Numerical Ship and Offshore Hydrodynamics
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Lecture - 15
Introduction to BEM (Contd.)

Hello, welcome to Numerical Ship and Offshore Hydrodynamics. Today we are going to discuss some basics of the boundary element method or panel method ok.

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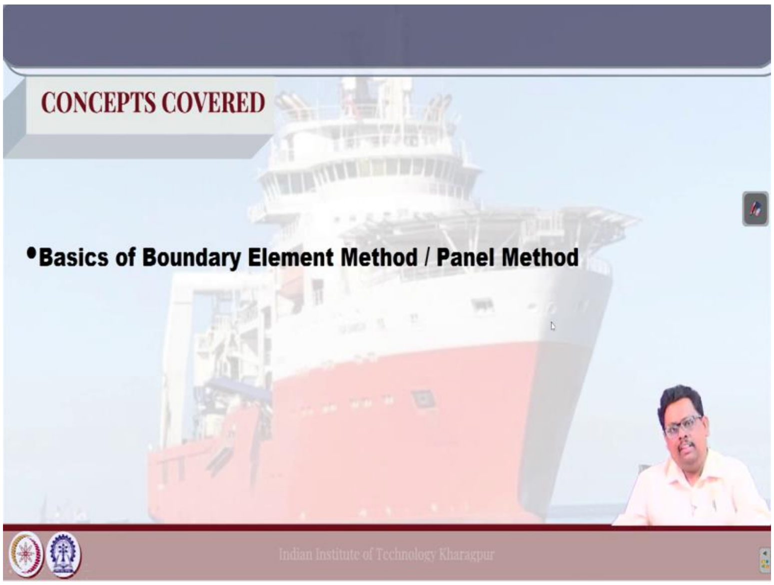

These are the key words that you have to place in order to get this lecture ok. So, let us start here.

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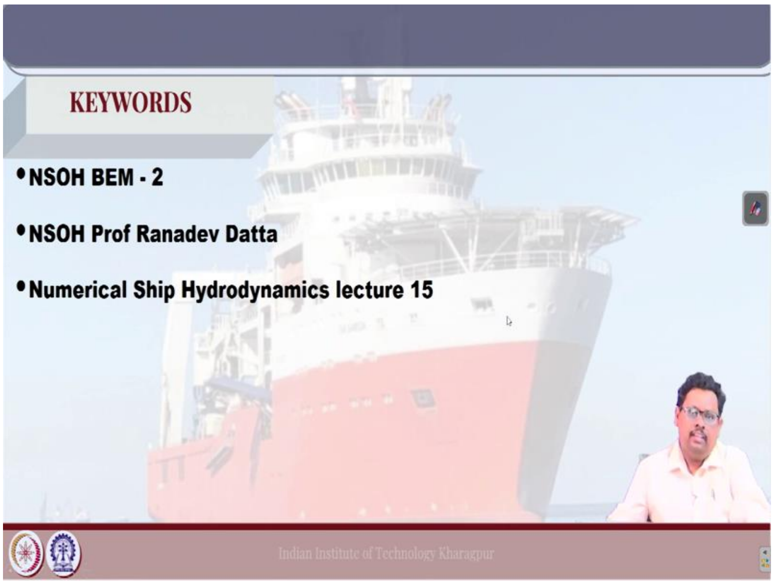

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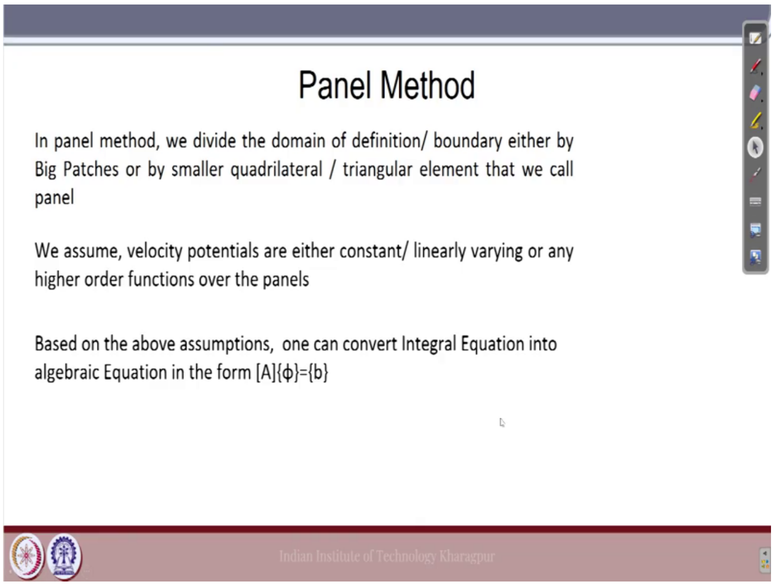

So, in last class I just typed these words like, in panel method, we divided the domain of definition or boundary either by the big patches or by the smaller quadrilateral or triangular panel like, but we really did not realize what I try to say. So, today we very carefully try to understand these statements ok. Now, when I say that this panel method, we divide the domain of definition or the boundary, so what is that mean? So, now, let us see when we start with this boundary integral equation method, we start with the equation which is ϕ P right.

