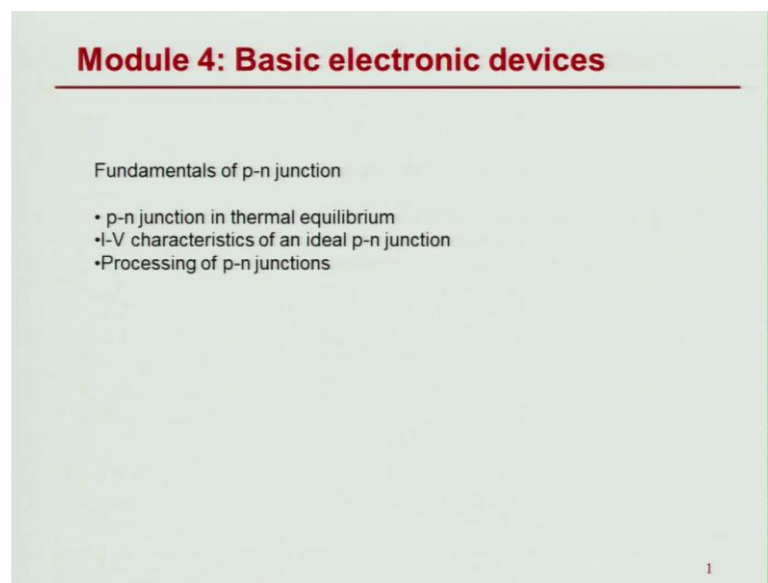


Optoelectronic Materials and Devices
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Module - 4
Optoelectronic Device Physics
Lecture - 32
Fundamentals of p-n junction

In the last lecture, we discussed some basic electronic devices, their characteristics, namely resistor, a diode, a transistor and then how these device characteristics are dependent on some material parameters. Today, we are going to focus on p-n junction. I think it is obvious why we should focus on p-n junction because it is a device, which is used in many electronics circuits as a rectifier, as a switch, as well as it as discrete device. You can use it to make solar cells or light emitting diodes.

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And hence, it is important device to understand more about it. I am going to cover three aspects of p-n junction. In the first aspect I will cover the p-n junction in thermal equilibrium, then the I-V characteristic of an ideal p-n junction. Finally, I will like to show how a p-n junction is processed and that will give some real practical information on what actual processing of the device is.

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p-n junction in thermal equilibrium

Built-in potential

- 1-D problem
- p-n junction at $x=0$
- Step junction
- Perfect ohmic contact
- Complete ionization of the dopants

Complete ionization of the dopants

n-type $n = N_D$; $p = \frac{n_i^2}{N_D}$

p-type $p = N_A$; $n = \frac{n_i^2}{N_A}$

To start with let me define what device structure I am going to study in the, in the first part where I am looking at the electrostatics of a p-n junction in thermal equilibrium. And the first assumption, that I am going to make here is I am going to take my problem, the first assumption that is used is, I am going to use a one-dimensional problem and by this I mean, I, when I say a p-n junction, it may have all the three dimensions where this is the p-type semiconductor and this is the n-type and I am basically going to study in the x dimension where we have the junction again.

So, my problem is then reduced to a one-dimension problem where my p-n junction would look in this manner, where this is the p-type, this is the n-type and I am looking at a junction in the x direction. And to further define things I am going to define, that the p-n junction is at x is equal to 0.

The third thing, that is assumed here is, that my p-n junction is a step junction and this we will see later, is not necessarily always true in a real device. By step junction I mean, if I look at the concentration profile of acceptors or donors as a function of x, the concentration is uniform in the n-type for donors as well as for acceptors. It is a uniform concentration in the p-type and at the junction there is a step junction. So, we are assuming this in our analysis.

The fourth assumption I am going to make is, that in my device I have perfect ohmic contacts and this is important because when I look at the I-V characteristic of the device,

the only junction, which is important to me is the p-n junction because if I have contacts here, I am assuming they are ideal ohmic contacts and they are not changing the device characteristic in a significant manner.

Finally, to make the analysis a little bit simpler for presentation I assumed that in my semiconductor I have complete ionization of the dopants. This basically means, that for a n-type semiconductor, n is equal to the donor concentration, the number the electron concentration in n-type and we know, that if this is so in equilibrium, p is equal to n_i square divided by N_D , the minority carrier concentration. And for the p-type, the whole concentration is equal to the concentration of the acceptors and along with that we know, that the electron concentration then is equal to n_i square over N_A . Now, with these assumptions I am going to start our discussion on how is the electrostatics in a thermal equilibrium of a p-n junction.

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p-n junction in thermal equilibrium

Device structure

$$qV_{bi} = (E_c - E_F^p) - (E_c - E_F^n)$$

$$= E_F^n - E_F^p$$

$$E_F^p = E_i - kT \ln \frac{N_A}{n_i}$$

$$E_F^n = E_i + kT \ln \frac{N_D}{n_i}$$

$$V_{bi} = \frac{kT}{q} \ln \frac{N_A N_D}{n_i^2}$$

So, the next topic I am going to cover is the built-in potential in a p-n junction. How is the built-in potential in a p-n junction developed? So, already described by me, the device in the device structure, my device structure is defined in this manner. In one-dimension, at x is equal to 0 I have the junction and if I want to see, bring these two together, what would be the built-in potential?

