**Numerical Ship and Offshore Hydrodynamics**
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**Lecture - 11**
**Wave and Wave Effect**

Welcome to Numerical Ship and Offshore Hydrodynamics. Today's topic is Wave and Wave Effects.

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So, today we are going to discuss something about the free surface waves and other things.

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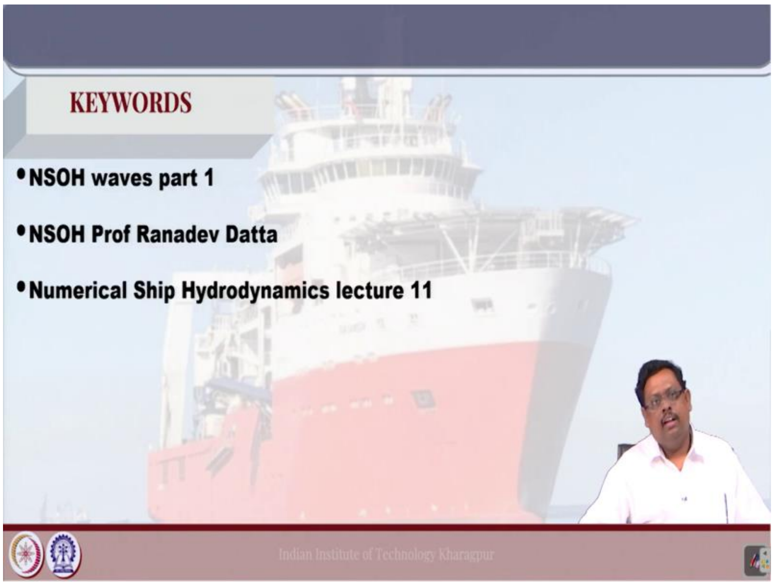


And, this is the keyword that we are going to use to get this lecture.

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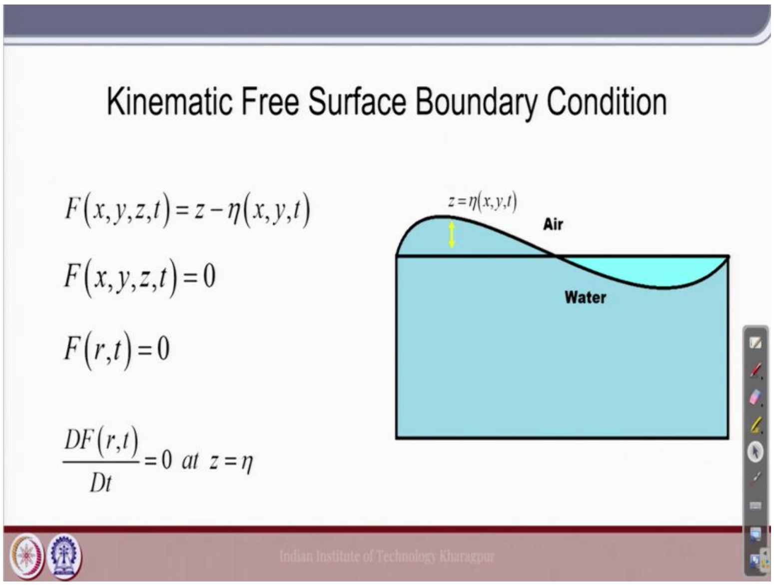


Now, let us coming back to this kinematic free surface boundary condition. Now, if you look at the wave then actually this water is and this the black line you can see is basically border line between the air and the water. And, this surface we call the free surface. Then, what is would be the equation for this free surface?

Now, if you assume that the coordinate axis that vertically upward is the z. So, we can assume this vertical distance z is basically a function η which is depending on x, y and t.

Now, here in two-dimensional case we cannot see the transverse axis, but one can see that if you travel along this x axis, this $\eta$ will change right.

Also, if you fix that x at some point, let us say we are fixing somewhere here the x and if you try to vary your time then you can see the same kind of sinusoidal wave. So, therefore, this $\eta$ is changes not only with respect to x, but also it will change with respect to t also and of course, with respect to y also ok.

Now, remember in my last class. So, when we called a surface to be a boundary surface right and the definition that we have learnt in last class. Now, here I can simply write a boundary surface F which is $z - \eta (x, y, t)$ right. Now, then this surface become $F (x, y, z) = 0$ or we can say it is $F (r, t) = 0$ right. So, till this point there should not be any problem.

Because, in last class we discussed about that F would be a boundary surface etc. And, here I can see this F we can call as a free surface because, it is the surface which is between air and the water. And, also I have discussed in a last class and this is the one of the key things or a or say unique thing for shift dynamics or offshore dynamics, that we have to deal with this free surface right.

Now, let us try to find out that what would be the condition that a water particle is stick on the surface right. So, the objective is to find out a condition so, that a water particle should stick on the surface. So, assume that this a water particle over here or somewhere over here. So, then what is the condition that this water particle cannot come out of the surface? Now, what would be the condition for this?

So, let us try to find out. Now, if you remember my last class what I said that a part water particle or a particle stick on a boundary if and only if the normal velocity of the surface should be equal to the normal velocity of the water particle right ok. So, therefore, here and this would be the condition that material derivative should be = 0 right. So, F be the boundary surface, then the condition should be $DF/Dt = 0$ right ok. So, this everything we discussed in the last class.

