Numerical Ship and Offshore Hydrodynamics
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Lecture - 12
Wave - 2

Welcome to Numerical Ship and Offshore Hydrodynamics.

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Today we are going to discuss about the incident wave and wave force ok.

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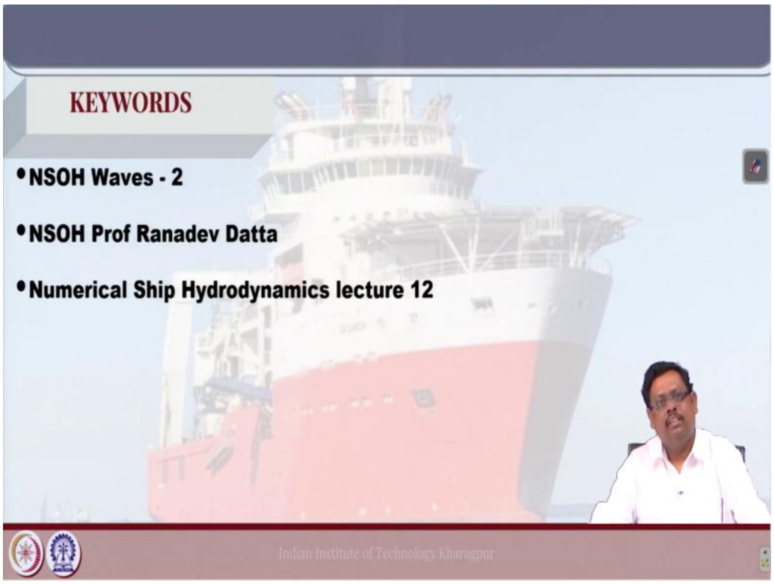

And, this is the keywords to get this lecture.

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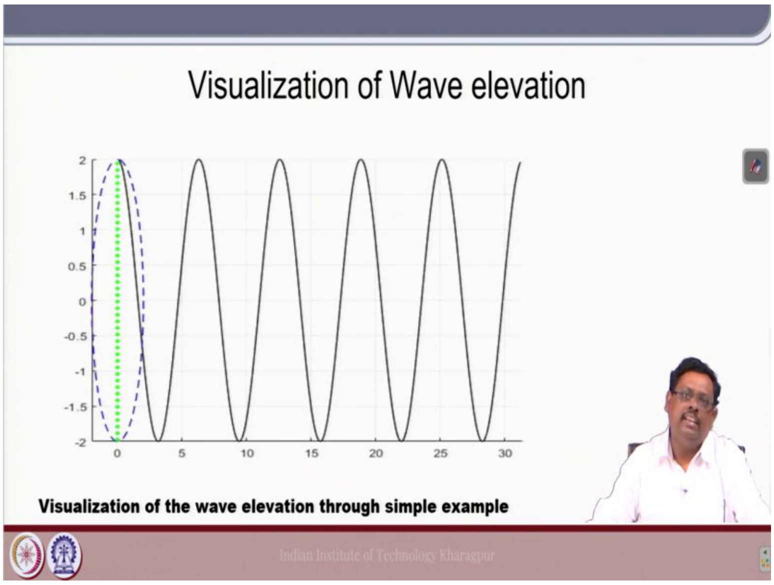

Before we start, let us see the visualization of the wave elevation ok. Now, we can see that water particle actually moving in this circular fashion and then that this wave is propagating in the horizontal direction as follows. Now, this green dot basically tells you about the wave amplitude and then when it makes the complete circle, let my green dot comes down and up; it actually travels from this peak to the next peak right, you can see that. Now, with this visualization let us move forward.

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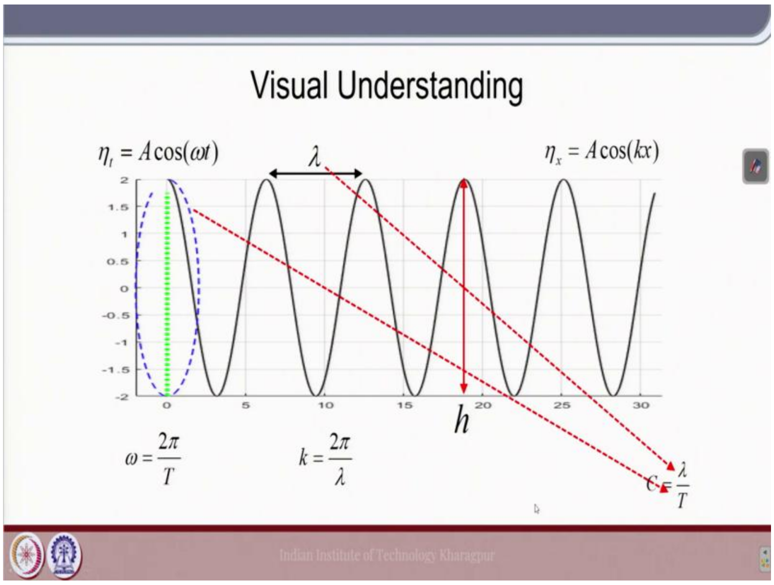

Now, here I can as I said that when this green dot makes the complex rotation, like it go down and then come up, then this waves moves this peak to the next peak ok. Now, this we call the wavelength; that means, the length of the wave when this water particle complete 1 circulation or 1 rotation. Secondly, we call from this 2 to - 2 that that complete that that height from the top to the bottom, we call this is the height of the wave ok.

And, this top part we call the wave crest and this part we call the wave trough. Now, from the classical that you know angular motion you know that this if omega is the frequency and T be the time period. So, both are the having a relationship with $\omega = \frac{2\pi}{T}$ right ok. So that means, that water particle takes the time capital T to get the complete rotation.

