Optoelectronic Materials and Devices
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Lecture - 27
Optical Properties of two interfaces: thin film case

So, today we will continue our discussion on optical properties, because this course is about optoelectronic materials and optical properties is a very crucial aspect of making devices.

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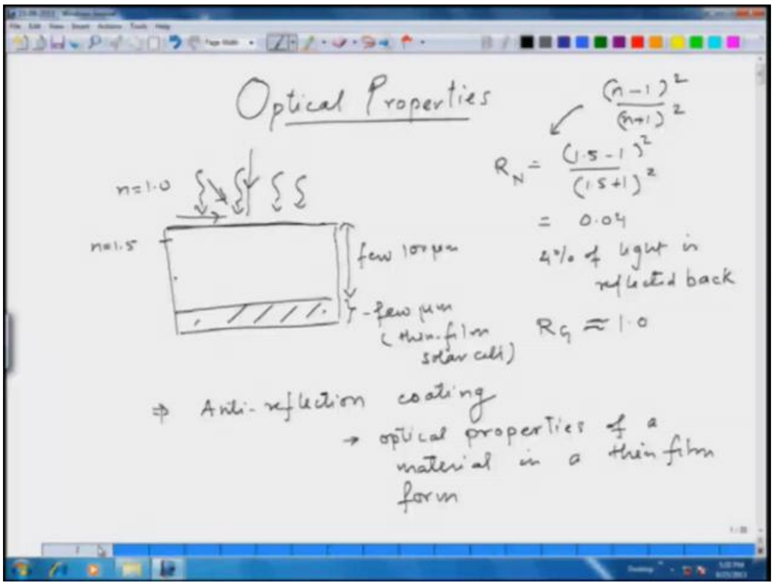

So, far we have seen how the basic optical parameters of a material, refractive index and extinction coefficient. They are based on how light interacts with the matter. Then based on those parameters, we derived what is the absorbance and reflectance of a surface is and and the fact that it is not just a n (( )), the surface roughness and so many other parameters are in homogeneity of them the of the material itself can can give rise to different effects like scattering. So, all these things have to be kept in mind. We are talking about interaction of light with the matter. For example, let us let me extend my discussion from materials to thin films because most of the optoelectronic devices we are going to discuss are going to be in the form of thin films.

So, in the last lecture I discussed the reflectance property of a surface. So, let me take an example, let us say if I am working with the solar cell, which is on a substrate which is made of glass, generally glass is going to be in few hundred of microns. Then the device would be in may be few microns. This could be an example of our thin films. For example, this film solar cell where you are making a solar cell on a glass substrate. On on this optoelectronic devices the light is shining from the glass site.

Now, my purpose here in this device if it is I am using it as a solar cell is to absorb as much light as possible in my active area of the device. Now, if we so far what we have discussed is discussed it if I have this glass substrate taking the refractive index from the ambient as 1 and for glass as 1.5, I have developed earlier formulation for finding out what is the total reflectance? How much light is reflected back, which is given by in normal incidence, let me say it is going to be given by 1.5 minus 1 over 1.5 plus 1 and square of these.

This is what we did in the last lecture. This basically comes from the normal incidence can be shown to be this for both polarization, we evaluate this, what we find is that this is almost 0.04, which means at normal incidence 4 percent of light is reflected back. Now, this may not be so bad. If you think, it is just 4 percent, but you know that if I take normal, if I take grazing incidence R g is approximately 1. It is very close to 1 which means that most of the light which comes at the grazing angle is reflected back. So, when I am talking about a device like solar cell, these considerations are important. Basically, I have I will be getting light which is not going to be directed only; it is going to be coming from all directions.

As you can see from normal as you go towards the grazing angle reflection can be as much as 100 percent or as low as 4 percent which means a lot of light is lost because of this effect. Hence, it becomes important to figure out how you are going to change this. One of the solutions which is used in solar cell in many other devices also is to use anti reflection coating. So, in order to understand anti reflection coating, what we need to do is, we need to understand how the optical property's how the material behaves or what are the optical properties of a material when it is in a thin film form?

So, this is what we are going to discuss today. We want to figure out how would be reflectance transmission. These are the two things we will discuss. You can even worry about absorbance in some case will change when your material is in the thin film form. Thin film form means that the materials one dimension is very, very small the other two dimensions might be all right. So, when we have this situation, what how would the material respond to the light. That is what we are going to start looking at today.