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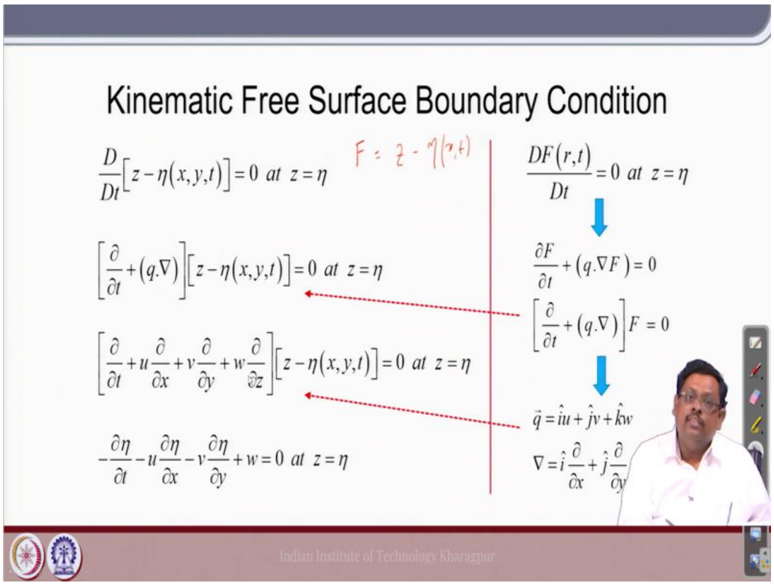


Now, here let us in the right hand side actually I just derived that DF / Dt is how I can break it in partial derivative. So, this is the vector notation DF / Dt can be written as dF / dt + q dot ∇ F = 0 right. And, then this q actually has 3 component that u, v and w right and ∇ is of course, i ∂/ ∂ x + j ∂/ ∂ y + k ∂/ ∂z right ok.

Now, we assume that material derivative z - η that should be is = 0. So, this is the condition. Now, here if you remember that in my last slide I said that D / Dt of F should be 0. Now, I substitute the F over here and we know that in here actually we write that F = we write F = z - η x, t. So, I substitute this F over here right and then I substitute that D / Dt also. Now D / Dt is nothing, but from here I know that it is ∂/ ∂ t + q . ∇ F ok.

So, now what I do is this ∂/ ∂ t, I replace over here. And therefore, if I replace this q then I will get you know this ∇ if I replace over here, then I get it is u ∂/ ∂ x + v ∂/ ∂ y + w ∂/ ∂ z right. So, it is it is very elementary. I just change that q . ∇, I just get q = ui + jv + wk. And, I call ∇ = ∂/ ∂ x + ∂/ ∂ y + ∂/ ∂ z and if I do the dot product, it should be u ∂/ ∂ x + v ∂/ ∂ y + w ∂/ ∂ z right ok.

Now, I just do the operation. Now, here this z is not function of t; however, this η is function of t right. So, therefore, ∂/ ∂ t of η I just write - ∂/ ∂ t. Now, again z is not the function of x either, but η is the function of x. So, I substitute over here. So, I get - u (d η / ∂ x). Similarly, z is not a function of y also; however, η is a function of y. So, if I do this then I get - v × (d η / dy). And finally, that ∂/ ∂ z this goes this goes to 1. So, I have that w, but then η is not the function of z so, therefore, it 0. So, if I do this operation so, I will get - ð/ ð t - u ð/ ð x - p ð/ ð y + w, that should be = 0 right.

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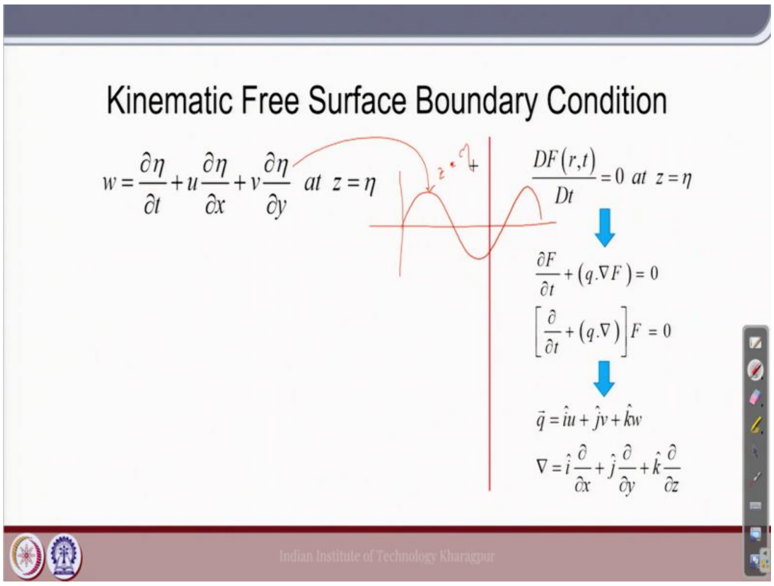


Now so, now, I understand that this is the equation. But, you know where I apply this; that means, that at which z this will apply? Now, if I write it this again the graph, I can see that this equation apply on the free surface. So that means, at the free surface your z is = η right. So, therefore, we understand that this kinematic free surface boundary condition, it is w = ð η / ð t + u (ð η / ð x) + v (ð η / ð y) and it should be apply at z = η.

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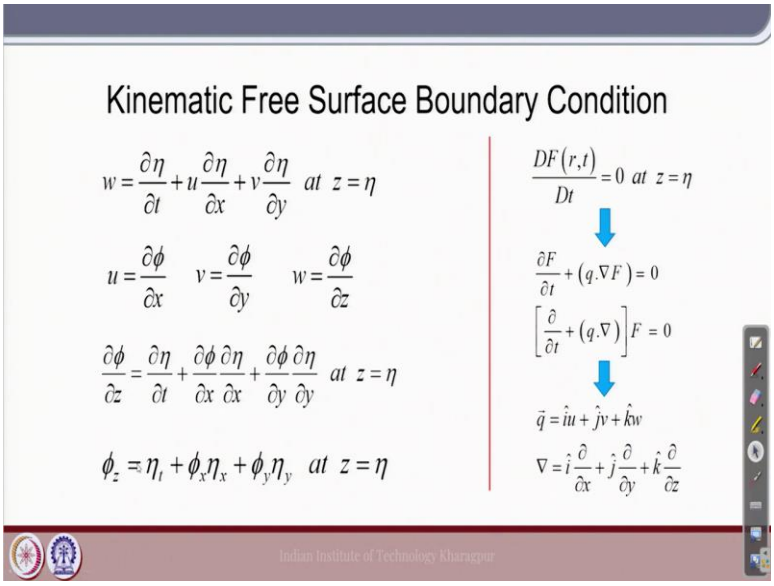