So, I know, in the p-type if I draw the energy band diagram, the fermi level is close to the valence band. This is the fermi level of the p-type and I know, in the n-type, the fermi

level is close to the conduction band. And if I need to know what is a built-in potential if I bring these two together? The requirement is, that the fermi level should become constant and if I do that part, this means, that I equate the fermi level on both sides, which means, on the n side I am looking at band diagram of this kind and on the p side I am looking at the band diagram of this type.

I have intentionally not joined is I am just joining them as a dotted line because we really do not know what is happening in that area and that is the, that is what I am going to show in this lecture, how is the field or the potential changing in the junction area. But from this equation where we are equilibrating the fermi level on the p and n side, I can calculate my built-in potential.

What is my built-in potential? My built-in potential is given by this energy difference. So, if the built-in potential is V_{bi} , this energy difference is my built-in potential. Now, how do we, I get this energy difference? It come from the fact, that this is, I know this for my p-type semiconductor and I know this information for my n-type semiconductor and $q V_{bi}$ is nothing but this minus this part. So, even if I just look at pictorially I can say, that my built-in potential is equal to E_C minus E_F on the p side minus, the E_C minus E_F on the n side or this will equate to E_F on the n side minus E_F on the p side.

So, this is my built-in potential, which comes only from the requirement that I need to equate the two side fermi level. I can calculate my built-in potential, I can further put it in terms of material parameters because we have earlier learnt, that the fermi level on the p-type is nothing but the intrinsic fermi level of the semiconductor we are considering. And from, from this point it is reduced towards the valence band and is given by N_A divided by n_i . N_A comes here because we have already assumed, we have completely ionized p-type semiconductor. Same thing I can write for the fermi level of the n-type and in this case, the fermi level moves up to the conduction band by the amount of dopant I have put in the n-type.

So, now I know, if I know this, I can easily calculate what is my built-in potential is in terms of the dopant concentration of the two types and this by subtracting the two I will get is... So, here we see, that if I, depending on how I dope my material and what is my initial intrinsic carrier concentration, I can control, have a control on the built-in potential of my p-n junction.

Just two points at this junction what is the maximum built-in potential that I can get? The maximum built-in potential, that one can get theoretically, can be only the band gap of this p-n junction. But even that is not true because we know, that all the expressions we are using here are for a non-degenerate semiconductor, which means, there is a maximum limit on how much dopant I can put in the semiconductor. If I cross that limit the semiconductor becomes degenerate and then these expressions will not be valid. Hence, this is a way of calculating built-in potential in thermal equilibrium of a p-n junction.

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p-n junction in thermal equilibrium

Space Charge and depletion width in the junction

Depletion approximation
- no free carriers

$$q N_D x_{n0} A = q N_A x_{p0} A$$

$$\frac{x_{n0}}{x_{p0}} = \frac{N_A}{N_D}$$

$$W = x_{n0} + x_{p0}$$

Next we would like to look at what is the space charge in this p-n junction area and what is a depletion width? So, we have already said, that when I make up junction, I have uniform doping of n-type on the n side, hence electrons are going to move, having higher concentration on n side to the p side and the holes are going to move from the p side to the n side. If the electrons and holes are moving, which means, a material, which was neutral all along has left some positive dopants behind near the junction and on this side, the acceptors with the negative charge. This region, which is affected by this motion, the junction region is what we are interested in finding out.

And here comes an approximation because when I diffuse electrons from n to p-type, effectively I, because of this charges I am generating an electric field from positive to negative, which will oppose the motion of the electron going from n to p and that is what

is the stabilizing equilibrium factor. The, the drift current due to the created field balances the diffusion current and in this situation, my purpose here is to figure out what is this width and in order to calculate this width, I do of, make a further approximation and that is known as the depletion approximation. And in this depletion approximation I assume, that there are no carriers in this depletion region.

Let us say, the depletion region extends up to x_{n0} in the n-type and x_{p0} in the p-type. In that case, the depletion approximation basically says, that in the depletion region you have no free carriers. Free carriers mean electron and holes and if that is the case, then if I want to look at the space charge in this region, I can simply plot charge on this axis. Here, I have, since it was uniformly doped, I have all dopants, the charge on the dopants, the donor concentration multiplied by the x_{n0} , that is, the total charge multiplied by the area, assuming the area for this p-n junction is A. So, this would be the positive charge and similarly, I will have the negative charge on the p side, which will be given by q, the acceptor concentration x_{p0} , the extent to which the depletion region is in the p-type and the area.

So, this would be the situation for the space charge inside the junction and this comes with the assumption, that there are no free carriers. Now, this assumption is, is justified. It is justified mainly because as we said, there is an electric field in this junction area, which means, if there are free carriers, they would eventually move the electrons, are going to move towards the n-type and holes are going to move towards the p-type. So, the assumption is not bad as such because because of the existing field any free carrier is going to be collected by the two n and p side and hence, the space charge situation inside the material is like this, which is basically consisting of the positive dopants on the n side and the negative ion cores in the p-type.

Now, if this is the situation we know, that overall we are looking at thermal equilibrium charge plus charge has to be equal to the negative charge, which means, that I must have the total charge on both side has to be equal. So, this must be equal to or I can say to the extent the depletion region is in the n-type is decided by the doping level on the n and the p side.