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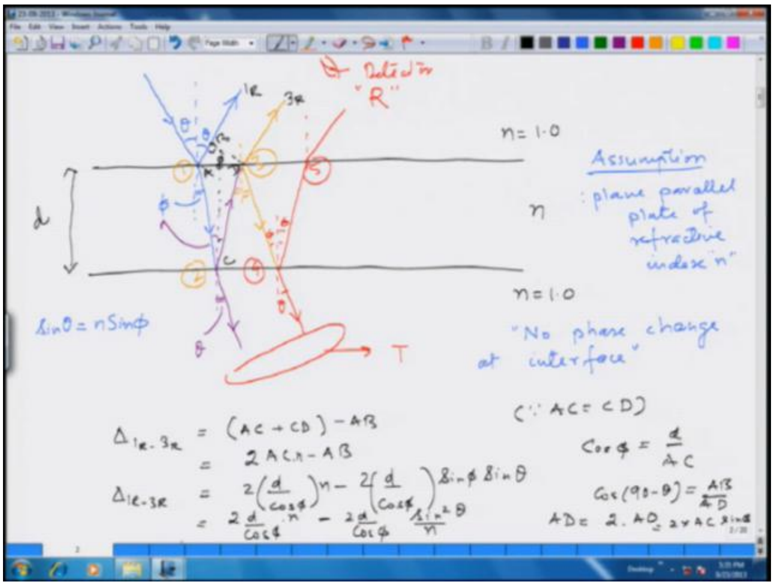

So, we are interested in finding the behaviour of a material, which is in a thin film form. Let me draw first a cartoon on what will happen when the material is in thin film form. So, let me take this material which has the refractive index n and for simplicity, let me say that this is a free standing substrate in a in air. So, the two interfaces are between air and the material and material and and the air itself. This dimension of this the material is thin enough the d is thin enough to the order of the wavelength of light then what what is going to happen.

So, let us first build up what all will take place when this is the case. Then you will discuss the the mathematical formulation for it. So, initially what will happen is, if let me think of a light coming at an angle and as we discussed let us say at this interface at incidence angle theta. Now, because of this n and one, some of it will be reflected back. Then we discussed the case that we know and we have shown that this angle will be the same as the incident angle. The angle at which it is reflected is going to be phi .We know the from a Snell's law that sine theta is equal to n sin phi.

Now, this light is transmitted. In the earlier description we just looked at this problem the reflection or refraction at one single interface, but when the film is thin enough, this light which is transmitted, then comes and sees the second interface. Now, what happens is at this interface also it is going to the experience refraction and reflection which means this angle which is again going to be same as phi at this angle it is sum of the light is going to be reflected back and the other part is going to be transmitted. From the angles you can easily show that this angle at which it comes out is again going to be theta. So, basically as the light passes through this slab, the light comes out at a parallel angle to the incidence light because these both the angles remain as phi.

So, now this basically your light is a still travelling, it is travelling outside. Some part of it is transmitted, but rest is is reflected back. Again it sees this interface and it is going to have another event of refraction and reflection. At this interface the angles being same again here and part of it is going to be reflected back. It can be shown they will all be parallel to each other. So, that is the so the first event, the second interface interaction, the third interface interaction and then it goes for the fourth interface interaction. Again at this interface, it is going to be reflected and reflected back the light going in this direction.

So you have all these angles as being phi and this angle being theta. So, this is what happens when you have a material in thin film form. You are not only limited to the one single event of refraction and reflection. There can be multiple reflection and refraction event because of this process. So, in the end what you observe what you observe far away at this at this point using a detector is superposition of all these events 1, 3, 5 again so all these events is what you would see as total reflectance of this thin film. Once again the superposition of all these transmitted light you are going to see as the total intensity has transmitted. So, this is what we are interested in when the when the material is in thin film form you want to calculate what this reflectance and transmittance is going to be. It is not only because of one reflection event, but number of them coming together.

Now, in in this picture I would like to make sure that we are clear about the assumptions that we have made here. So, the assumptions in drawing this picture are that I am assuming parallel pit geometry. So, the geometry of the thin film is a plane parallel plate of refractive index n. We we are going to take take up this problem we at each interface we are not going to assume any phase change, so no phase change at interface. The reason we are saying that is, because when we develop this formulation we are going to use fractional coefficient that we derived in the last lecture, where the fractional coefficient by themselves will be positive or negative and will give you, if there is a phase change at a particular interface. So, you will assume no phase change while developing this formulation and it will be corrected and be given the equation for the fractional coefficient.