So, now I replace the u = ∂ ϕ / ∂ x, v = ∂ ϕ / ∂ y and w = ∂ ϕ / ∂ z in this equation. And then finally, I get the complete expression as ∂ ϕ / ∂ z is = ∂η / ∂ t + ∂ ϕ / ∂ x × ∂ η / ∂ x + ∂ ϕ / ∂ y ×∂ η / ∂ y and this apply at z = η. So, this is my kinematic free surface boundary condition which states that water particle has to stick on the free surface. If it is so, then this condition should apply or this condition should hold.

Now, I just writing this or writing in a little bit compressed way or smart way. So, I just write instead of ∂ ϕ / ∂ z I use $\phi_z$, instead of ∂ η / ∂ t. I use $\eta_t$. So, in that way I can write $\phi_z = \eta_t + \phi_x \eta_x + \phi_\eta$ y and this apply at z = η. So, this is known as the kinematic free surface boundary condition ok.

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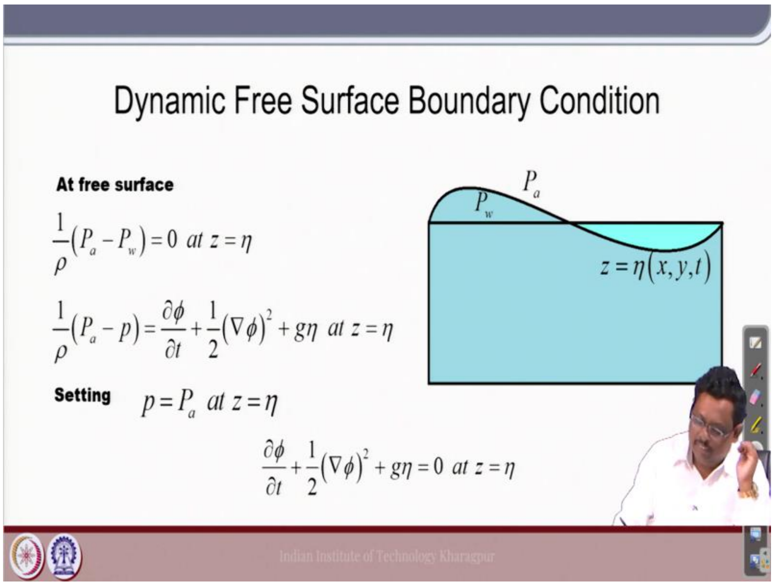


Now, let us move on the next part is called the dynamic free surface boundary condition. Now, kinematic free surface condition explain you that at this the water particle kinematics; that means, water particle to stick on the free surface right. Now, this dynamic free surface condition is telling you that at free surface there is equilibrium of the pressure. So that means, the difference of the pressure I mean the air pressure water pressure balanced each other and this goes to 0.

Now, if we apply the Bernoulli's equation or my pressure equation. So, this is the equation that we learn the pressure = ∂ ϕ / ∂ t + 1/2 ∇ $\phi^2$ + g η, we have discussed in the last class. So, here you know we are not deriving it right. So, this is one and then this left hand side should goes to 0, because this one balance each other. And then finally, we will get the dynamic free surface condition as ∂ $\phi$ / ∂ t + 1/2 $\nabla \phi^2$ + g $\eta$ = 0.

And of course, it apply at z = $\eta$. So, it apply over the free surface. So, therefore, I have two boundary conditions. One is we call the kinematic free surface condition which tells about the kinematics of the water particle. And, second one is the dynamic free surface boundary condition which talks about the pressure over the free surface ok.

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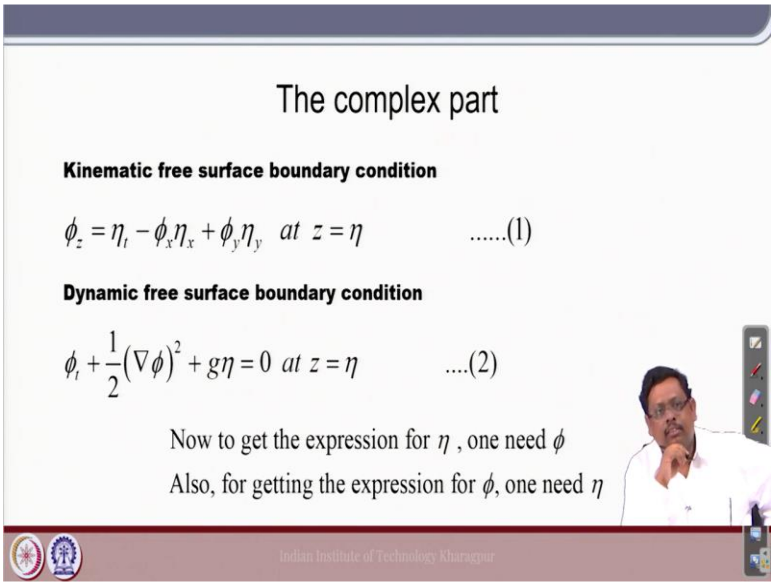


So, what is the complex part of it? Now, this is the equation the derivation of equation is fine. But, what is the complex part? The first one is let me this is your the non-linear kinematic free surface boundary conditions right. And, then we have the non-linear dynamic free surface condition. Now, here needless to say in this equation 1 and 2, both $\eta$ and $\phi$ are actually unknown to me right.

