So, if

we write if we equate equate the earlier equations to zero, you will get for the case of electron you will get c of n p t in equilibrium I am defining it with the subscript zero.

The concentration of electrons in equilibrium will be given by e and this captured cross section, let me for the time being also define for the case of equilibrium as n zero. So, emission coefficient also with e n 0 times n t 0. Now, this equilibrium statistics basically is giving you a relationship between the emission rate and the capture rate. So, I can write the emission rate coefficient and let me define them here basically what we had used in the last equation c n is known as the capture coefficient, electron capture coefficient and e of n is electron emission coefficient.

So, now by invoking the equilibrium condition, we are able to find a relationship between the equilibrium coefficient in equilibrium and emission capture coefficient emission coefficient and the capture coefficient in equilibrium. By the same thing I can also write for the holes this will give me e p o capture for holes in equilibrium n of t o and p o over p of t. Now, although there is equilibrium capture coefficient and the emission emission coefficient for electrons and holes may change as things change, but we are going to make an assumption here and you are going to assume, that the e n remains similar to what is it? What its value is in the equilibrium or c n also remains similar to its value in the equilibrium. Similarly, for e p for the holes its value is close to what it is in equilibrium and for capture of holes also its value is close to what it is in equilibrium with this simplification.

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Then I can write that e of n is equal to the emission coefficient which I say remain similar to the equilibrium coefficient will be c of n times n in equilibrium P T o over of n T o and for holes the emission coefficient or R G statistics will be c of p and t o P o over t o. Now, these values one can write this part and give it name as e of n is this being called as n one for electrons and for the holes the same thing is being called as this this p one then it becomes a short hand and we can write this as c n n 1.

Now, n 1 is a new parameter, what this parameter is n 1 is equal to n naught, which is equilibrium concentration of electrons. The number of equilibrium traps which are not occupied by electron divided by the number of equilibrium traps which are occupied by electron and p 1 is p naught n of t o over p of t o. So, we have a shorter notation for writing the emission coefficient in terms of the capture coefficient. Now, with this expression that I get from the equilibrium statistics, I can write my equations for R G statistics in a more compact form and this will be then Del and Del t due to R G centres.

So, I have got in the requirement so the general equation can be written now as negative of c of n p of t n minus n of t and n 1. Similarly, this is for also for R G centres will be written as c of p n of t p minus p of t p 1. Now, this expression let us discuss a little bit more about what this n 1 n p 1 is N 1and p 1is something that we can calculate, if we know how many traps, how many capital n t traps we have in the material? How we can do that? You can see very easily what is n naught? n naught is nothing but the

equilibrium carrier concentration. We know that this equilibrium carrier concentration is given by n naught n p naught are known n naught will be given by n i n i exponential e f minus e i this we know from our carrier statistics in equilibrium divided by k t.

What is n t o and p t o? That is the equilibrium n t o divided by capital N t is basically the number of traps which are number of trap levels or fraction of trap levels this is fraction of trap levels with electron. Now, this we have earlier had an expression for when we are looking at the dopant, we had come up with an expression how many dopants are ionized at a given time. So, if we know the dopant level and we know the Fermi level we can calculate how many dopants are going to be ionised.

Using the same equation we can also evaluate how many n how many trap states are going to be ionized. And p t o is nothing but total of n t minus n t o. So, basically this expression n 1 and p 1 can be calculated and now we have done R G statistics, which can be written in this term. Where we have basically starting from four rate constants, we have using the equilibrium condition. We have reduced it to two constant which is the capture of electron and capture of the holes by the R G centre and this expression here.

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Now, equilibrium state is good to get certain simplification here, but in general most of devices when we are looking at the R G centre mechanism we are finding that most devices operate in the steady state condition. So, the steady state condition provides interesting simplification of the R G mechanisms, which is of interest to us and that is

why we would like to spend some time to figure out what happened to the R G centre mechanism in the steady states. Now, in a steady state like in equilibrium each process had inverse and it balanced each other so electron generation and recombination was balanced by its generation and recombination process are balanced. But in case of the steady state, the average values in a steady state of all macroscopic observables remain same.

So, in such problems basically what we are saying is if the device has a electron concentration, then the electron concentration is not changing with time or hole concentration is not changing with time. In this case, we are talking about R G centre processes, basically then what it says means in steady state is that the concentration of R G centres which are occupied by electron this is 0. Basically, the concentration of the trap centres which are occupied by electron is 0.

So, if you have R G centres which is a case that we are discussing here, the number which is occupied n of t and the one which is not occupied which is p of t, this is not changing with time in a steady state. What does that mean? In a steady state, it means that the processes the overall processes balance each other. Basically, in order to have this condition that there is no change in Del n t then what we are saying is that overall this means, that this is equal to 0, which means that any change in the carrier concentration due to R G centres of electron plus any change in the carrier concentration of holes.