So, let us first before proceeding further, let us look at when I add up when I am looking trying to see the total reflectance because of this film thin film and total transmission. I am going to add up the intensities or the electric field vector of all these reflected rays. I am using a single ray picture you know as a light is going to be in the form of a beam, but for clarity we we follow a single ray here. Then I am going to add up all the electric field vectors for transmitted ones and see what is the total interference I get. That would be my total r. So first thing I want to do right now is to calculate what is the phase difference between for example, let us say the ray 1 and 3.

So, the ray 1 which comes and if it is after the first thing the one amount reflected is 1r and then it follows through as a reflection at interface 2 continues to 3 and gets transmitted at 3. So, this will be 3 r that is what I am I need to add up so when I am trying to add this up, I need to know the phase difference between 1 and 3. So, the phase difference I am going to calculate by looking at the path difference in these two rays. So, this ray has gone has moved from this point a to b, if I am trying to look at it far away at at in some time t in the same time the one which was reflected has gone to 2 and come back to 3 so basically the path difference here is going to be, let me make some marks here this is going to be c and this is going to be d so the path difference between 1r and 3 r.

So, let me give it this name 1 r and 3 r. The path difference is going to be two times. We will make it first it is going to be A C plus c d minus A B. So, this path that it the ray has travelled and this path the that 1 has travelled and this path that 3 has travelled. So, if I am looking at a interference in these two at time t, I am looking at the interference because of this path difference that is written here. Now, we can simply simplify these equations because we know that what is theta and we know the relationship between phi. So, basically A C, A C is what, it is related to d. So, A C plus C D because this is phi and this is also phi are same. So, I can write it as 2 A C minus A B because A C is equal to C D. What is 2 A C? 2 A C is nothing but if I take this angle which is phi, so it will be cos of phi is going to be equal to the d over a c. So, a c is going to be d over cos of phi.

So, that is going to be the A C. What is going to be A B? Now, we can look at A B also A B is because this is perpendicular in order to look at the path difference, this is perpendicular on to the ray 1 from this point. So, a b is going to be basically a d and this angle is going to be 90 minus theta, so it is going to be cos of 90 minus theta. Incidence angle is going to be equal to A B over A D. So, we know that cos of 90 minus theta is sine theta. So, I can write for A B basically A D, A D sine theta. Now, what is A D? A D is double of this this point. It is double of this exactly. So, a d is nothing but d by sine theta, 2 times a d is going to be, let me just first write it in the form A D is going to be 2 times, let me mark this as O at this point. So, A D is going to be two times A O and A O is A O is cos of cos of sine of phi is A O by A C. So, this is going to be a c times sine phi.

So, the way we are proceeding as for A B is going to be A D two times A D two times A D two times A C times sine phi. So, it is going to be two A C, we have already shown is nothing but d by cos of phi, then this multiplied by sin of phi and I am writing about A B. So, this is A D times sine of theta. So, this is what I get as a phase difference phase difference between 1 and the 3 r is going to be 2 d by cos phi minus 2d by cos phi sine phi sin theta. Now, there is something which is very important here. This is actual path difference, but when in last lectures we have shown that the light travels at a lower, smaller, slower velocity inside the material. So, when we compare the path difference inside the medium and outside I have to take the optical path. Optical path is n times the actual path difference. So, for the A C, this is going to be actual path difference time. I should if I am looking at the path difference here, it should be this times n A B is in air so that will remain 1.

So, it will it is going to be n here. So, the path difference between 1 and 3 is going to be 2 times d by cos phi n because we are looking at the travelling of the wave ray inside the medium. For A B it is not inside the medium n is 1. So, this remains the same. So, this would be my difference in the 1 r and 2 r. Now, this can be easily reduced into a more easier solution. If we do that we can write this as we will use the Snell’s law here. Let me take this to the next point, so this will become 2 d over cos phi times n minus cos of phi. Sine of phi can be taken as sine theta over n. So, it will become sine square theta divided by n.

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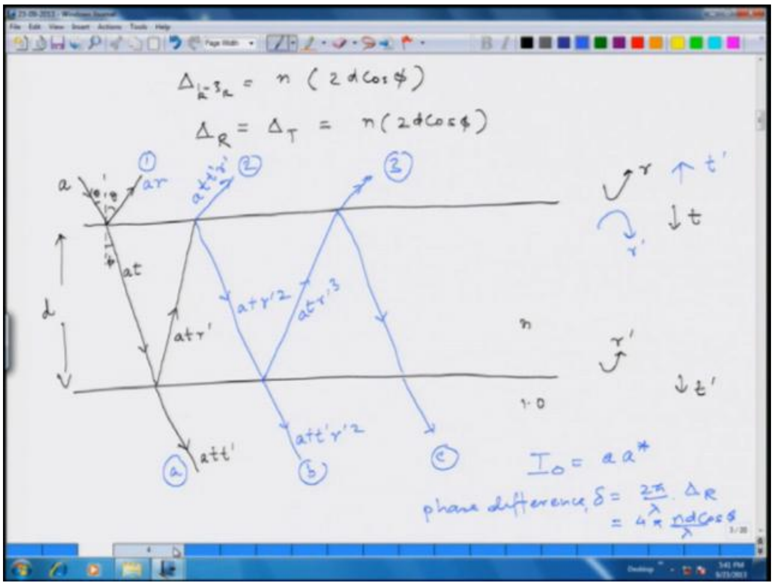