Now, this is an interesting equation because it says that if I have high doping on one side, the depletion region in that side will be smaller. If I have lower doping, the depletion

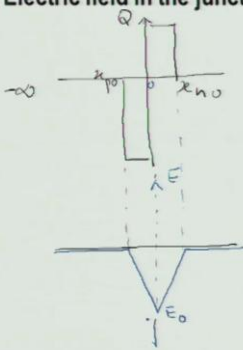
region would be larger. Now, interesting concept also for further application of device is, if I want large depletion width, I should have lower level of dopings. If I want a smaller depletion width, I should have high level of doping.

So, just from the considerations of space charge and depletion approximation we can estimate the relative depletion width. And here I would also like to define, then the total depletion width of a p-n junction is nothing but x_{n0} plus x_{p0} . So, now, we know how the space charge in the p-n junction looks like.

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p-n junction in thermal equilibrium

Electric field in the junction



Point form of Gauss's law

$$E(x) = \frac{1}{\epsilon_s} \int_{-\infty}^x \rho(x) dx$$

ρ : charge density

$$E_0 = -\frac{q}{\epsilon_s} N_A x_{p0}$$

$$= -\frac{q}{\epsilon_s} N_D x_{n0}$$

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We can take it to the next level, where, to see in this junction area what is a situation with the field. So, if I take the earlier picture where I have charge because if you have charge distribution, which means, there must be a field. What would be the sense for this field? For this we will use the point form of the Gaussian law, point form of the Gauss's law, which basically says, the field at any point x can be represented by 1 by epsilon S , which is the dielectric constant of the semiconductor coming from negative infinity to that point, the charge density times dx , the integration of this will give me the field at any point and ρ here is the charge density given in the units of charge per unit volume.

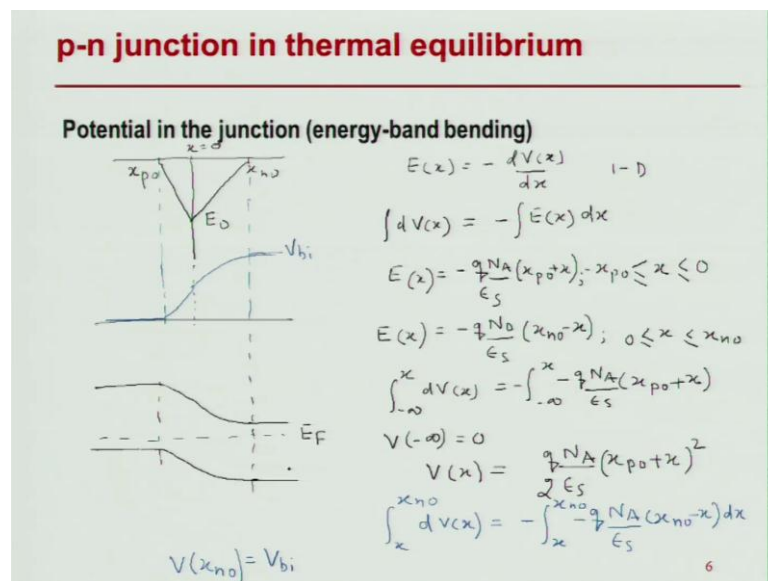
Now, if I apply that equation and try to follow what would be the field in this junction area as I am coming from minus infinity to x , I need to have charge density multiplied by dx , up to point x_{p0} I have absolutely no charge. So, if I were to plot this, the field is 0 up to x_{p0} , now what is happening is, as I go further, integrated for values at less than x_{p0}

up to 0, I am going to have a negative charge multiplied by, because this is the uniform doping as we have assumed, multiplied by the distance, I am going to move and since that is a linear equation, it is going the, the field is going to increase in the negative direction up to some maximum value because as I continue further in the positive direction of x , then the negative charge is going to be compensated by the positive charge and hence, the field is going to further come down and again beyond this point field is going to be 0.

So, what we have done here is, knowing our space charge situation inside the junction and depletion approximation, we have calculated the field in the junction and it goes, it becomes maximum at the junction, at the step junction itself and decreases in both directions. What is the value, the maximum value of this field? The maximum value of this field is nothing but the total charge, that you have, which can be obtained from this equation is going to be x_{p0} or since all the charges are equated, it can also be written as $N_D x_{n0}$. So, this tells me how the field is changing inside the junction.

Now, that I know the field inside the junction, I need I can take it to the next step. Now, having understood the space charge and the field inside the junction, we move next to understanding how the potential changing inside the junction.

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So, what we have seen so far is, that we have a depletion region and field in that depletion region and we want to, to find out how is the potential changing in the junction

area. And we use the expression then for that because field is nothing but the negative gradient for the potential and hence, we can use this. Then this is written for a one-dimensional situation, which we have here. Hence, we can use this to find the potential at various points by integrating it and using the appropriate limits. And the reason I say appropriate limits because E_x is not a continuous function in the whole, entire x range. So, we need to define E_x from $x = x_p$ to $x = x_n$ and then from there to $x = x_0$.

So, let us first define that function. And so this function is basically defined in two regions, the first region is going to be from $x = x_p$ to the value of x , which is greater than equal to x_p or less than equal to 0. In this region the value of E is going to be defined by the equation for this line, which is nothing but the maximum value plus x and similarly, we can define the field in the n-type region, which is defined for the, the line that gives the electric field in the n-type region, which is defined for values of x greater than equal to 0 and less than equal to x_n .