If I add both processes together there should be no change, which means n of t remains constant so that is the steady state condition. Now, if I apply the steady state condition to the R G statistics, then we are going to have equal number in words. Basically, we can say that there is equal creation and annihilation of electrons and holes. So, if you apply that condition I can write my equation now for Del n by Del t. I had a equation which says minus of c n this is taking it from the previous pagem where I got this particular expression. So, I would write that this plus this is equal to 0. So, minus c of n p of t and minus n of t n 1 minus c of p n of t p minus p of t p one is equal to 0.

So, we simplify this expression and see what we can get. Basically, I am trying to get expression in terms of n of t, so n of t is here I can take that out. I would say n of t is basically c of n n 1 plus c of p p minus plus c of minus c of n p of t n plus c of p p of t p

1. But we know that p of t is nothing but capital N t minus small t. So, if we substitute that here what i am going to get is I will write the final expression here. I will get that the number of traps which are occupied trap states with the electrons is going to be equal to c of n small p t n plus c of p p of t p1 divided by the capture coefficient times n 1 plus hole capture coefficient times the hole concentration.

Now, this expression is this is a total number of trap levels in a steady state, which occupied by electron this is expression for that. From this particular expression I wish to go to an expression, which would change this in terms of since I have both on both sides. We have n of t and p of t I would like to change that by utilizing this into only n of t expression. So, let me write the simplification for this, so if I do that I will write this whole equation as after putting this condition in...

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I will get expression, which would give me n of t in terms of the total number of trap states and that is given by c of n n, which is known to me which is number of electrons in a steady states times p 1 total number of trap states divided by c of n n 1 plus n plus c of p p plus p 1. So, I know what is the concentration of trap levels which are filled, so if I now want to calculate what is Del n Del t due to R G centres, the overall effect of annihilation and recombination generation in the case of R G centres and Del n of del t was nothing but minus of c n p of t in the general case minus n of t n 1.

Now, I have the expression of n of t so I would write this as minus of c n capital N t minus n of t n n small n of t n 1. This now I substitute the expression for n of t. So, I substitute this expression into this equation to find out what would be my Del n Del t. Now, the equation becomes as you can see the number of traps with electrons is a long equation so that the equations start running into three four lines. So, instead of writing all the lines let me give you the final answers and which you can check that you will get the final answer. In order to get the final answer I am going to make the substitution that n p is equal to n i square, which we know in equilibrium is always true.

So, if we make that substitution what I will find is that the del n del t due to R G centres I can write an expression which is given in terms of n p minus n i square divided by n plus n 1 c of p capture coefficient for holes times total number of trap states plus p plus p 1 p being hole p being that constant. We have discussed earlier divided by c of n capital N t. Now, this is an expression which is going to be generally true this is an expression that in a steady state it is going to define in a steady state. The recombination rates for electrons are due to R G centre recombination generation rate due to R G centre will be given by this.

This gives us the opportunity to define another parameter. That is basically this capture coefficient times n t this we are going to define as one over tau of p what is tau of p? Tau of p is nothing but it is the hole minority carrier life time. Similarly, we will define this quantity as one over tau of n and that is tau of n is electron minority carrier life time. So, in order to physically understand what these parameters mean, one has to think of what we have done. We have done we have looked at our recombination generation net effect due to our R G centre on electron concentration.

So, let us take an example of may be if we have let us say a semiconducting material which has some n naught p naught and it also has R G centres and by some means, I change the carrier concentration access carrier concentration delta n is equal to delta p, which means at some point t is equal to 0. Let us say I increase the carrier concentration to n which is n naught plus delta n. Change the hole concentration to p naught plus delta n plus delta p and delta n and delta p are same for this particular mechanism that I am talking about.

Now, since I have put in some access carrier concentration depending on what kind of material it is and let me since in most times in a most cases the semiconductors will either deal with one n or p type. So, if it is n type then under the condition that I have low injection either it is going to holes, which are the minority carrier. There will be low injection level. So, minority carriers are going to be the delta p. So, if when I have increased the electron and hole concentration it is a n type material access electron is not changing the c of the electrons, which are there already in the material, but the access holes that I have put in is changing the minority carrier concentration to p naught plus delta p and that minority carrier concentration.

If I leave after increasing this carrier concentration with time would eventually relax back to n i it would like to go back to its equilibrium carrier concentration and with the life time of tau p. So, that is what it means, when a carrier is in minority role, what is the life time of the carrier? How long can it survive in the c of the majority carrier? So, tau n and tau p parameters, which are basically being defined by the mechanism of R G centre thermal generation or recombination are signifying. If I create access carriers in a material how long the minority carrier can survive in the material before it is annihilated by the majority carriers?