So, the difference the path difference then comes out to be for for any consecutive rays is going to be n times 2 d cos phi. This can be easily seen here that the path difference that you get between 1 and 3 and the path difference that you get between 2 and 4 is same. So, basically all consecutive rays which are either being reflected or transmitted in this case are all going to have the same path difference, which will given by the path difference between two rays that we are adding is going to be either in reflection or in transmission is going to be equal to n times 2 d cos of phi.

So, we know the path difference. Now, we need to add up all the rays in order to calculate the total intensity. So, we need to figure out what would be the intensity of each reflected ray. So, we will be using the fractional coefficient that we have developed in the earlier lecture to figure out what is the total intensity that I am going to use. We will spend some time to to develop this formulation in this on this figure and then it is basically a mathematical addition. So, let me say that let us say the amplitude the incoming amplitude is a.

Then I know from fractional coefficient if this surface if it is getting reflected from this surface, let me call that fraction as small r, if it is getting transmitted in let me call that as t. So, the light which is then reflected back is going to be the its amplitude is going to be a times small r. The amount which is reflected at angle phi is going to be a times t. Then this will be at angle phi. Now, again when it comes to this n it is going to be reflected.

What is reflected back, let me that is going from n to this is a refractive index n to 1 so going from n to 1 let me call this n prime.

This was going from 1 to n so that was r. What will be transmitted, we will call that as t prime from this interface. So, what is going to be coming out here is going to be a times t times t prime. What is going to be reflected back here is going to be a times t times r prime. Now, this gets reflected to this n and again it is a same interface going from n to ambient. So, this is going to be r prime coefficient for whatever is reflected and whatever is transmitted is going to be t prime. So, this then what is going to be collected at reflection is going to be a times t times t prime r prime that is a transmitted one what is reflected here is going to be a times t times r prime square.

So, you can follow this again at this interface, it is going to be reflected and transmitted back again. So, the amount which is transmitted is going to be a t t prime, r prime square and amount which would be reflected is going to be a t r prime q. So, this series will continue in that direction .We have already calculate when I am going to add up the intensity from ray, let me number them now as 1, 2,3 on this side and on the transmission side as a b and c. So, when I am adding intensity, I will have to take the amplitude that is reflected plus the phase difference with the between 1 and 2 and 2 and 3. I know all that phase difference is going to be n times 2 d cos phi.

So, having done this, now we can write the expression for the total intensity. So, in this expression since I have taken the magnitude to be a for the incoming incident wave, the incident intensity is a times the conjugate multiplication a a star and ray 1 is going to be a r, ray 2 is going to be a t, t prime r. The exponential i delta the phase difference and phase difference is phase difference. We have already calculated between consecutive rays is going to be 2 pi by lambda times the delta that we have calculated for r or for t it is going to be the same value which is n 2 d cos phi. So, phase difference is effectively 4 pi n d cos phi over lambda. So, we will keep this picture in mind and develop what would be the total reflected and transmitted ray is going to look like.

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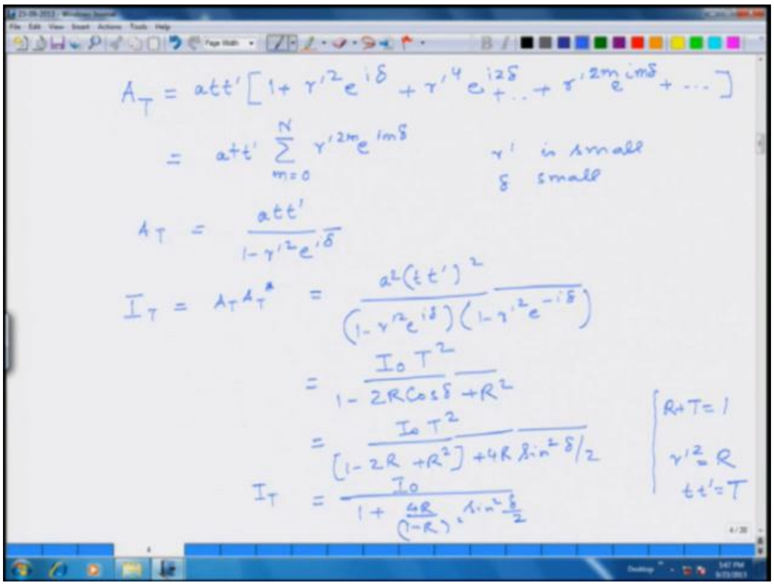