Now, having done this I can first calculate what is the potential in the p-type region. For that I will take integration from minus infinity to values of x and the expression here is... Now, if I am taking the potential from minus infinity to x , it is convenient to assume, that the potential at minus infinity is 0. So, what I am saying here is that the potential up to here is 0 because there is no field up to the point $x = x_p$. The potential remains 0 till this point and beyond this point I will find, that the expression becomes V of x is equal to, which is a parabolic equation and says, that I have potential changing in this manner from $x = x_p$ to $x = x_n$.

Now, I need to do the same integration for the n side and the integration now will go from value of, from value of x to plus infinity of $V_x = x_n - x$ dx, Now, we know, that there is no field beyond $x = x_n$, hence there is no need to take this integration all the way up to infinity. It might work out better to do this integration only up to $x = x_n$ in both cases and I have already shown in the beginning, that what is the potential at $x = x_n$, the difference in the potential at $x = x_n$ and $x = x_p$ that is the built-in potential.

So, I know the final point. I know that if I assume this to be the reference point, I know at this point my potential should be V_{bi} and all I need to know is how it is changing within the junction. So, I know that V of V at $x = x_n$, assuming the p side to be the reference potential, 0 is going to be V_{bi} . Now, I can solve for this and it also comes out

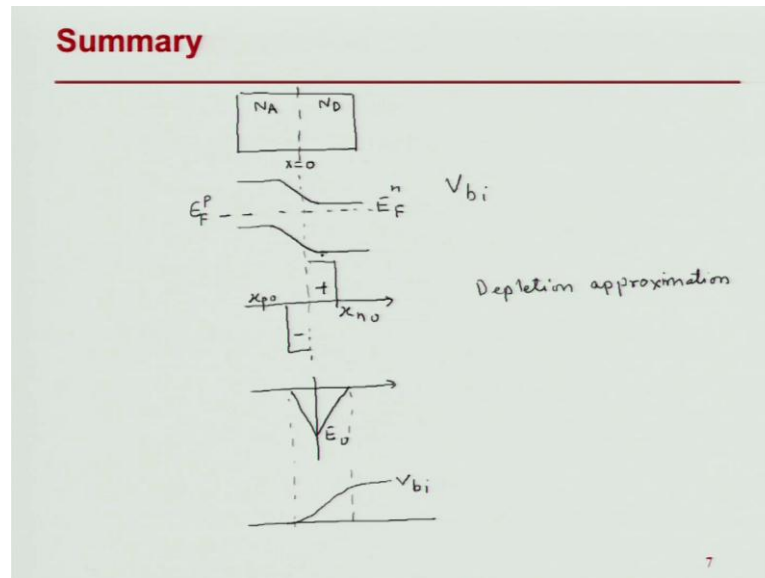
to be a parabolic of an inverted nature and hence, this will become like that. So, this is the potential inside the junction and it makes sense the, the slope is always positive, hence the field is always negative. The field will be maximum somewhere at this point and it reduces as it is shown in the earlier calculation from the space charge. So, this is the potential inside a p-n junction.

Now, using this potential we will generate the energy band diagram. I told you earlier, that I am not plotting how the bands will be inside the junction because I do not, do not know what the potential change is. Now, I have calculated how the potential is changing inside the junction, so I can complete that band picture. So, what was my band picture in equilibrium I had? What I knew was that I have fermi level, which is same in both the direction. Here, away from the junction I have what is there in the material. In the junction now what will happen?

The built-in potential is increasing, right. Now, this is the potential is plotted with respect to all with the positive charge and I am plotting energy band diagram always with respect to the electron energy. Hence, the energy band diagram bending is always going to be, one will have to take, subtract that from here and it is going to be absolutely mirror image of the potential change. So, now I am able to draw in this depletion approximation of the p-n junction for a step junction what is the energy band bending inside the junction.

So, that completes my electrostatic picture of a p-n junction in thermal equilibrium. I know what is the built-in potential of my p-n junction; I know what is a space charge, how is the field and now, I can plot my energy band diagram, and how is the band bending of the energy band diagram inside the material.

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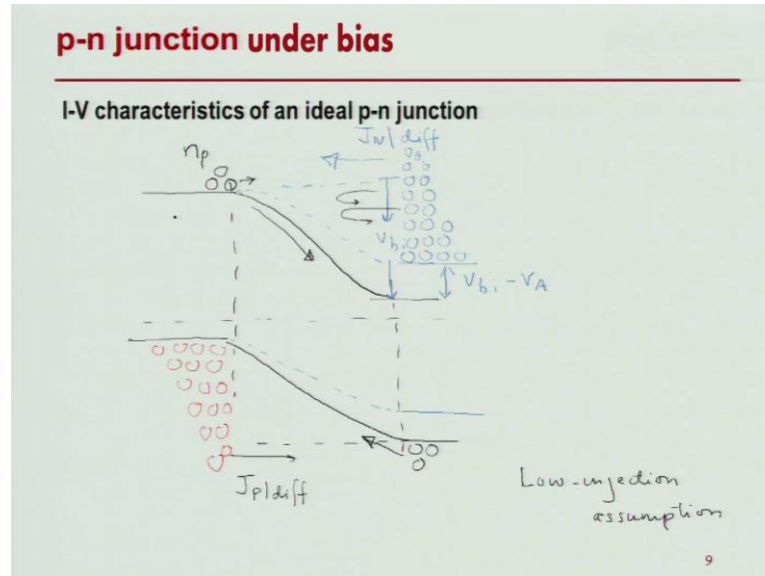
So, at this point I would like to summarize what we have learnt about the thermal equilibrium of the p-n junction. And so what we started with? We defined a p-n structure, which had uniform doping on both sides. Without getting into too much detail, we could calculate the built-in potential just from the requirement, that the, when I bring these two together, the fermi levels have to align on the n and the p side. From that we could calculate the built-in potential.