So, these terminologies which we have is started with in trying to define the R G process from here we get a physical meaning for what this term means what this capture cross section then signify for the minority carriers.

So, if you have a minority carrier in a material, then this tau p is the average life time that minority carrier is going to survive. Now, then let us take this parameter as a material parameter it is generally evaluated and experimentally obtained. One experimentally obtains this parameter and so for a given material tau n and tau p are calculated. Although, you can see if I know n t and I know the capture cross section for a particular type of impurities that we are talking about. This is something which can also be calculated, but generally one measures it because c p and c n have to be measured.

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So, in terms of this then minority carrier life times my R G statistics starts looking like, change del n del t due to R G process will be given by sorry there is a mistake here it should be n 1 p 1 otherwise this will become zero. So, n 1 p 1 minus n i square so this will be given by n 1 p 1 minus n i square divided by tau of p n plus n 1 plus tau of n p plus p 1. Similarly, you can say that in since we were discussing it in the steady state condition. This is for the steady state condition this would also be same as for the R G process for the holes it will give that the whole concentration as well. So, this is so whenever you have device consideration for steady state. This is the expression one uses for finding out the carrier concentration change due to the R G processes.

Having discussed that, let us spend some time on what this parameter tau n and tau p are? These are basically electron or whole minority carrier life time. They are being defined with respect to the presence of R G centres in the material. The important part of it is that most of the semiconductors this R G process will always exist and hence this is important parameter for materials. What is this dependent on? It is generally proportional to as you can see one over some constant times capital n of t.

So, it is very interesting it does not depend on how many electrons or holes are there, it depends on how many trap states are there. So, the minority carrier life time is not depending on the carrier concentration, but it depends on the trap concentration. So, how many impurities or traps you have in a material it will decide the minority carrier life

time. So, typically if I take silicon I would find the minority carrier life time can range. Basically, tau n p or tau p can range almost sub seconds sub micro seconds in that range to the range of mille second.

So, depending on how many impurities, what is the purity level of the material is? The material parameter tau n and tau p can have a huge difference. Depending on what the application is you would like your carriers either to live for a very long time or you do not want them to get annihilated, even before they get collected by either electrodes or have some subsequent action in terms of band to band transition or emission of light. Hence, these two parameters for a given material tau and tau p are very important, but they are important in the sense they are not intrinsic material property they depend on n of t.

So, they can be changed they are dependent on the processing of the material. So, depending on how pure your material is, you will find the minority carrier life time can change in within four orders of magnitude. That would change your that would define what kind of devices you have. So, if you have if you want high speed electronic devices you want very, very few numbers of traps in it you, want high carrier life times even if you want a good l e d you want high carrier life times. So, that the light can come out by band to band transition rather than a thermal R G centre mechanisms for annihilation. So, this is a tau n and tau p and it comes out from the R G mechanisms statistics of recombination generation.

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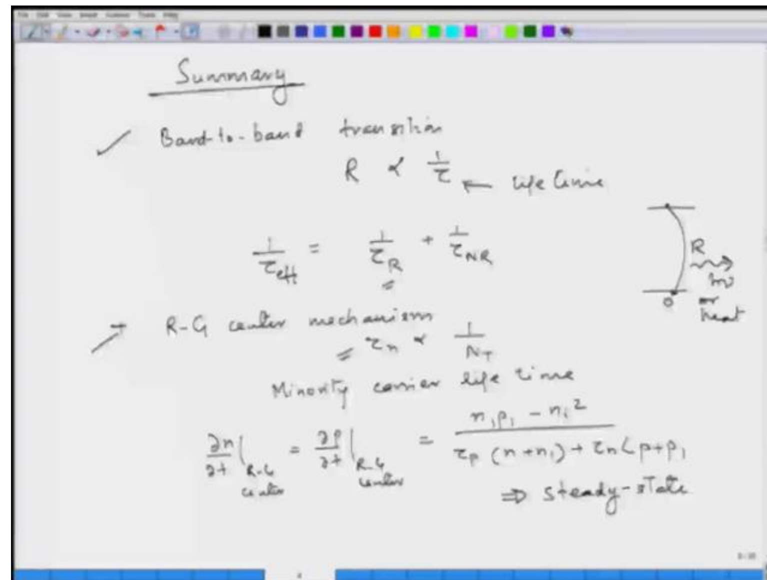

So, to give a summary of what we have discussed and to connect it with some of the things that we have talked about earlier also when we talked about the recombination process earlier for band to band transition, you are talking about band to band transition there also we had a recombination rate which was proportional to one over tau. Now, this tau which is the life time for the band to band transition and we have discussed that what are fact this tau will have in terms of recombination rate.