So, we first write the total amount of the amplitude which is transmitted. So, we are looking at total A T on this n by adding up all these here. Then I will also look at what is the total amount reflected from this thin film later. So, a of T and following the same picture is going to be basically a a t t prime plus a t t prime r prime square. So, we are going to add up this times the path difference, then times of path difference with respect to this this will be twice then folds it will be 3 times path difference. So, we will have a t t prime and this is going to be constant throughout, so I can take it out of the bracket and the other terms are going to be 1 plus r prime square exponential i the phase difference.

So, this one has a phase difference of i delta the second is going to be r to the r prime to the power 4 exponential i 2 delta and so on so forth. It is going to go in this in this direction the let me write the n-th component that is going to be 2 times n exponential i n delta we are going to add it up it is going to be a series. So, just writing it in more simplified form, I can write it in a series as a t t prime summation of the series for, we make it m because m is also for refractive index, m both going from 0 to m going to may be large number of rays that we are going to add up. So, this is going to be n is going to be very large 2 n exponential i m delta.

So, for for the problem where r is r prime is small and also that you know that delta is small. Basically the point we are making here is, although I have drawn it so clearly, as if this way so much far of from from the next and so on. In actuality, they are very, very close. So, basically the path difference between 1 and 2 is very, very small. They are very very close to each other that is why they are able to interfere and give you an effective reflectance and transmission value. So, delta is also a small we have shown it enlarged in the in the earlier figure. So, if I do that, this is basically a geometric series, so I can write a a of t as a t t prime divided by 1 minus r prime square exponential i delta.

Then this now, the I know the amplitude what I am interested in is calculating what fraction of the light is getting transmitted through this thin film of thickness t. That is basically nothing but the complex conjugate multiplication of the overall amplitude. We can do this easily and this will become a square t t prime square divided by 1 minus r prime square exponential i delta and the complex conjugate of it exponential i delta. Now, one can simplify this easily. This will become what is a square, that is basically the incident light i naught t t prime. We are going to define this as parameter capital T, which is which is based on the 2 interfaces that we have. So we call it I naught T square and in this symmetric interface, we will see there is a relationship which holds true and r prime square, I am going to call that as capital R. This is n going to be 1 minus 2 R cos delta plus R square.

Because, the way we have taken our symmetric problem as the film is sandwiched between the two similar ambient surfaces, what we can what we know is from fractional coefficient, we can easily show that R plus T that we are showing here is equal to 1 for this particular case. So, R plus T is equal to 1. So, we will we will use that relationship and if we simplify this further, we can write this as i naught over 1 minus 2 R I naught T square is 1 minus 2 r plus r square. I want to use the algebraic expression here to write sin square the phase difference divided by 2. Now, this becomes then 1 minus R square and T square, which basically means I can write this as I naught divided by 1 plus 4 R over 1 minus R square sine square path difference.

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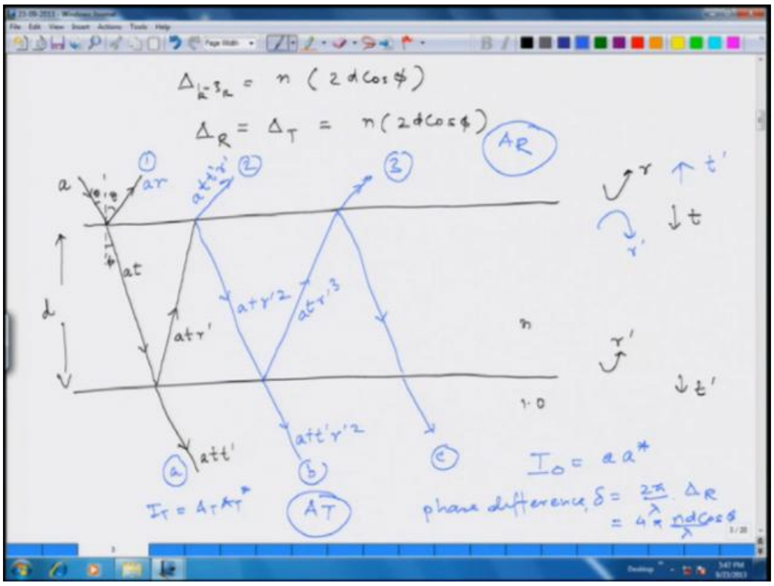

So, this is my total transmitted intensity, which is basically the total intensity by adding all these rays. So, I T is nothing but A T A T star and I have shown this comes out to be this value.