Then, we use the depletion approximation to show how is the charge distribution inside the junction because we know the electrons and holes have diffused from high concentration to low concentration, thereby creating a field and in that approximation I can plot my charge and define a depletion width for the junction, which depends on how I have done the doping. And based on this, using Gauss's law we found the field in the p-n junction and based on that I could plot the potential change in the p-n junction going to V_{bi} . And once I had that, I could complete my energy band diagram. What is happening in the junction is a mirror image of what is happening to the potential and this would be my p-n junction in thermal equilibrium.

Now, the importance of p-n junction in thermal equilibrium is, that all the parameters, that we have defined will be useful when we apply a bias, when we start studying the behavior of the diode and its I-V characteristics and you want to apply a bias, we will see how the depletion width changes, how the fields will change and how will that affect the

current passing through the diode. So, next we will look at the p-n junction in forward and reverse bias and first let us look at the situation what is happening qualitatively.

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So, this is the bulk p-type region. In thermal equilibrium we had diffusion and drift current balancing each other in the depletion region. Now, if we forward bias it, effectively we are saying we have changed the built-in potential; we have reduced it, which was this earlier, by the applied bias. As a result, we can now see band bending is going to be different inside the depletion region due to this forward bias.

What is happening to the various currents at this time? At this time if you look at the electrons, now there will be lot many electrons, which will have enough energy in the conduction band of n-type, which get cross the reduced barrier. So, there will be an increased diffusion current for electrons in the forward bias from the n to the p-type because earlier there was, they were getting reflected back from the built-in potential. But now, that the built-in potential is reduced by the applied forward bias, there will be an effective increased current. Similarly, there will be a change in the current for the holes because now there will be more number of holes, which can go from p to the n-type or diffuse from p to the n-type.

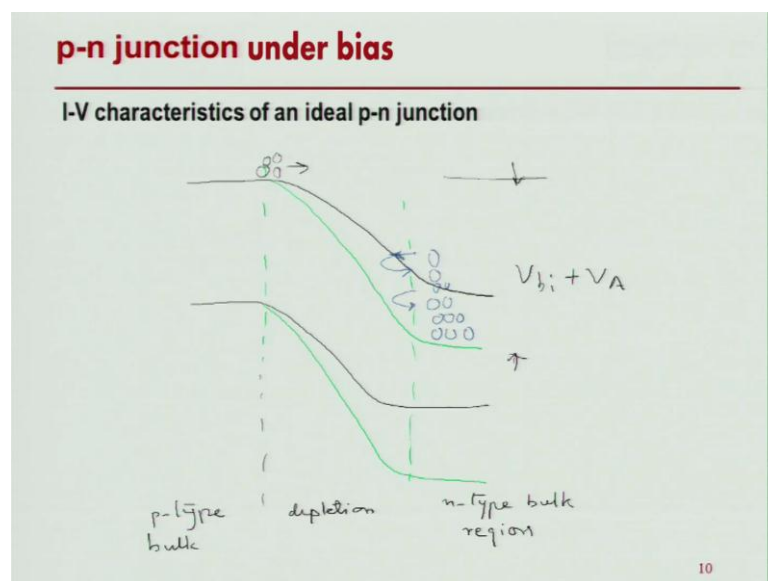
What will be happening in this time to the drift current? The drift current is mainly coming into the picture because we have a built-in potential, which we have already taken the assumptions for during thermal equilibrium, that it is a step junction and in the

depletion region we do not have any free carriers. So, it basically means, since there is a field, if there is generation of any electrons in the p-type at the edge of the depletion region, if it comes closer to the, within the diffusion length of the depletion width of minority carrier, it is going to be swept out due to the drift field. Now, by changing the, putting the forward bias, there is no change in this drift current.

Similarly, on this side, the additional holes, which are getting drifted due to thermal generation and creating a drift current by applying forward bias, that is effectively no change in that current. So, bias in terms of the current picture, what is changing is, that we, we, I have an additional diffusion current, which is giving rise to, to whatever the current I am going to calculate.

Now, in this calculation we make certain assumptions to keep solution simple and that is a low injection region. This is to say, that there is injection of the electrons from n to p side, but the amount is so small, that it is not changing, it is only changing the minority carrier concentration and it is not changing the thermal equilibrium concentration into any significant amount of the majority carrier. So, if I have a forward bias, I would have diffusion current because of this extra injection into the bulk region of the p-n diode and the reverse, the drift current does not change too much.