Now, similarly as I said that for a full rotation these also travel the lambda length. So, then we can define one parameter k which is called the wave number. It means that similar to the wave I mean the wave frequency omega. So, meaning is same; one is in the respect to the time and one is the respect to the space. Now, how to understand this? Basically, that if you take this snapshot here. So, we can have a harmonic; that means, you are fixing at any point of time suppose you can see some progressive wave and then at each point of time you take a snapshot, you can find that this sinusoidal pattern.

Now, similarly what we could do is that if you fix a point x, let us you fix a point over x like this green dot and if you see the oscillation of the green, then you can see a harmonic motion. So, this is harmonic in the direction of x, if you take a snapshot, if it is harmonic in the direction of the T also. See, in case of a direction of T, we have the frequency omega in direction of a x, that mean direction of the wave propagation we call this since this is harmonic; so, we call this as a wave number.

So, here it is $\omega = \frac{2\pi}{T}$, here is $k = \frac{2\pi}{\lambda}$ ok. Now, as I said if you fix a take a snapshot, if you take a snapshot then this signal is $\eta_x = A\cos kx$ Now, if you fix the space and if you just look at the motion of this green dot, then you know η is basically you can see it is $A\cos(\omega t)$. Now, you see here this η is a function of both, if you take a snapshot you can find a sinusoidal curve which is the this one.

And, also if you only follow the motion of this the green dot which is comes ups and down. So, if you try to plot that then again you can get a sinusoidal curve right. So, therefore, η is actually function of both function of space and as well as function of t. Now, let us try to understand some more definition which is called the speed of the wave.

Now, here you can see that this T basically the time period it says that as this water particle which is the blue dot moves a complete circle, if I take a time T; at that particular time T, these waves move horizontally the lambda distance. So, therefore, the wave speed should be $\frac{\lambda}{T}$ It means that if the particle you know water particle takes a complete circle in time T second, then this wave travelled along the x direction is λ meter. So, therefore, we can define the wave speed $C = \frac{\lambda}{T}$.

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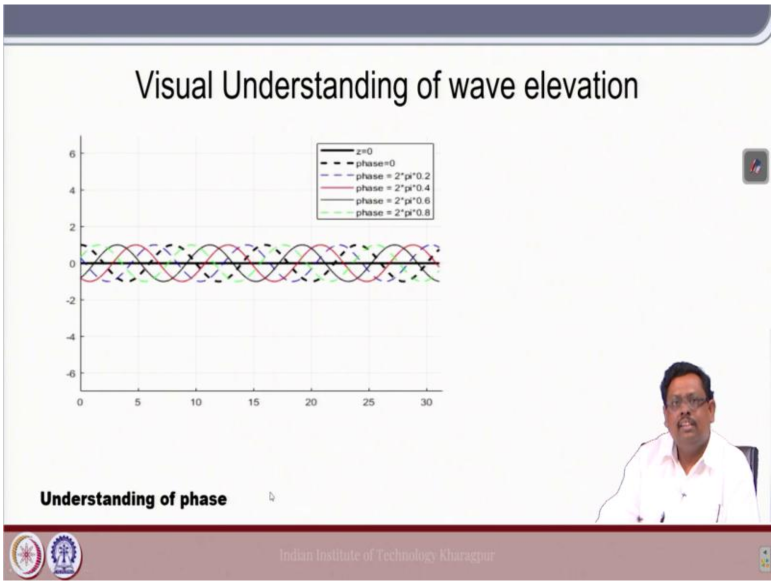

So, now, let us try to understand that what would be the equation of the wave in that case. We understand that it is function of x, it is function of T, then what would be the combined equation; that means, a wave which is propagating in the positive x direction. Now, before that we need to understand the s concept of the phase. So, let us see the video. So, we have the same wave; however, the frequency is same, but it has some phase.

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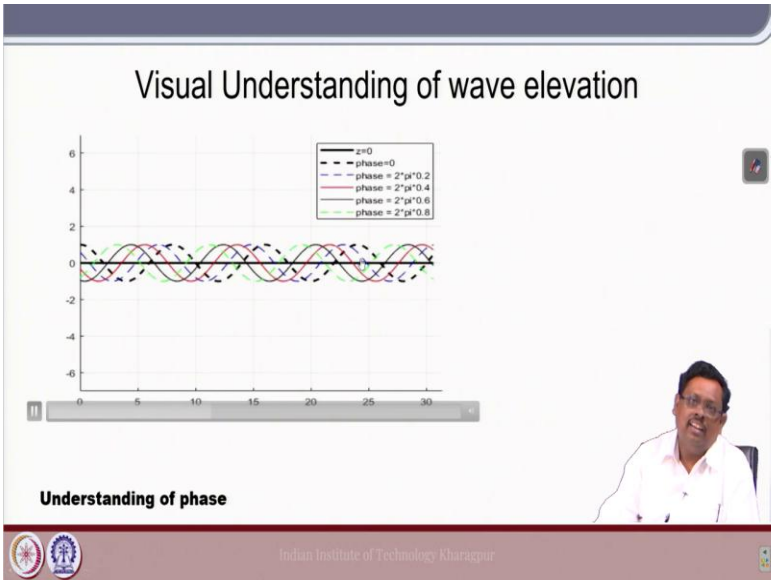

Now, if you look at this carefully, you can see that as if this wave profile is actually slowly moving forward right.