So, if you have more than one processes band to band processes you are going to lose some light. So, if you have more than processes then one over tau effective for band to band transition would be may be a radiative transition and if may be there is some non-radiative transition. So, this was the life time for band to band transition, this was for electron joining with the hole it is a recombination process giving you a photon or heat. And that was the life times that tau effective that we had looked at.

But as we discussed even at that time in the semiconductor there are probably more mechanisms and if it have direct band gap semiconductor you would have band to band plus you would always have this R G centre mechanism. In this R G centre mechanism we just looked at what these R G centres do so the thermal R G centre mechanisms can affect at a given what would be your minority carrier life time going to be. It is inversely proportional to that, so the tau n or tau p is inversely proportional to one over n t and

there is a life time associated with it, which is different this is the minority carrier life time minority carrier life time is related to the R G centre mechanism.

This should not be confused with the radiative life time which we discussed earlier. Basically, what we have been doing is we have been looking at single processes and giving you what how to deal with it mathematically. But if you would have a material where more than one processes are available, then you will look at the band to band transition which may be giving you the light out and also add the R G centre mechanism to that, which we will see. Then if you have more than one mechanism you would write the equation for del n del t for different mechanisms.

So, basically in this lecture we saw that if we have R G centre mechanisms, then in a steady state we have an expression that would be very useful. We have a general expression also and we then derive for a steady state. You can write this as this expression in terms of minority carrier life time, which is given by tau of p and plus n 1 p plus p 1. So, this is a general expression in steady state that can be used while calculating device characteristics.

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So, let me now take an example to show you that this general condition, that we have derived for steady state R G mechanism. If we look at in our special condition it will give you exactly the same solution that we derived for looking at band to band transition or some other mechanism. So, the special condition that we want to look at for the steady

state solution is special case we will look at for low level of injection. So, this shown example to show that this whole equation can gives you finally the same answer for the special case. So, we have the recombination rate in a steady state, which is being given as n 1 minus n i square divided by tau of p n plus n 1 plus tau of n p plus p 1.

So, generally as we said we have a normally one majority carrier. So, let us say this is n type semiconductor. Basically, that is saying that my n is much greater than p n with this condition, I also see that n and p, which we know the expression for it. So, n 1 is we say is roughly close to p 1 is also very close to n i n 1 is similar to p 1 is approximately close to n i then i can expand this for a n type semiconductor. The n 1 p 1 is going to be equal to n naught plus delta n time's p naught plus delta p and I am creating a situation where injection level is low. So, Del n or Del p is much less than n for n type semiconductor.

So, for low level of injection if this is true then this would be n naught plus delta n p naught plus delta p minus n i square divided by tau of p n can be written as again n naught plus delta n plus n 1 and p as p naught plus delta p plus p 1. Now, applying the condition for low level injection and the other conditions for which we know that n naught p naught is equal to n i square. If I expand this I will get n naught p naught plus n naught delta of p plus p naught delta of n plus delta n delta p minus n i square. We immediately see that this cancels and in the denominator I still have n naught plus delta n.

Now, since this is a low level of injection, one can immediately see that only this is going to be very negligible. Since, it is a majority carrier device p naught is much smaller. So, this is going to be very negligible the only thing that is left is, n naught delta p from the numerator because other quantities are going to be very small. The same if you see the denominator, since it is a majority carrier device, this is going to be always much larger than this value and within this. Also this is going to be always much larger that these values for n type device.

Hence, you would get n naught tau of p and you can see this nicely. This expression for steady state recombination rate reduces to what we have earlier derived for band to band transition to exponential decay of access carrier concentration with the life time. So, you can see why you have similar expression obtained in the special case of low level of injection. It is also important here to mention, that if we did the dimensional all the time

or if we looked at the dimensionality all the time, that tau of p which is defined as one over capture cross section for c p times n of t the dimensionality is such that this is in terms of seconds.

This is in terms of per centimetre cube, which means the units for the capture coefficient is basically 1 per centimetre cube per second. You can check this that even in earlier equation, that this particular dimensional units for c p will would be correct with the earlier processes. As we have discussed from the beginning so tau actually comes up getting a unit of a second and that is kind of a life time for the carriers, that we get and in the low level of injection our general expression for steady state does reduce down to what we have derived earlier. For special cases of a low level injection when we have certain rates, this gives us a good consistency with whatever we are deriving has similar expressions coming out or they come down to matching with what we get in this special cases.