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What happens qualitatively if I put a reverse bias in that situation if this is a thermal equilibrium? Then in the reverse bias, this gets further increased. Now, what is that

doing? It is effectively in again in these three regions that I can plot. Whatever little bit of diffusion current I had in the beginning due to the, due to the holes here, now even those will not be able to find enough energy to surmount and go to the p side. They will all be reflected back because of this higher potential. The potential now has changed from V_{bi} to $V_{bi} + V_A$. So, the diffusion current has, has considerably reduced in this case.

What is happening to the drift current? Drift current would remain similar because the field does not make too much different into this current. It is basically decided by the thermal behavior at the edge of the depletion width is how much drift current I am going to get. Same thing is going to be true for the holes in the reverse bias.

So, now our strategy is going to be to find a quantitative I-V characteristic. You will try to solve for the, the current equation in three regions. One is this p-type bulk region where there is very little field, this is a depletion region, and this could be the n-type bulk region, again with negligible field here. So, I will be looking at continuity equation in these three region, solving it and trying to figure out what would be the current flowing when I put a device in a forward or a reverse bias, that would be the strategy. So, let us start looking at how we look at the current here under different assumptions.

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p-n junction under bias

I-V characteristics of an ideal p-n junction

1-D system
p-type region

$$J_n = qn\mu_n E + qD_n \frac{dn}{dx}$$

for minority
this is small

$$J_n = qD_n \frac{dn(x)}{dx} \quad (1)$$

Low injection level
 $n = n_{p0} + \Delta n_p(x)$
Small compared to p_p

$$\frac{dn}{dt} = \frac{1}{q} \nabla \cdot J_n + \frac{\partial n}{\partial t} \Big|_{\text{thermal R-G}}$$

$$\frac{d\Delta n_p}{dt} + \frac{d\Delta n_p}{dt} = \frac{q}{q} D_n \frac{d^2 \Delta n_p}{dx^2} - \frac{\Delta n_p}{\tau_n}$$

$$0 = D_n \frac{d^2 \Delta n_p}{dx^2} - \frac{\Delta n_p}{\tau_n} \quad (2)$$

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So, in the forward bias, where my built-in potential is reduced by the applied potential, I want to figure out what would be the current due to the forward bias. And as earlier, I am

going to assume a one dimensional system and in this one-dimension system, let us first look at the p-type region, what would be a current equation in the p-type region and let us look at the equation for the electron. Why electron, because we just showed you qualitatively, that it is the, the injection of the electrons from the n-type to p-type, which would be giving rise to the diffusion current. So, it is the electron diffusion current in the p-type which is what we are interested in.

So, if I am looking at the electron current here, it is going to be given by what the drift component, which is given by q times the concentration times the mobility of electrons times the field plus the diffusion constants for the electron and its concentration gradient. This is the one d system, the equation for the current in the p-type.

Now, immediately we can see, that some of these terms are going to be not significant and first of that is going to be this one because we have already shown, that in the bulk region there is hardly any field. Moreover, in this bulk region in the p-type, the minority carrier concentration is also going to be small. So, effectively, for the minority carrier this term is going to be very, very small. So, we are left within the p-type for electron current, only the diffusion component of the current.

Now, in this diffusion component of the current we need to figure out what is the electron concentration, which itself is a function of x . Remember, x was defined earlier where x is equal to 0, at the junction at the depletion width it is x_n and it x minus x_p .

Now, we will use the continuity equation. Continuity equation basically tells us the balance in number of electrons overall is going to be given by the gradient of the current and since this is a simple p-n diode with no additional generation terms, we need to only know the recombination generation term due to thermal sources. So, this would be the continuity equation for the p-type region.

Now, immediately we can use the assumption of low injection level, which basically means, that n is given by, n in the p-type equilibrium concentration plus the excess n concentration in the p-type, which is a function of x and this is small compared to the majority carrier concentration. So, there is a very low amount of injection in the p-side. In that assumption I can always change my electron concentration in terms of equilibrium minority carrier concentration and the amount, extra amount, that have been injected, now I know this on the p side, that there is no change in the concentration of the

p side, it is uniformly doped and in the, in a, in trying to find a solution, steady state solution, there will be no change in the time also. So, I can replace n by this expression, which will basically give me... And further I can use the expression for diffusion current and this gives me...

Now, here since I am using low injection level I can assume the mechanism for thermal recombination generation, which can be represented by the lifetime of the minority carrier in the p side. Now, once again, since the overall situation is the steady state solution, that I am looking for this basically says, this anyway is not changing with time. This is equilibrium concentration of the electrons and we are looking at a steady state solution. So, my continuity equation will become simply a second order differential equation in the excess electron concentration.

So, I have one equation for describing the electron current in the p-type and which is the, which is, as we discussed, coming because of injection of electrons. And then I have the continuity equation, which I can solve to get an expression for electron concentration in the p-type and all this is because of the low injection current approximation where I can put the electron current as the equilibrium electron current plus the excess electron, equilibrium electron concentration and the excess electron concentration. In a similar manner I can write the equation for the p-type, which I will just write on the next page.