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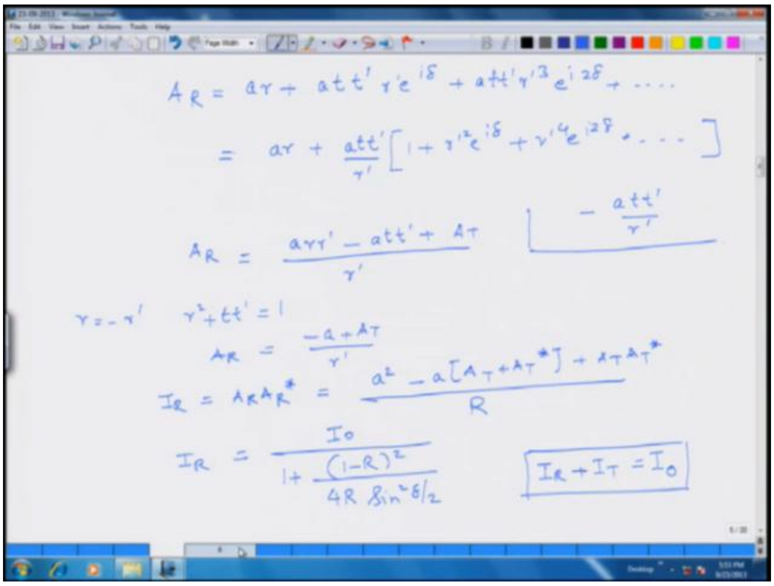

Similarly, I can do this for A R and you can do the whole exercise, let me write just the first term here. So, A R is coming out to be nothing but if I add up all the cases 1, 2, 3 and so on so forth, I will get a r plus a t t prime r prime exponential i delta plus a t t prime r prime cube exponential i 2 delta plus so on so forth. Then I can write this expression as if it is going to be, I can write it as a of r. I am trying to make it simpler to do the final analyses. So, I can take a t prime over r prime out. So, there is no r prime which means, I have to multiply the rest with r prime. So, if I do that, if I take a t t prime and multiply the whole thing with r prime,

Then I am I am going to get here r square r prime square 1 plus r prime square exponential i delta. Say if I have done 1 here, I should overall subtract a term a t t prime over r prime. So, that takes care of the 1. This becomes r prime square e to the power i, delta plus a t t prime will become r prime to the power of 4 2 delta and so on so forth. So, I am doing this some manipulation, so that I can write it in a form which I have already calculated. So, then this becomes a r r prime minus a t t prime. This comes from here, plus this is something which we have already calculated. This is this is what we had a of a t transmitted was this term. So, we can write that as it is and this will be a of t overall r prime.

So, this is the total after interference the amplitude of all the rays which are reflected. Now, as we mentioned earlier, because the way we have taken this problem the r is equal to negative of r prime and r square plus t t prime is equal to 1. So, using these conditions I can easily show that A of R is equal to negative of a plus A T, which has been calculated divided by r prime. Now, if I know this, my interest is actually to calculate what is total reflected which is nothing but A R A R star. If I try to calculate that this will give me a square minus a A T plus A T star the complex conjugate of it plus A T A T star, which we have already evaluated divided by r prime square which is capital R. Now, one can do a number of simplifications here to bring it in the form where which is similar to what is i t is. So, we are going to use the same relationships that we have used and use a relationships for I T that has that have been shown earlier.

Then basically I T is equal to this value. so wherever we need to put in cos delta we can change it in terms of I T in R. Eventually we can show that I R comes out to be nothing but I naught divided by 1 plus 1 minus R square divided by 4 R sine square the phase difference delta by 2. So, we have not done the simplification steps, but you can see that I R will become equal to this. Now, we can notice few things here. If you look at the I T and I R that is the total light, which is transmitted from a film or reflected, you can easily see that I R plus I T is equal to I naught. So, the total there there is there is no amount of light which is lost.