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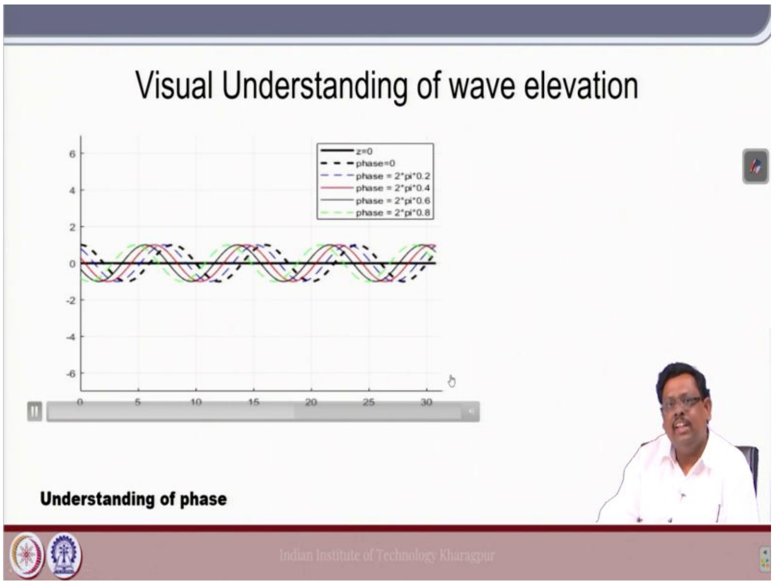

This is something we can call as a progressive wave, just see carefully. This slowly it is moving forward.

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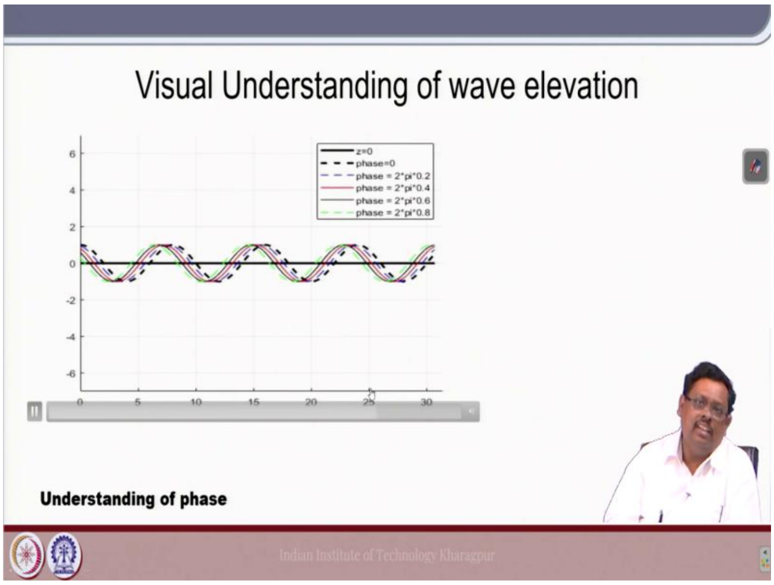

So, now, we can see what I am trying to do here, that I am making the phase from 2 π into 0.8 to 0 and I can see that what is actually happening.

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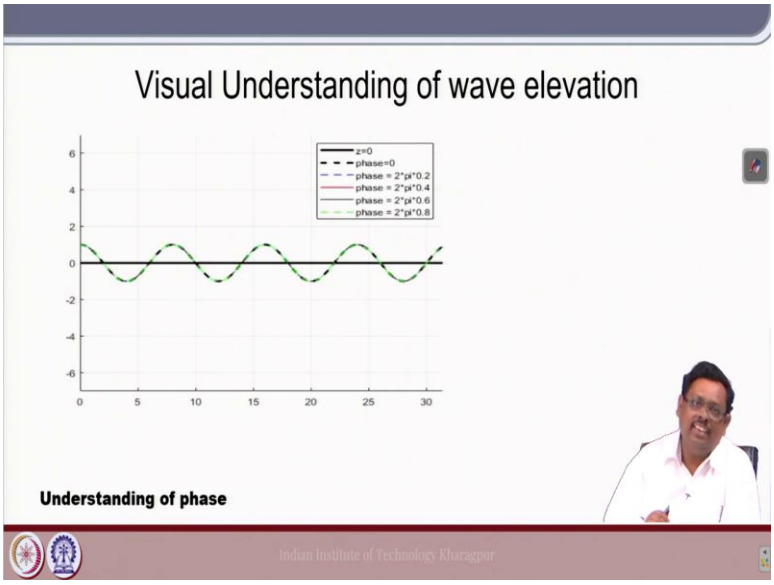

Now, let us see this video fully and then we can understand. Now, you can see here earlier there will be a bunch of waves are travelling and now only a single monochromatic wave is travelling. So, what is happening?

Now, let us see again this video one by one. Now, here what I can see there is all the waves have some phases and moving. Now, as if you can see that this wave is basically the propagating in the forward direction. Let us see again, it appears you like this, this wave is propagating a wave is propagating in this fashion right from the left to right, right.

Now, what I am making slowly, I make all the phases equals to 0. So that means, I am initially is 0.8 then 0.6 then 0.4 then 0.2 and then finally, I can get the phase equal to 0. So, once I do that I can see that this, see now slowly this wave is not propagating you see like, if I make this video little bit here, maybe here; now we can still you can see that wave is like propagating slowly.

Now, you see that as we are making phase is equal to 0. I can see that wave is not propagating as such in it is like only a single motion. Now, we can see when the phase equal to 0, it acts like a monochromatic a single wave. So, it is you do not have this visual impression that wave is propagating.

So, this visual that thing will only coming when I can see lot of phases are there. So, if I make everything 0, then it is as if there is a one single wave is moving. Now, this let us try to understand what is this.