Now, now then the problem is how can I obtain $\phi$ if I do not know the information about $\eta$ and how they obtain this $\eta$ if I do not have the information about the $\phi$? Now, how to solve this problem? So, because you see this is a very, this is really a complex part because now we get an expression for $\eta$.

So, in order to find the $\eta$ we need $\phi$. Also I have $\phi$, but I need to find out in order to get the $\phi$ I need $\eta$. So, the best way is just eliminate either $\phi$ or $\eta$ right. So, how to do this?

That means, we have here the both $\phi$ and $\eta$ both are present in the equation, I need an equation where that either $\phi$ or $\eta$ is absent.

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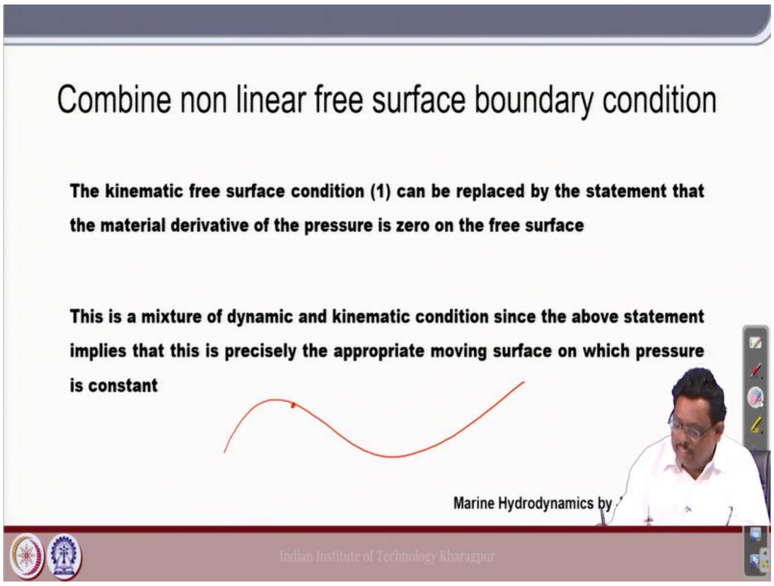


Let us see that. Now, in a classical book by Marine Hydrodynamics by J. N. Newman; he said that the kinematic free surface condition 1 can be replaced by the statement that material derivative of the pressure is zero over the free surface. So, actually you can say this is a mixed boundary condition. So, it means that the pressure is zero on the a varying surface.

So, when I consider the a varying surface so, definitely it is a kinematic part and then we are considering the pressures is a dynamic part. So, what he said is very interesting. He says that I take the pressure along this free surface and that pressure should be = 0. So; that means, I am considering. Now, what I am doing as per his statement I mean this statement; I am considering a surface of pressure.

I am considering a surface of pressure where in this particular surface at any point of time, the pressure variation = 0. So, this is very strong argument actually and with respect to this we can easily eliminate the $\eta$ right. How?

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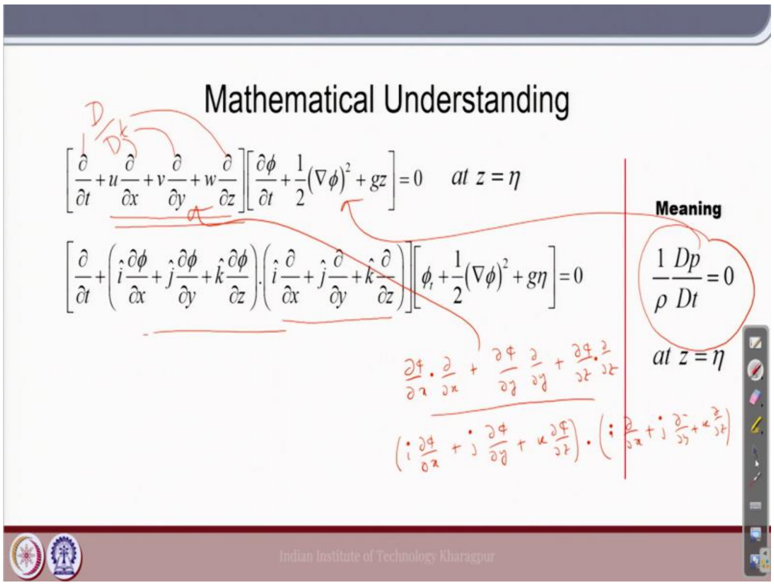


Let us see. Now, you see here what I said that, that what I said the meaning mathematically meaning is that I just considering a surface where the material derivative of pressure should be = 0. Now, if I say that then this is the equation for the D / Dt the material derivative. We know that is ∂/ ∂ t + u. ∂/ ∂ x + v. ∂/ ∂ y + w. ∂/ ∂ z right and then this pressure is basically this one, our pressure right.

Now, let us continue and let us find out what we can get out of this. Now, here you can see that in this equation there is no existence of η right. So, now, if I break it So, I just write in terms of you know here actually what I did is, this term I makes × two component right. Now, you can see here I replace u. ∂ ϕ /∂ x, v.  ∂ ϕ / ∂ y and w. ∂ ϕ / ∂ z right. And, apart from that what I do that I write another component is you know i ∂/ ∂ x + j ∂/ ∂ y + k ∂/ ∂z.

Now, you see if you do this that; means, that means that is how I just make this u ∂/ ∂ x + v ∂/ ∂ y + w ∂/ ∂ z. I break it this way. Now, if you do this then actually you can get; now if you use a dot product its become ∂ ϕ / ∂ x × ∂/ ∂ x + ∂ ϕ / ∂ y × ∂/ ∂ y + ∂ ϕ / ∂ z × ∂/ ∂ z. See this is nothing, but your this term right.