The total which is incident is either reflected and transmitted, which is the case because we have assumed a non absorbing film. The film is not absorbing. Since, there all the rays are getting interfered, so there must be energy conservation. So, the total intensity which is reflected plus total which is transmitted is equal to the total that is incident that is seen in this formulation also. Now, what is the implication of this this effect is the total I R from the from the film. Now, if we if we look at the total effect of reflectance, it is basically showing and I am going to make some general generalization.

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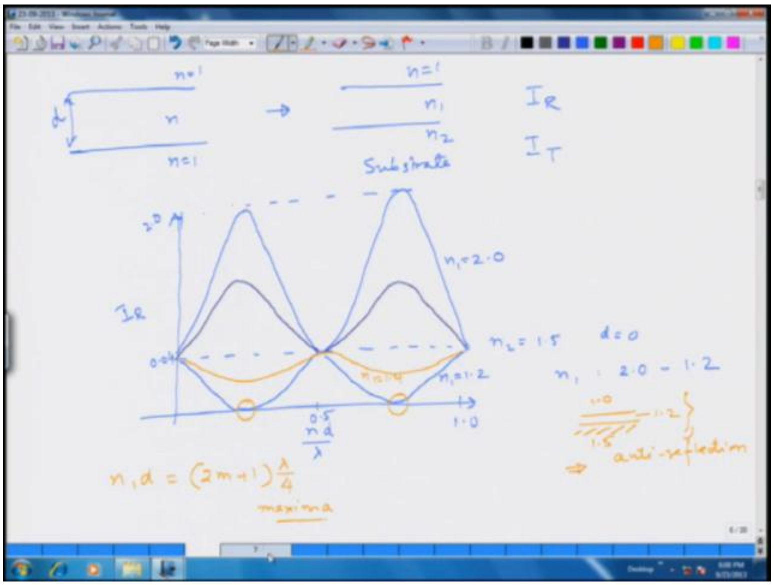

Now, so I started with the film which had n is equal to 1 on both side and the film was inside, but in general when I am dealing with devices probably one side is the ambient, the other side is the film. Here you will have a substrate or some other film which would have n 2. So, generally you you will have this case, I am not going to re derive this case, but if have this case you can see that in instead of writing r prime you would be writing it between n 1 and n 2.

So, formulation will be developed similarly. Only difference is r prime and t primes are going to be different in this particular case. So, if we you take this case and try to plot for this case derive i of R term and i of T terms and plot, what will happen to the reflectance as a function of thickness of the film. So, I I will use the same term. The only thing is r r prime is going to be different. Now, we will be using n 1 and n 2 for a a no symmetric geometry. So, I am going to plot i R and I am going to plot the optical path length n times t the film thickness. The optical path length will be refractive index times d divided by wavelength lambda. So, it is important to realize that all those equations that we have written, the n as it is written for a particular wavelength since refractive index is the dispersive property with wavelength it changes.

So, I am going to plot this in terms of n d over lambda and I am interested in 1. We will plot it up to ratio 1 when optical lens is equal to the, to the wavelength of light, then what will happen? So, if if I had for example, just the glass let us say there was no film and there was only glass, then what would be the reflectance? We calculated in the beginning this reflectance is going to be equal to 0.04. So, this is for n 2 equal to 1.5 and d is equal to 0. So, there is no film. I have 4 percent reflectance. Now, let me put a film on my glass for n 1 and let me change n 1.

Let us say it is changing from 2.0 to about 1.2. What will happen to the reflectance? Using these equations that we have developed and but using the right t and t prime depending on what problem we have developed? If we plot this value then what we get here is, this goes almost all the way up to 2 for this particular case I am going to get for 1.2, this is when n 1 is equal to 1.2 when n 1 is equal to 2. Then I am going get a very high reflectance almost almost going to the fraction twice to up to 2. This is for n 1. If the film is 2.0, if it is somewhere in between, it is going to be maximizing. Then if it is let us say something like 1.4, so n 1 is equal to 1.4.

So, this is not surprising because we know that this interference effect is going to be oscillating with thickness. So, when when you have certain thicknesses where the path difference is interfere constructively, you can see that the intensity of the light increases quite a lot, whenever you have the path differences that I would cancel out with each other if the refractive index is low, you would also get the lowest intensity. So, you can easily see that the maxima and minima in the intensity come at certain values depending on what n 1 is...