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p-n junction under bias

I-V characteristics of an ideal p-n junction

In the n-type region:

$$J_p = D_p \frac{\partial \Delta p_n}{\partial x}$$

Low injection $p = p_{no} + \Delta p_n(x)$

$$0 = D_p \frac{\partial^2 \Delta p_n}{\partial x^2} + \frac{\Delta p_n}{\tau_p}$$

$J = J_N + J_P$

$J_p = J - J_N$ $J_N = J - J_p$

No generation of carriers

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So, same continuity and the current equation for the n-type region of the forward bias p-n

junction is going to be given by, now it will be the diffusion current due to holes and that is going to be given by J_p , the excess holes in the n side. And in the low injection approximation, the hole concentration in the n side will be given by, the equilibrium hole concentration plus the excess hole concentration in the n side. And the continuity equation will be similar to the continuity equation earlier, which is a second derivative of the excess hole concentration in the n side plus the thermal recombination generation. So, now we have these equations and what we need to do is solve the second order differential equation for the boundary conditions in the p region and the n region and then once we have solved it we can calculate the current.

Now, you, you may ask what happens to the other current? We have calculated so far the electron current in the p region because of diffusion and the hole current in the n region because of diffusion. So, that is quite simple because in the device if I look at the current, the current has to be constant, the total current has to be constant and this is equal to the total electron and total hole current at any point. So, what we have just shown you is this expression for the, for the hole current is obtained in the p side and similarly, for the threshold for an electron current is obtained on the p side and hole for the n side.

And so what, what would be the hole current in the n side? It will basically be the total current minus J_n . So, I need not calculate the hole current here. If I know the total current, it would just be negative of that and what would be the electron current here? It is simply going to be J minus J_p . So, what we have done is, we have written an expression for calculating this and we have written an expression for calculating this and we do not know exactly how it will go on. In, in this we can try to figure that out.

Now, this it might look weird because we have kind of ignored the depletion region and which is not surprising because what we have assumed is low injection and throughout our qualitative description we said, well there is nothing much changing in the depletion region in terms of the valence. All I am doing is I am injecting some little amount of carrier from p to n or n to p.

And so in the low injection region you are saying, that whatever was a status, the depletion region is maintained and whatever current I will have here, since there is no generation and recombination in the depletion zone, no generation of carriers basically means, whatever current I am getting here for the electrons or whatever current I am

getting here for the holes is going to be same in the depletion region and so I need not calculate what is happening in the depletion region. So, basically in the depletion region J is equal to J_N plus J_P . So, so now, let us find a solution for J_N and J_P from which we can calculate the total current in the device.

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p-n junction under bias

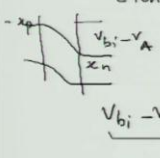
I-V characteristics of an ideal p-n junction

Boundary condition (low-level-injection)

$$J = q\mu_n E + q D_n \frac{dn}{dx} = 0$$

$$E = - \frac{D_n}{\mu_n} \frac{dn}{dx}$$

Einstein relationship $\frac{D}{\mu} = \frac{kT}{q}$



$$E = - \frac{kT}{q} \frac{dn}{dx}$$

$$V_{bi} - V_A = - \int_{-x_p}^{x_n} E dx = - \int_{-x_p}^{x_n} - \frac{kT}{q} \frac{dn}{dx} dx$$

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So, in order to do that, we first need to find the boundary condition to solve the second order differential equation. What will be the boundary condition? And here again we will use that low injection, low level injection approximation to say, that depletion region has not really changed too much and I can use the same expression here. So, in the, if I write an equation in the depletion region for current, it is basically the drift current and in the low injection regime I said, this is still 0. If I say this is still 0, I can, I have an expression for E , which is basically...

Now, we have earlier looked at E from electrostatic point of view and calculated the field in the junction. Now, here we are doing the same thing, like to do that, but now we have apply, we are looking at that situation where there will be a bias or there could be a bias also. Now, in this situation I will now use the Einstein relationship, which relates the mobility of the carriers to its diffusion constant. The diffusion constant and mobility is related by some constant $K T$ by q , the charge. And if I use that, all I am saying is that the field is given by $k T$ by $q d n_x$ over n .

Now, we already know, that field is nothing but negative differential of the potential.

Hence, I can integrate this equation to relate it to the potential. So, if I do that my potential, built-in potential minus v_A , what I am saying is, in the forward bias I have written the equation for the depletion zone. This is V_b minus V_A , so if I integrate the field from x_p , negative x_p to x_n , I will get the potential, which could be, the difference would be V_b minus v_A . So, what I am doing is, I am integrating field starting from minus x_p to x_n and that is the V_{bi} minus V_A . So, if I integrate that, this specifically means, this is negative, second negative coming from the expression for E going from minus x_p to x_n .

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p-n junction under bias

I-V characteristics of an ideal p-n junction

$$V_{bi} - V_A = \frac{kT}{q} \ln \frac{n_n(x_n)}{n_p(-x_p)}$$

$$\frac{n_n(x_n)}{n_p(-x_p)} = e^{\frac{q(V_{bi} - V_A)}{kT}}$$

$$n_p(-x_p) = n_n(x_n) e^{-\frac{qV_{bi}}{kT}} \exp\left(\frac{qV_A}{kT}\right)$$

$$= n_n(x_n) \frac{n_i^2}{n_{no} p_{po}} \exp\left(\frac{qV_A}{kT}\right)$$

$$= n_{no} \frac{n_i^2}{n_{no} p_{po}} \exp\left(\frac{qV_A}{kT}\right)$$

$$n_p(-x_p) = n_{po} \exp\left(\frac{qV_A}{kT}\right)$$

$$n_{po} + \Delta n_p(-x_p) = n_{po} \exp\left(\frac{qV_A}{kT}\right)$$

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So, let us solve this and the n we will get an expression, which would relate V_{bi} minus V_A with the concentration of carriers at the edge of the depletion zone. So, this is the electron concentration at the n -type region and this is in the p -type region. What I am interested in, I am trying to figure out the boundary condition when I try to solve my continuity equation in the p or the n region. So, I would like to reverse the order and look at the electron concentration in terms of, we can write that exponential qV_{bi} .