And, then actually what I do is I just use this as a vector product. So, I split it out × vector component. One component is i ∂ ϕ / ∂ x + j ∂ ϕ / ∂ y + k ∂ ϕ / ∂ z and multiply by the another component i ∂/ ∂ x + j ∂/ ∂ y + k ∂/ ∂ z. So, I did that. Then what is the benefit of doing all these things? Let us see.

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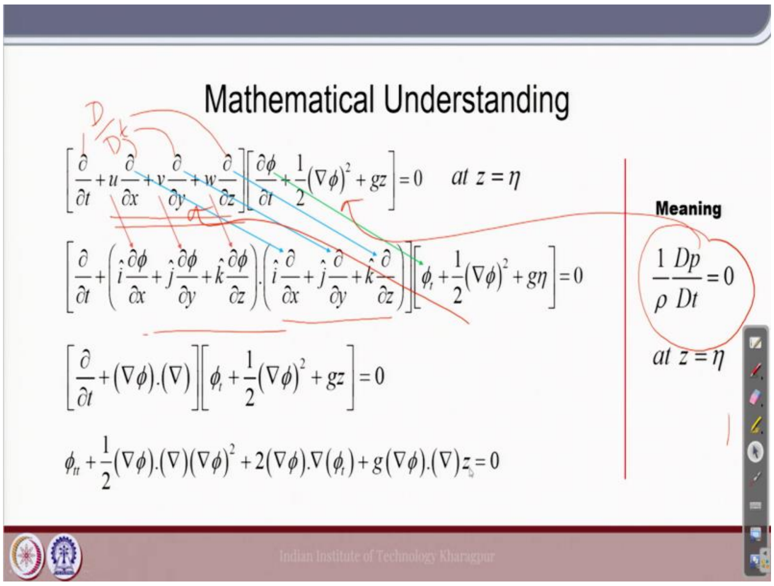


Now, if I do that, if I do that then you can see that I actually write this term as a $\nabla\ \phi$. And, then I write this term as a $\nabla$ right and again just in case of $\partial\ \phi\ /\ \partial\eta_t$, I make at $\phi_t$. So, therefore, this is the I write in a vector form. So, I just changes this into it nothing it is just a notation right. So, now, if I apply the differentiation over there.

So, if I use $\partial/\ \partial\ t \times \phi_t$, I will get you know $\phi_t$ and then if I get $\partial/\ \partial\ t \times 1/2\ \nabla\ \phi^2$. So, I will get $\nabla\ \phi.\ \nabla\ \phi_t$ and also if I make $\nabla\ \phi\ .\ \nabla\ \phi_t$, again I am getting $\nabla\ \phi\ .\ \nabla$ of $\phi_t$. So, therefore, I get $2\ \nabla\ \phi\ .\ \nabla\ \phi_t$ and finally, I multiply this with this. So, I will get $g \times \nabla\ \phi\ .\ \nabla \times z$ ok.

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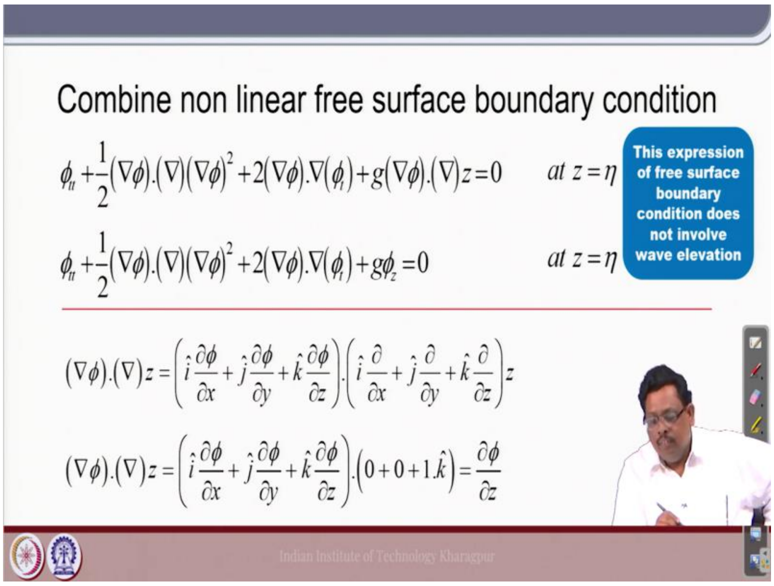

So, now I get a combined free surface boundary condition which is the absence of η right. So, in this equation you can see there is no η is present. So, now, if I solve this equation then actually I really do not have to worry about my η. So, this is called the combined non-linear free surface boundary condition. Now, here you can see there is no presence of η right ok. Now, let us just little bit do some maths quickly.

Here, I can see that I just replace that ∇ ϕ over here, I replace the ∇ over here and then if I multiply by the z. So, I can get here, now you can see that only ∂ϕ / ∂z term exist right. So, therefore, therefore, I can replace this as g × $\phi_z$ right. So, this is basically the I mean the most simple simplified case for the combined non-linear free surface boundary condition.

These are elementary mathematics that you can follow very quickly like this ∇ ϕ is this one, then ∇ is this and then I multiplied by the z. So, it is k ∂ ϕ / ∂ z multiplied by the ∂ / ∂ z. So, therefore, we will end up getting ∂ ϕ / ∂ z and then we replace the ∂ ϕ / ∂ z over here ok.

So, math part is elementary. So, the, but the important part is here this boundary condition we do not have η. So, if I try to solve some problem and try to find out the what is my free surface or is the velocity potential of the free surface, the water particle. So, then you can use this equation ok. So, now, this exercise as I said it is free surface boundary condition absence of the wave elevation.