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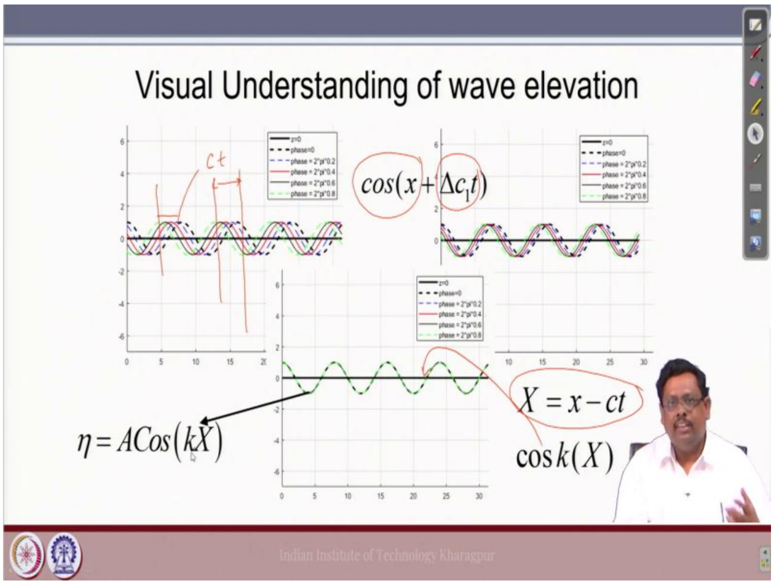

Now, this is the first thing you can see let us say I assume that wave is moving slowly from here to here. Now, this phase is let us see the in some other way, like in this phase if I assume that basically wave is travelling with a speed c right. So, what is happening in this particular picture?

Now, I can assume that this along with this the dotted black wave, this dotted black wave basically it is travelling progressively from green to t ok. So, again I can consider let us see that all these waves have the different speed let us say ok. So, therefore, I can assume that if one wave is cos x, another wave may be cos x plus and then ∇ct that you can think of a phase ok.

So, idea is very simple. Now here from here to here, this let us say the wave travelling in progressive wave travelling forward, it will if I take the time t. So, therefore, this distance definitely ct so; that means, after time t, the wave travel x. I mean the wave travel ct distance. Now, if I make (x-ct) basically, what I do? I cannot see this wave is progressing either, that is from the visual the MATLAB that you can see from here; I can see it is actually a monochromatic wave.

And, this wave basically can have a unique definition right. So, here the point is if we assume this x-ct, then actually visually I can see there is only single wave and this equation of these waves can be written in forms of $\eta = A\cos kX$. So, this is the idea, this is the idea that I am assuming that the velocity if the velocity the wave is c. So, at t time it is moving x - ct that is how it is progressing.

Now, if I take X=x-ct capital X equal to x - ct, then actually I am getting a simple wave which is travelling along x and with a frequency k. And, then equation for this particular wave is $\eta = A\cos kX$ right. Now, if we understand this visually then remaining part are simple algebra. We really do not have to think much. Now, if I understand this really this physics part where I am trying to say that, x-ct is as if a monochromatic wave with the frequency k, then the signal should be A cos kX ok.

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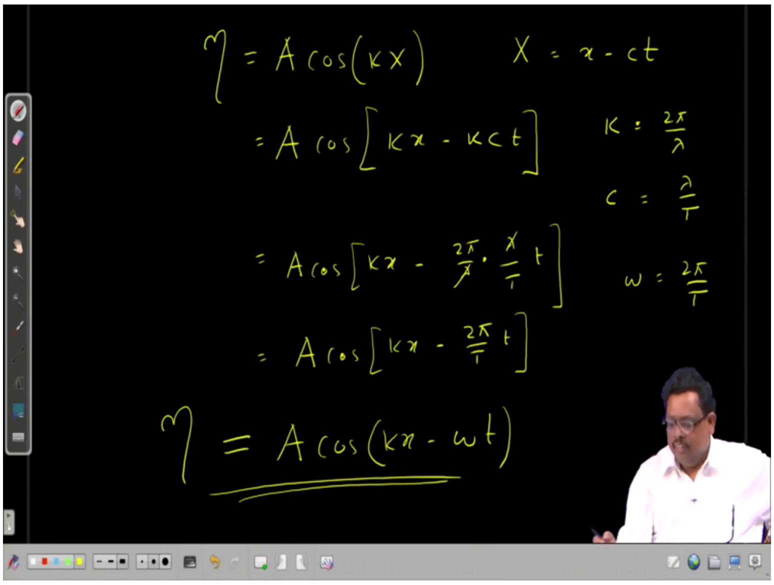

Now, what is happening here, I can write then $\eta = A\cos(kX)$. Now, this capital $X = x - ct$. So, therefore, I get $A\cos(kx - kct)$ right. Now, I replace the value of k here. So, $k = \frac{2\pi}{\lambda}$ and you know the $c = \frac{\lambda}{T}$, see if I replace over here.

So, I will get So, $A\cos(kx - \frac{2\pi}{\lambda}\frac{\lambda}{T}t)$ lambda lamda cancelled out. So, I will get $A\cos(kx - \frac{2\pi}{T}t)$. Now, I know that $\omega = \frac{2\pi}{T}$. So, therefore, finally, I can find out my

$\eta = A\cos(kx - \omega t)$ ok. So, this is my final equation of the linear progressive wave ok fine. So, now let us go back to here and try to solve the boundary value problem ok.