Now, it has very interesting implication, which means that if I have n 1 at 1.2 at this value and at this value my reflected light is 0. So, which means that if I take a glass substrate and if I have put a material of wavelength 1.2, then this is basically acting as a antireflection coating. There will no reflection from this surface. So, this is how I create my antireflection coating which is basically in going from 1.0 to 1.5 I put in my intermediate refractive index material in between. If the thicknesses are right, if I have the right thickness then I will have reflection. The total reflection from this surface is going to be 0. This is actually used in solar cells in order to reduce the reflection. You one does use antireflection coating to minimize the reflection from the surfaces.

The other thing which is also important to notice is that when n becomes 2, see we can also increase the total reflectance. So, this can also be used in some cases where you want to increase the total reflectance if the refractive index of this thin layer can be changed it can be increased to more than 1.5. So, this this is the importance of of the optics in thin film form is by playing with the thickness. The optical thickness of the material and the refractive index of different layers you can change the overall optical behaviour of the device structure, whether how much it is reflecting or how much it is transmitting through the layers.

So, this is a way of creating antireflection coating. One can see very easily that for all values of n 1 d you can find a maxima, if you have this at 2 m plus 1 lambda by 4. So, this is like a quarter wave plate that we have studied earlier, where if you have a quarter wave plate, the path difference will add up to lambda by 2 and it will be cancelling completely. So, this would this would be the values for the maxima in in this case. Now, having done this I hope it it tells you that when we are dealing with devices which has large number of layers for example,

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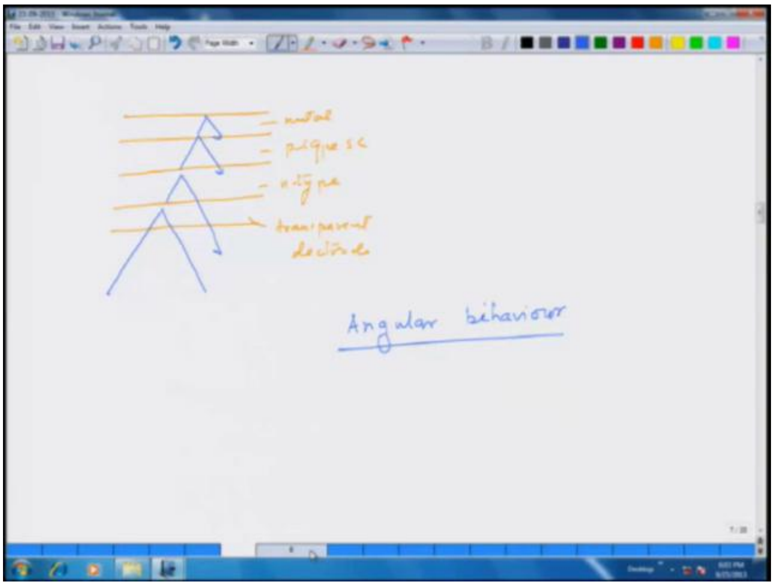

A typical device may have contacts, semiconductor, another semiconductor. So, you could have a metal contact, you could have a p type semiconductor, you could have a n type semiconductor. Then you can have may be a transparent electrode. So, a a particular device may have many different kind of layers. Then this becomes even a much more complicated problem, which we are not going to do in this course. But it is just to appreciate that whatever we have developed with the first single interface. Then we looked at the behaviour, then we had two interfaces.

We saw some drastic changes and how we can play with the thickness of the film and create very interesting optical effects, but when we have multi layers these effects will also have to be added up. So, one has to be one has to know that behaviour of the thin film stack optically, how it is going to be in when we are working with the devices in particular case. A very important part point that I want to mention here is about the angular behaviour. This is just to highlight that even when we have single interface, you saw that that the optical performance of the interface is highly dependent on the angle at which light is coming.

Even from daily experience, you may have noticed that your LCD screens or sometimes some displays you can see it only from a certain angle. That is because when you have all these multi layers thin films, they are highly directional. It is only in particular directions they satisfy the rule for maxima or minima, but in other directions they do not and hence they are their their response is highly direction dependent. This has been a major issue of development in LCD type of displays. So, one needs to be aware of this effects of the thin film on the optical performance of the materials when dealing with the optoelectronic devices.

This course, we do not go beyond this point, but we need to keep this in mind. So, to summarize what what we have done on optical properties is, we have looked at what is the reason for optical properties just like we have done for an electronic structure. We have looked at the how light interacts with the material, the basic parameters that decide the reflectance transmittance, the in homogeneities, the roughness of the surfaces, all those things which add up to scattering. One has to be aware of when in homogeneities, one is talking about optoelectronic devices. In addition to that, one has to be aware of when we have in thin film forms different layers? What would be the overall effect of these layers on the optical response of the device?