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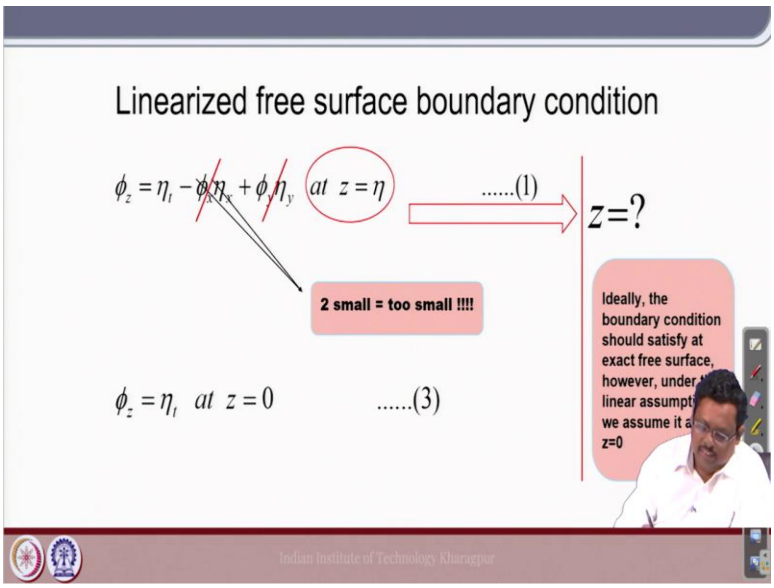

Dealing with the non-linear free surface boundary condition is very complex and it is very difficult to solve the practical, real practical problem. So, therefore, we have to make some kind of approximation, some assumptions and then we need to drop some terms and we have to get a simplified equation and then we are ready to solve the equation.

So, let us now see that in case of the non-linear boundary condition what approximation we make; so, that I can find a simplified linearized equation ok. Now, this is your non-linear free surface boundary condition. Now, here you can see we have non-linear terms such as $\phi_x$ $\eta_x$, this one and $\phi_y$ $\eta_y$. So, we need to drop this to get some linearized free surface conditions.

So, what is the approximation? Approximation is we assume $\phi_x$ and $\eta_x$ both are very small. Now, if both are very small then multiplication of the product should be too small so, that we can ignore. So, this is the underline assumptions. So, under this underline assumptions we ignore the contribution for $\phi_x$ $\eta_x$, we ignore the contribution of the $\phi_y$ $\eta_y$. But, the question is at which z actually you are applying it? This is a very important question.

Now, ideally speaking we should apply at $z = \eta$; that means, you should apply this equation at the exact free surface. However, in case of a under the assumptions of the linearity, we apply at $z = 0$ ok. And then finally, we end up getting this kinematic free surface boundary condition, what is called the $\phi_z = \eta_t$.

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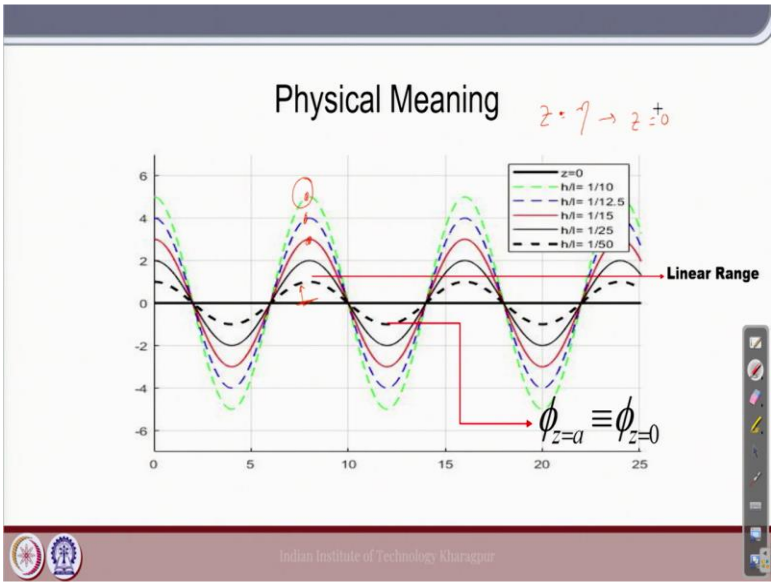


Now, let us try to understand what is the meaning of this linearity. Now, here I can you can see that this is a wave elevation with various z / l ok or h / l; that means, height divided by the λ, the length. I mean the length between this from here to here. So, now, you can see this is called the in its frankly speaking this is called the wave stiffness.

Now, we can see that this h / l it is 1 / 10, means is very stiff. So, I decrease the step slowly slowly. See, first is 1 / 10 and after that it is 1 / 12.5, after there is 1 / 15, 1 / 25 and then finally, it is 1 / 50. And, then actually you can see when is 1 / 15, this elevation is really really small.

Now, what is happening that this I can think of a linear range. So, what we are logically what we are try to say as follows. We are trying to say that in case of this small amplitude; we can assume what is happening at z = 0, the same thing is happening at z = a or z = η ok. So, this is the underlying assumptions of the linearity. We have to understand this right.

We assume that when you say it is a linear problem; we assume that what is happening at z = 0, the same phenomena is happening at z = a because the elevation is very very small. So, that assumptions not hold. These assumptions we cannot make. If we take this point, if I take this waves I if it is this wave we cannot say. What is happening here, the same thing happening here not possible.

But however, in case of a linear range we could say that what is happening here, the same thing is happening here. And, that is why we change the range the z = η to z = 0 right. So, this is the underlying assumptions of the linearity and one has to understand very well. Because, otherwise it is very difficult to find out that what is the linear range; when you say it is a linear range, it is a non-linear range.