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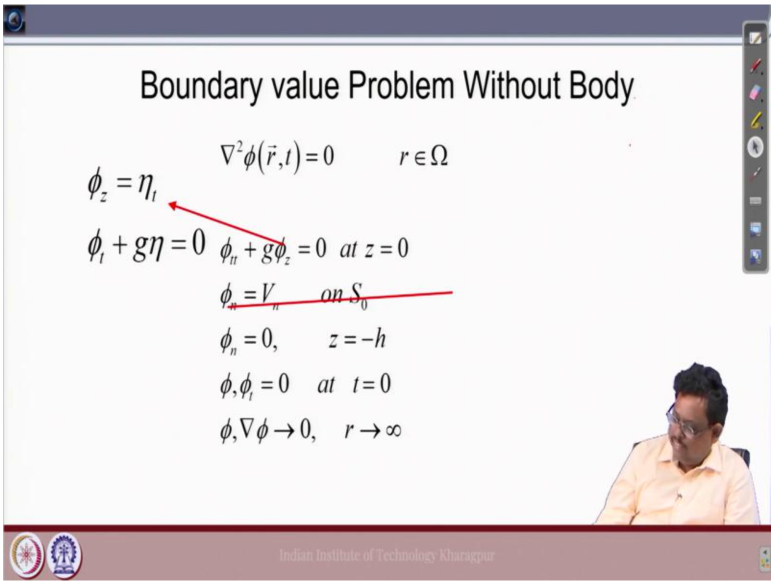

So, let us try to solve this boundary value problem. Now, our governing equation is $\nabla^2\phi(\vec{r},t)=0$ and these are the you know boundary conditions. Now, we have this free surface boundary condition $\phi_{tt}+g\phi_z=0$. We have the body boundary condition $\phi_n=V_n$ and we have the bottom boundary condition $\phi_n=0$. And, then we have the initial condition and the radiation condition.

Now, here since the problem is without the body right; so, we can discard this boundary condition, because we do not have the body here right. And, also let us do one thing, this combined free surface boundary condition let us you know split into a kinematic free surface boundary condition which is $\phi_z=\eta_t$. And, then the dynamic free surface boundary condition which is $\phi_t+g\eta=0$ ok.

So, this is my boundary value problem. I am going to solve $\nabla^2\phi=0$ with $\phi_z=\eta_t$ $\phi_t+gt=0$ ϕ_n equal to I mean this of course, at $z=0$. And, then the normal velocity $\phi_n=0$ at $z=-h$ and then we have the initial condition as well as the radiation condition ok.

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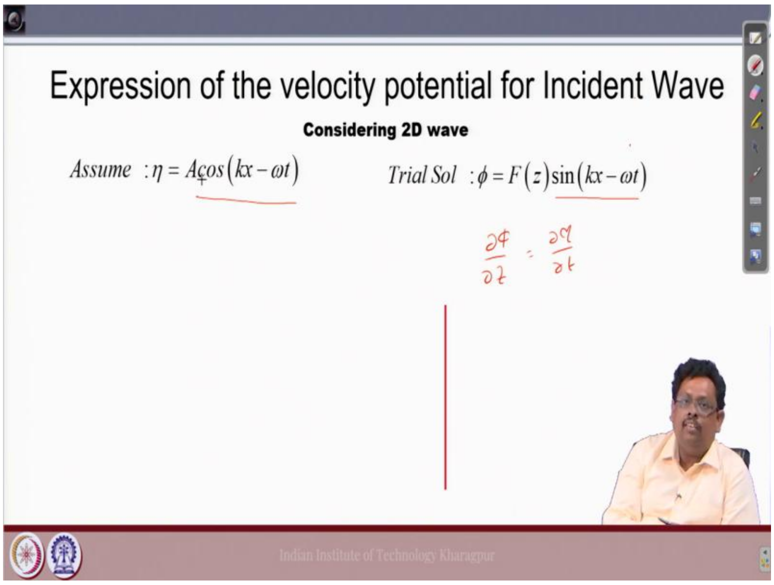

So, now we are assuming the plane progressive wave $\eta = A\cos(kx - \omega t)$, where A is the wave amplitude, k is called the wave number, ω is a frequency right. Now, to solve this I am assuming the trial solution .

Now, the question is why I select this as a trial solution? Answer is very simple, actually I know that that it is harmonic in x direction right, the wave is harmonic in x direction not in direction of the z. So, therefore, the Φ must be harmonic in the x direction. So, either this should be the $\sin(kx - \omega t)$ or $\cos(kx - \omega t)$, then why I select it is $\sin(kx - \omega t)$ right.

Now, this is the answer also very simple here. Now, you have let us find out just guess that my kinematic body boundary condition which is nothing but $\frac{\partial \phi}{\partial z} = \frac{\partial \eta}{\partial t}$. Now, if I differentiate η with respect to t, I will get here is sin function and if I differentiate Φ with respect to z, again I am getting the sin function. So, therefore, looking at this I know that if my $\eta = A\cos(kx - \omega t)$, definitely my solution become $Fz\sin(kx - \omega t)$ ok. So, with this understanding let us go forward.

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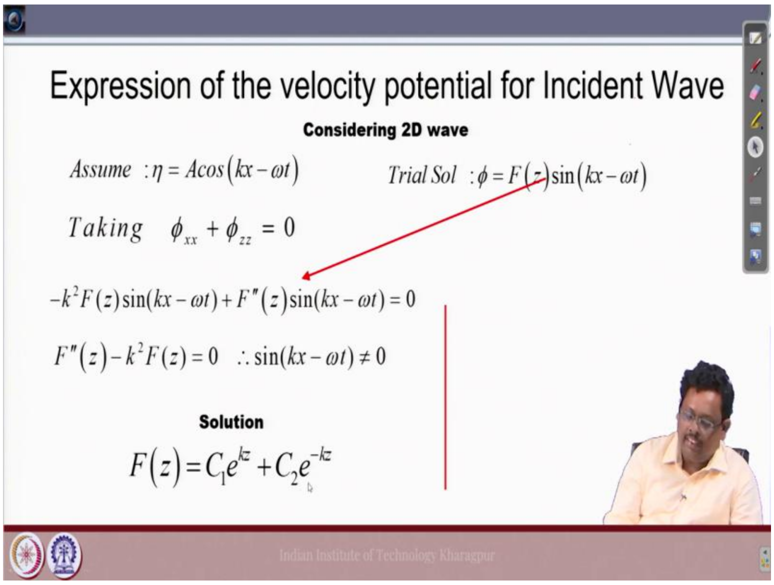

Now, I am taking the Laplace equation $\phi_{xx} + \phi_{zz} = 0$. So, there is many ways of writing the same thing, I can write $\frac{\partial^2 \phi}{\partial^2 x}$ or I can write Φ x s or ϕ_{zz}, both are the same thing. So, now, if I substitute this Φ here so, what I get? If I differentiate Φ with respect to x 2 time, then this - k^2 will coming out.