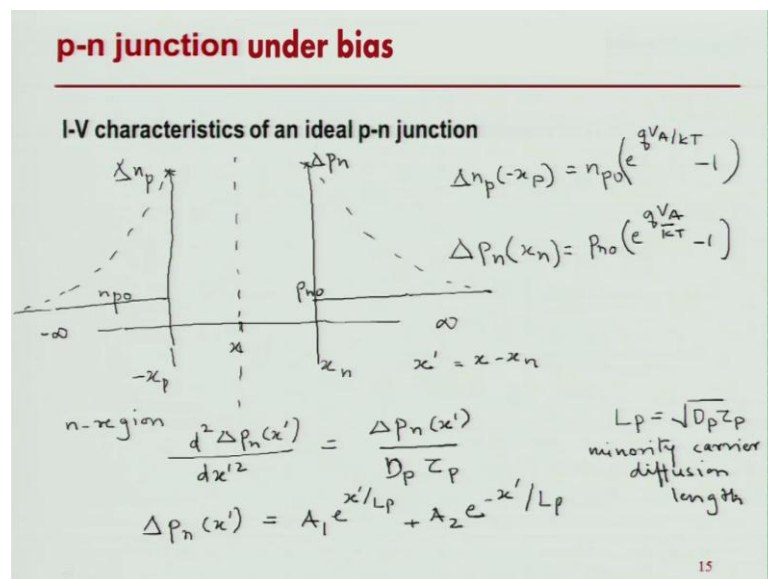
Now, let us first look at what is, when I am trying to solve for the p -type continuity equation. I need this as the boundary condition, so further writing it in this form I can write it as n of n and this is going to be negative, now q of V_{bi} . And I can separate the V_{bi} and V_A terms now. And the electron concentration in n -type at x_n is nothing but the majority concentration in n -type. And earlier, from electrostatics we had a relationship

for V_{bi} , we can write the built-in potential in terms of the carrier concentration on the two sides.

So, we will take help of that and that expression was basically, V_{bi} was nothing but $kT/q \ln(N_A n_D / n_i^2)$, but $N_A n_D$ in the approximation, that we assumed all the donors are ionized, is nothing but n and p , n in n and p in p . So, I can write it in this form, then that basically gives me n concentration, majority concentration in n -type and majority concentration in p -type. So, we already know, that n of x_n is nothing but n_0 and this will then become n_i^2 divided by p_0 , which is nothing but concentration of electron minority carrier concentration, equilibrium concentration in the p -type along with this exponential factor is my minority concentration at the edge of the depletion region.

In a, in a very similar manner I can write now this is a minority concentration, but when I wrote my equations for solving I wrote it in terms of excess carriers. So, this is nothing but $n_{p0} + \Delta n_p$ at $x = -x_p$, the equilibrium concentration plus excess carrier concentration. And what it is at the point $x = x_n$? And this will be, so I can take n_{p0} on this side and I will get an expression for the excess concentration at the edge of the p -type depletion region.

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So, using this low, low level injection we have found the boundary conditions for my solution to the differential equations. This is the depletion region beginning and I have

calculated that x 's hole concentration at this point Δn in p at minus x_p is going to be given by the equilibrium concentration of n in the p region given by $K T$ minus 1. This would be the x 's concentration here. Similarly, I can do the same derivation for the hole concentration on the n side at point x_n and that would be given by the equilibrium concentration was there in n side. So, I know what is the x 's concentration in the forward bias at depletion edge.

And what is happening at infinity? Infinity x 's concentration is going 0, basically whatever was p and o, and then there was a minority carrier in the p side. Eventually, this is has to go at infinity, has to become same an equilibrium. So, I have two boundary conditions, one at the edge of the depletion zone and one at infinity; same thing, one on the p side at the edge and at the infinity.

Now, that I have the boundary condition, all I need to do is the differential equation that we have set up for p and n's type. We will try to solve that. So, let us solve it for the n region. For the n region we are going to make slight change in the differential equation because the way we have written earlier is written with respect to x . It becomes easier if we change the coordinate to something x prime, which is x minus x_n because then solution is obtained from the edge.

So, if we do that we can write my modified differential equation in terms of the excess hole concentration in x . Whatever its distribution is in x prime $d x$ prime square, that is modified differential equation, the hole concentration in the n region divided by the diffusion constant for the hole in the n-type and its life time in the n-type.

Now, this is a very standard differential equation by using Laplace transform. You will find solution for this, which can be given in terms of, even is a constant, which need to, need to find out by applying the boundary condition. A^2 is the other constant. So, two constant needs to be obtained and two boundary conditions are there. Here, we have used the minority carrier parameters, diffusion constant and lifetime and used the, the minority carrier diffusion length, which is given by square root of, this is known as the minority carrier diffusion length. So, we applied the boundary condition and see what the solution we will get in the next lecture.