Everything depend on the fact that as long as we can assume what is going to happen at z = 0, the same thing is happening at z = η; we really cannot progress ok. So, let us stop here and in the next lecture we are going to discuss about the linearity of the dynamic pressure. And, then we have to deal with like how we can solve this problem and how we can obtain the ϕ.

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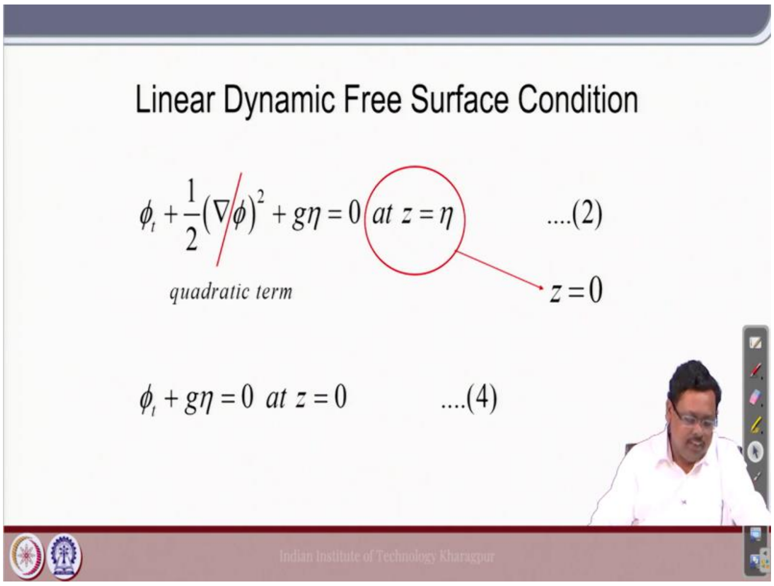


Let us now talk about the linear dynamic free surface boundary condition ok. Now, this is the non-linear equation as you know. Now, here also we are going to apply the same thing. We are going to drop the quadratic term which is this one right, half of ∇ $\phi^2$.

So, if I drop this term so therefore, we can get and also ok; also, we have to apply at z = 0 in this case also. Again, the logic is similar what is happening at z = 0, same thing is happening at z = η. So, therefore, I can drop z = η and I can write z = 0.

So, therefore, my linear dynamic free surface condition becomes $\phi_t + g\,\eta = 0$ at $z = 0$ right. Now, when I linearized my kinematic free surface condition and dynamic free surface condition, actually I can combine the both.

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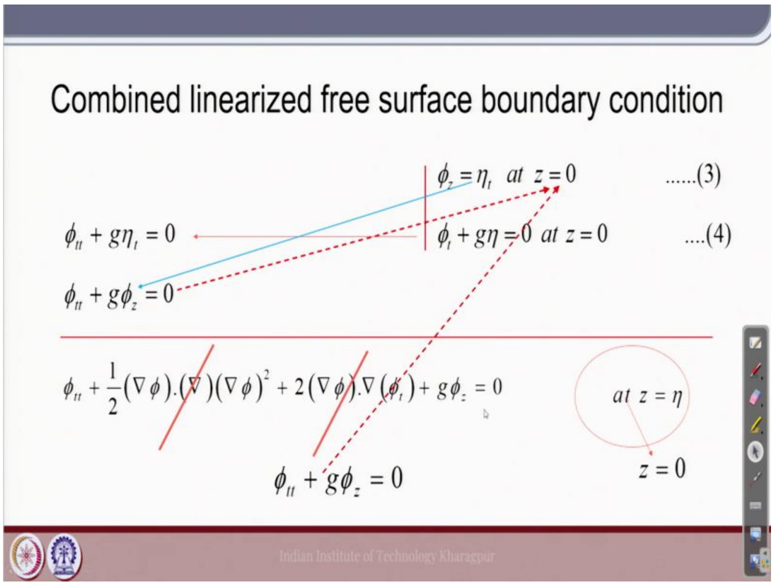


So, my idea again I need to eliminate either $\eta$ or I need to eliminate either $\phi$. So, let us see. Now, this is my linearized kinematic free surface condition $\phi_z = \eta_t$ and this is my linearized dynamic free surface condition $\phi_t + g\,\eta = 0$ right. So, now I differentiate this equation with respect to t. So, I get $\phi_t + g\,\eta_t$ should be = 0 right.

Now, here I can further replace $\eta_t = \phi_z$. So, if I replace this then finally, my equation comes $\phi_t + g\,\phi_z$ equals to 0 right. So, this is basically my combined free surface boundary condition. And, remember the next one which is in the bottom is basically the combined non-linear free surface boundary condition.

Now, here also if I drop the quadratic term right. So, and if I apply it is at $z = 0$, again I can get the linearized free surface boundary condition right. So, I drop this term, I drop this term and therefore, from here also I get $\phi_t + g\,\phi_z = 0$. So, you can see that whatever the approach you are going to take; finally, when you linearized both the things are same ok.

So, now in the linear problem I am going to deal with this combined free surface boundary condition $\phi_t + g\,\phi_z = 0$. And, if I want to go with the non-linear so, then I have to deal with the $\phi_t + (1/2)\,\nabla\,\phi.\,(\nabla\,\phi^2) + 2\,\nabla\,\phi\,.\,\nabla\,\phi_t + g\,\phi_z = 0$, this one ok.