So, I have here – k^2 and then if I you know differentiate Φ with respect to z, then we have this function $F(z)$ it should be the F double dot z right fine. And, then I know that since $\sin(kx - \omega t) \neq 0$ throughout; so, definitely the other part should be 0. And therefore, I can get a very simple you know differential equation $F''(z) - k^2 F(z) = 0$.

Now, also we know what is the solution for this particular problem, I know that you know there is the solution technique that you can take z equal e to the power, I mean $F = e^{kz}$ and you can substitute over here. And, then we can find out the solution is very simple $F(z) = C_1 e^{kz} + C_2 e^{-kz}$ ok right.

So, this is very elementary till this point, I mean the solution itself is very simple, but you understand that how to get this solution for $F(z)\phi_n$. Now, in the next what I going to do is I replace this F_z, this value $C_1 e^{kz} + C_2 e^{-kz}$ into the solution of Φ ok.

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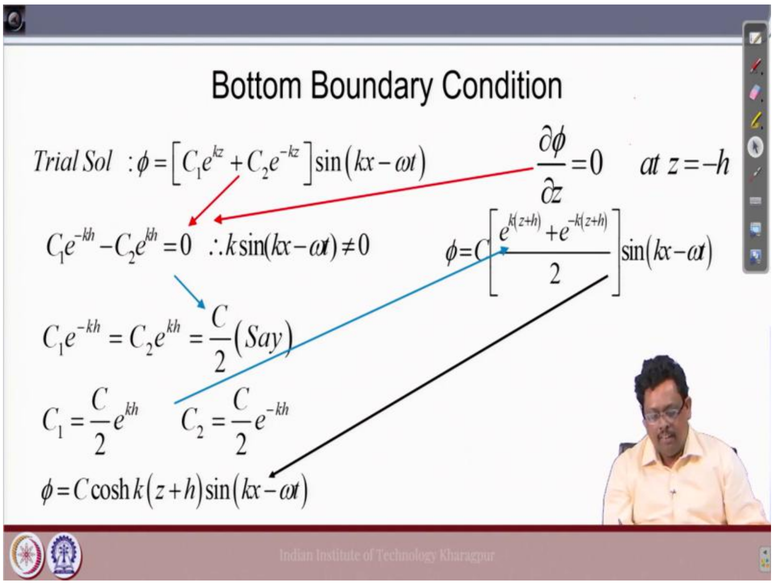

So, now my trial solutions becomes $(C_1e^{kz} + C_2e^{-kz})\sin kx$. Now, here I am going to apply the bottom boundary condition which is $\frac{\partial\phi}{\partial z} = 0$ at $z = -h$ right. Now of course, you know that at the bottom that normal $\frac{\partial\phi}{\partial n} = \frac{\partial\phi}{\partial z}$ right, because you know that in bottom is horizontal so, normal is the z direction. So, it should be del Φ del z.

So, if I apply this; so, if I apply this what I get? So, if I differentiate with respect to z, if I divided Φ with respect to z. So, definitely k will coming out right and also I put $z = -h$. So, it should be the kz should be - kh and then - kz should be kh right and if I define the second term, then - should be coming out. So, definitely I am going to get this.

So, from here what I get? I can get $C_1e^{-kh} = C_2e^{kh}$ right. The both the value is same and let us take that both the value is same and it is same as $\frac{C}{2}$, where C is it is an unknown to me, I can do that right. There are many ways to do that I can replace $C_1 = \frac{C}{2}e^{kh}$, that also I can do and I can proceed further.

But, there is a different way of solving the same thing. So, this is the way that we are going to solve. So, I am taking $C_1 e^{-kh} = C_2 e^{kh} = \frac{C}{2}$ and from here I can get the value of C_1 and C_2. So, I get the $C_1 = \frac{C}{2} e^{kh}$ and I am getting $C_2 = \frac{C}{2} e^{-kh}$ right.

So, now I know the value of C_1, I know the value of C_2 so, definitely I am going to substitute over here right. So, once I substitute and I do some mathematical you know adjustment, I will get the first term $Ce^{(kz+h)}$, I get $\frac{-k(z+h)}{2}$. Now, this is a hyperbolic function right, it is cos hyperbolic.

So, therefore, I can get the solution Φ equal to $C\cosh(kz+h)\sin(kx-\omega t)$ right. Now, here still I do not know the value of C. So, I need to find out the value for C. So, we are we need to use some more boundary conditions to get the value for C.

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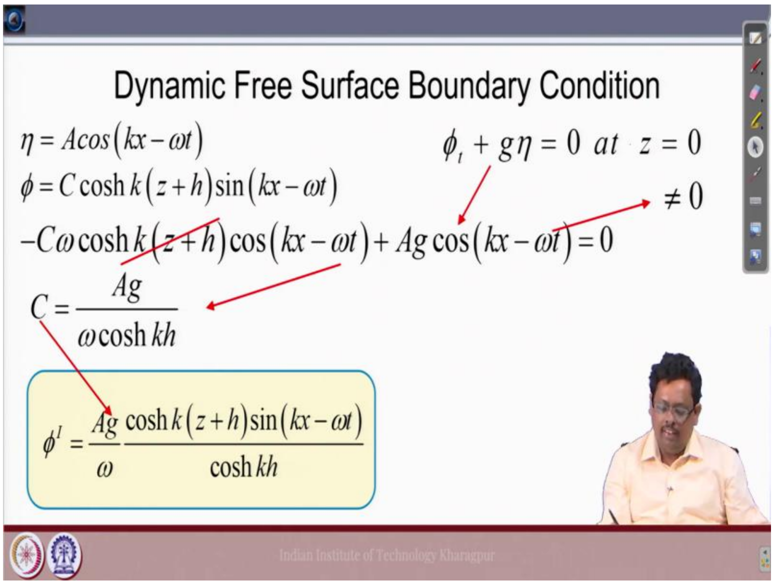

So, in next I am use the dynamic free surface boundary condition to get the value of C. Now, you see this is my η is $A\cos(kx - \omega t)$ and then we have the Φ which is my velocity potential or you can say the it is incident wave potential, that we say it is $C\cosh k(z+h)\sin(kx-\omega t)$. And then I so, I differentiate Φ with respect to t. And, then I substitute the value for η and I put $z = 0$. So, once I do that, I can find out this expression right.

So, this of course, this you know if you differentiate with respect to t, that this - omega will coming out right and then you put $z=0$. So, here you can put the $z=0$ so, I cancelled this. So, I put z equal to 0 over here and then I can solve for and also here I can make an argument that $\cos(kx-\omega t)\neq 0$ of course. And, then we can obtain the value for C which is $\frac{Ag}{\omega}\cosh(\text{kh})$.

Now, this value of C, I need to substitute over here in Φ and once we do that I get the expression for the incident wave potential $\phi^I = \frac{Ag}{\omega}\frac{\cosh k(z+h)\sin(kx-\omega t)}{\cosh kh}$ .Now, this is for the finite depth right. So, you see like we have the another boundary condition.

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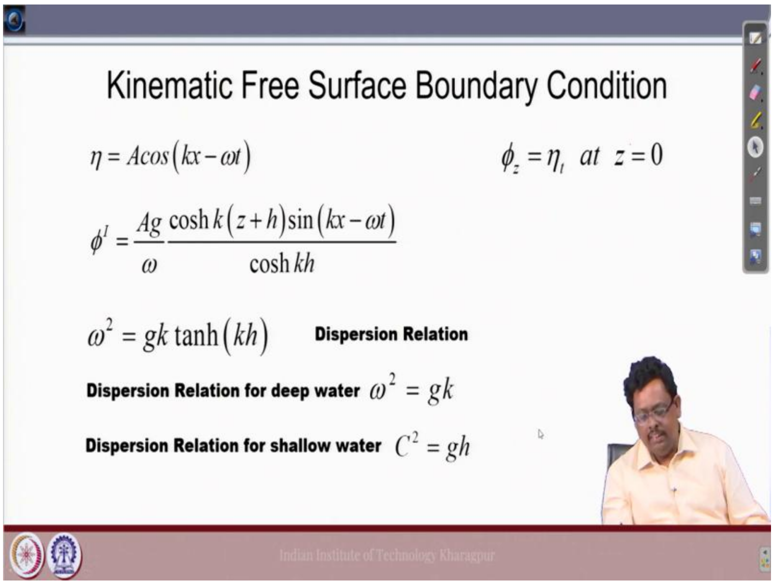

So, if I apply this another boundary condition what we get? Now, if I apply this kinematic free surface boundary condition and you can work out; that means, $\phi_z = \eta_t$ at $z=0$. If we use it, then you will get a relation between the omega and k which is called the dispersion relation right.

So, this is of course, very important relation for us, I mean and this is very important because this tells you very nice phenomena about the waves the difference between the water wave to sound wave or electromagnetic wave. But, at this in this course we really not going to discuss the physics part of it rather we are much more interested to the solving this problem. And so, therefore, we are going with the deep water situation when actually you can take h tending to infinity, then $\tanh(h) = 1$.

So, then the disperse relation for deep water equal to omega square g can you know we are going to use this results in future, because in ship hydrodynamics we are mostly dealing with the condition where you can assume that the water depth is infinity; I mean the. So, therefore, we are going to take this dispersion relation. However, the physics part is definitely very important and separately one can discuss in other courses.

But, here we are focusing only on the numerical part of it and also the dispersed relation for the shallow water is $C^2 = gh$, that also very limited take h tending to 0 then $\tanh kh = kh$. And, if you solve this you can get the $C^2 = gh$ this thing. But, we are not going to use this second thing, we are definitely going to use the .

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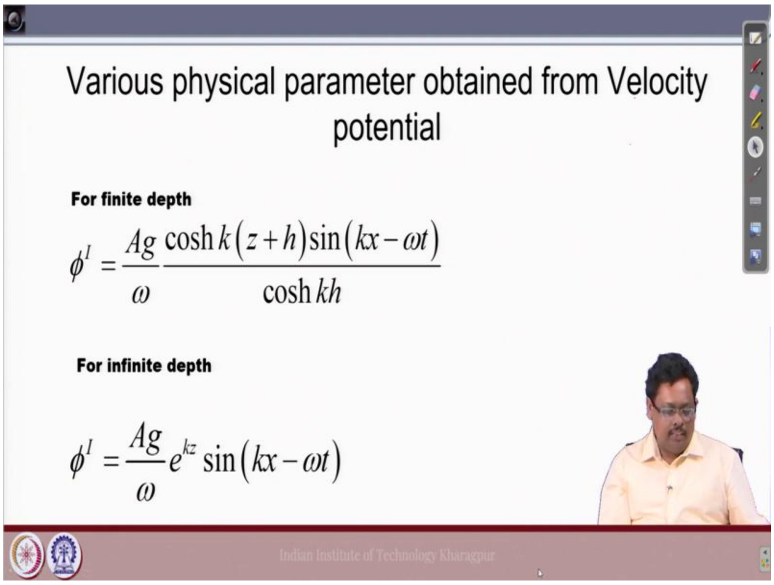

So, now so, this is to the velocity potential we are going to use, mostly I am not going to use this ϕ^I in the finite depth rather we assume our all ship structure interaction I mean ship wave interaction is on that deep water. So, therefore, we are going to use the incident wave potential ok.