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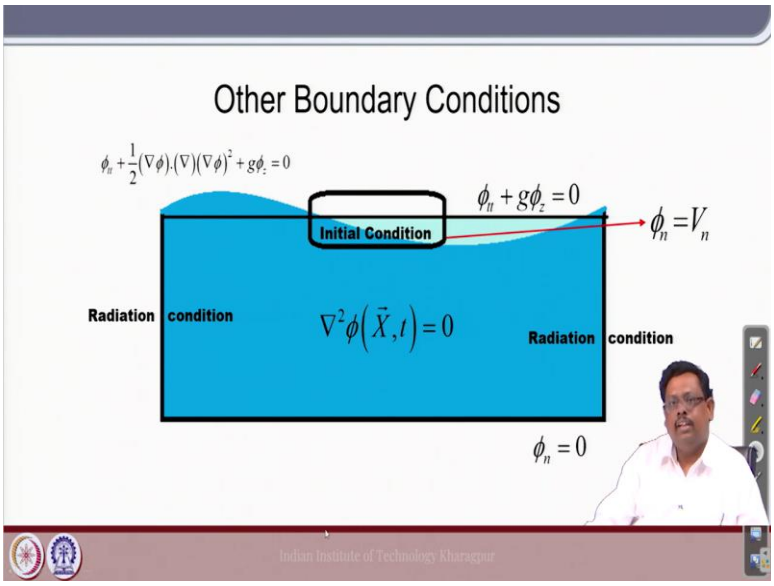


So, now let us see mathematically what are the boundary conditions that we are talking about. So, this is the $\delta^2\,\phi = 0$, that actually it is applying the whole fluid domain. We have discussed this in last class and then the initial condition we will discuss again later.

And, now this is your non-linear free surface boundary condition, we apply though the free surface. And, then this is the linearized free surface boundary conditions which applied at $z = 0$ right. Also, we have the body boundary condition, we discuss this body boundary condition later on. Right now, we are not going to discuss this.

And, also we have the bottom boundary condition which is basically the normal velocity, the fluid particle in normal direction is 0. So, fluid does not have any velocity in the direction of the normal. So, this is the bottom boundary condition right and also with this we have the radiation condition. So, basically when we are going to solve for $\phi$, we have to apply this $\delta^2\,\phi = 0$ and then all other boundary conditions.

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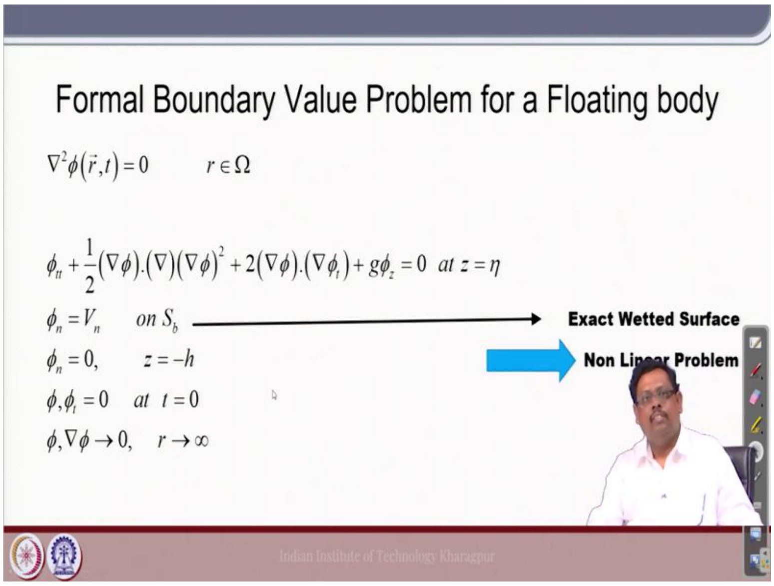


So, now let us quickly check like the what is the floating body problem and what is the conditions and what is the you know the equation that we are going to solve. Now, in case of a non-linear problem, we are going to solve this.

This Laplace equation with non-linear free surface boundary condition and apart from that the body boundary condition, bottom boundary condition, initial condition, the radiation condition right. So, this is actually $S_b$ is called the exact weighted surface which is the exactly the free surface and we call this as a non-linear problem.

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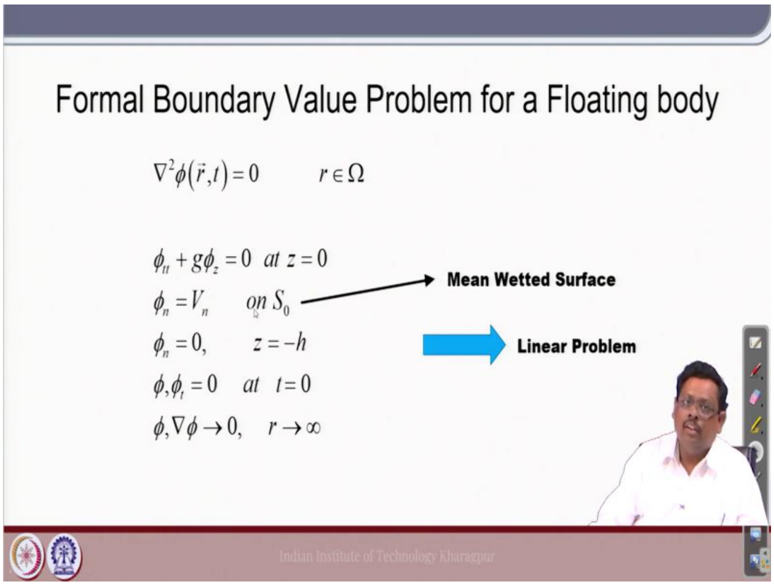

And, in case of a linear problem everything is same, only the free surface boundary condition is linearized. And, also this we are talking about the $S_0$ which is basically the mean wetted surface, remaining thing are same. Just what is this? From this diagram I can see this is basically your $S_b$, where the blue one and this black line is basically your $S_0$, ok fine.

So, now this is basically the boundary value problem that we need to solve to get the solution for the $\phi$ ok. So, from the next class we are going to get how we can get the solution for $\phi$ solving this problem ok. And, today we are going to finish here.

Thank you.