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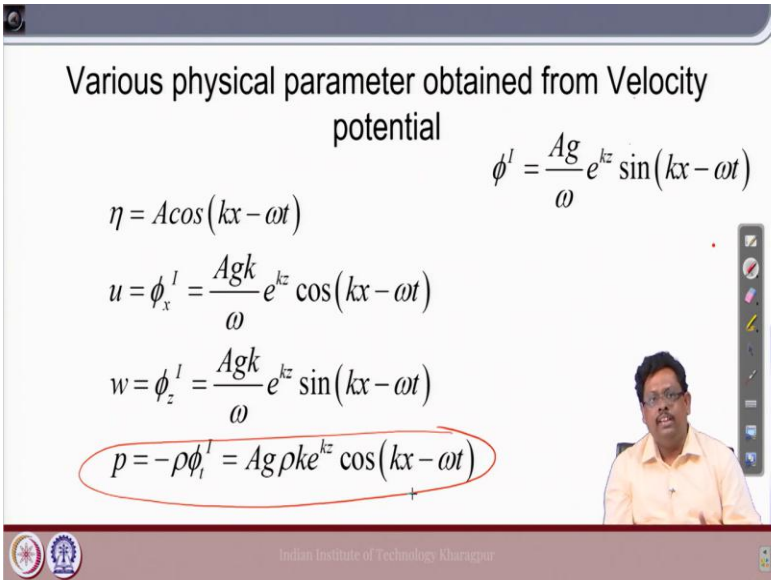

Now, let us see that how I can find out the various physical parameter from this velocity potential $\phi^I = \frac{Ag}{\omega} e^{kz} \sin(kx - \omega t)$ from here. Now, here this $\eta = A\cos(kx - \omega t)$ and then I can get the velocity in horizontal direction, it is $\frac{\partial \phi}{\partial x}$.

And so, this is the expression for $\frac{\partial \phi}{\partial x}$. So, I can get the horizontal velocity, I can get the w also; that means, the velocity along the z axis. And, also I can get the most importantly, I can get the dynamic pressure also using this relation $p = -\rho \frac{\partial \phi}{\partial t}$ and which is the expression is $Ag\rho e^{kz} \cos(kx - \omega t)$.

Now, this is extremely important for this course. This is how I can get the pressure around the body right. What is the pressure, because of this instant wave potential around the body and this is the expression? Now, once I integrate this pressure I can get the force and there that force is called the (Refer Time: 28:10) force and this is the one of the major force for my you know the ship wave interaction ok.

So, we understand that, now I have the expression for ϕ^I and I can get you know I can get the pressure over the body so easily, because analytical expression. And, if I integrate this pressure, I can get a force which is basically the (Refer Time: 28:38) force.

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Now, again let us go back to my; so, here some visualization that you know to tells you about the fact that everything cannot be you know peak at the same time ok. So, let us see here, you can see that that u velocity, w velocity, acceleration and acceleration at w direction; all is not getting peak at the same time, it has is a phase.

So, this tells you that importance of the phase, not necessarily that force is always all the forces is peak at the same time, because, sometimes that u velocity will be maximum, then w velocity is not maximum, when the acceleration in that horizontal direction is maximum that time acceleration in the vertical direction also not maximum. So, this visualization helps you to understand that what is the importance of the component of the forces, because all the forces not become peak at the same time ok.

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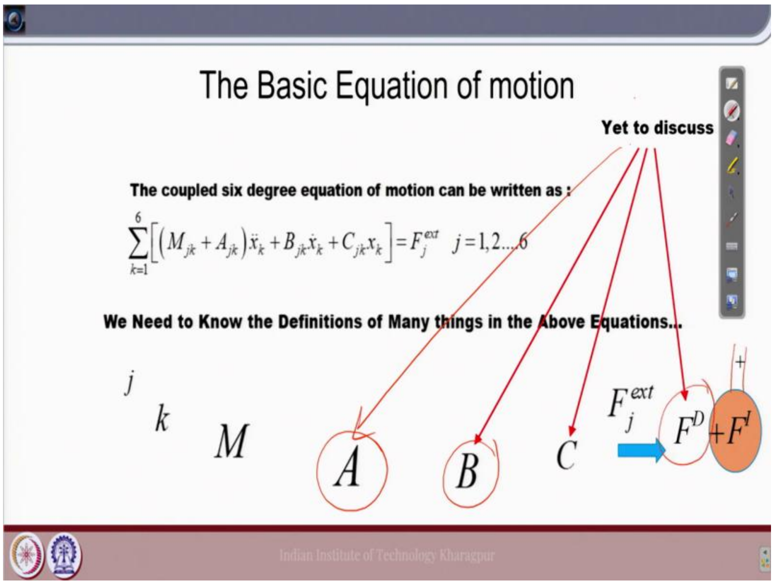

Now, with this let us finish here. Now, you can see this equation again I am coming, now here we now discuss all this component right. We discuss the j modes, k, M, A, B, C, x, now exciting force also. And, then this exciting force I now divide into different component ok, which is the force coming, because of this way which is called the (Refer Time: 30:18) force; this we just discussed. Once we get the ϕ^I, integrate the ϕ^I get this force.

But, we not yet discussed the other three things which is this A, B, C; we discussed, but we did not discuss the another component which is basically the A. So, now discussion of the A and discussion of the B is not yet done. However, discussion of the F^D also is not complete, only we discuss about this (Refer Time: 30:53) forces. So, in our coming lectures, we are going to discuss this how we can obtain the this A, how we can obtain this B and how we can obtain the diffraction force which is F^D ok.

Till this point, thank you.