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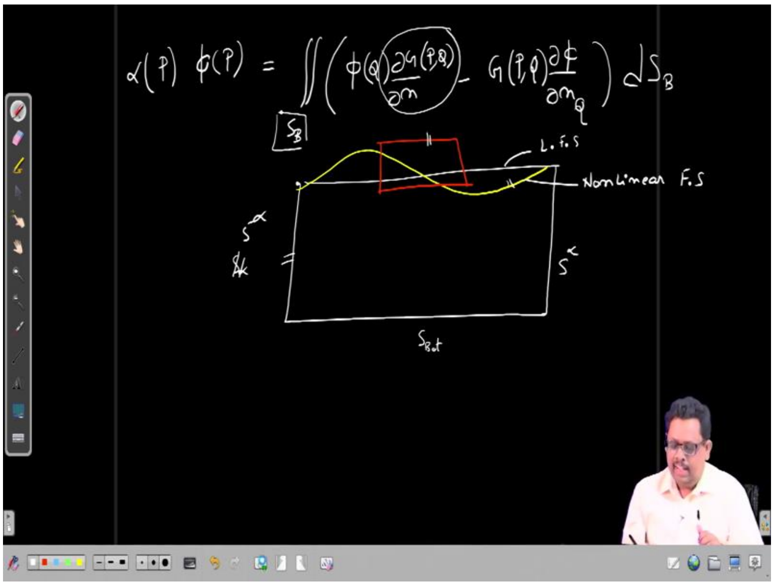

And you just if you want you can multiply it by the α(p) ok, this equals to with some boundary right some boundary. So, I just called S_B, B stands for the boundary. So, some boundary and then it is ϕ and then you can see that I just right here Q right. And then I write ∂G/∂n and then G also I write as G(P, Q) right and then - it is G(P, Q) and then multiply by the ∂ϕ/∂n and I called here as Q, and then I integrate over this surface.

Now, this is the integral equation that we are going to solve for this panel method. Now, very important that we need to discretize this boundary ok. Now, if I see, if I try to draw a let us say a 2-dimensional boundaries. So, we discussed this is my boundary and then may be the top part we can see that there is maybe we can consider this some waves are there right, and then in this waves you can put some body also here and we can say that this is basically my non-linear boundary, non-linear free surface and this is my linear free surface. So, we understand this.

Now, also we can call this as S infinity you can say because S - infinity in fact. You can call as S - infinity you can call as S infinity and this you can call the bottom or you can say, just I say this bot bottom. Now, here this S_B stands for all these boundaries. Now, here we need to discretize the whole boundary right; however, however based on the choice of this Green's functions, based on the again I repeat; based on the choice of the Green's function sometimes we only need to discretize the free surface and the body or sometimes we need to discretize the body.

If you remember I, in last class I said some mesh a 1, some mesh a 2; in mesh a 1 we have to discretize only the body, in case of a mesh a 2 we need to discretize not only the body, but also the free surface.

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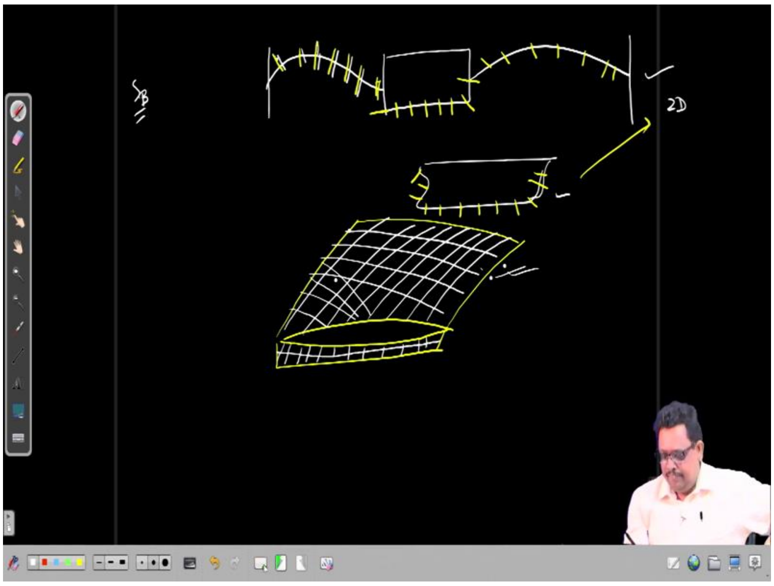

So, what happened? Finally, we have to either we need to model the free surface let us say and the boundary or we need to model only the body without the free surface. So, when I talked about this S_B. So basically, we are trying to say that we need to discretized either this whole thing or we need to discretize this one right.

Now, in case of a 2-dimensional problem this is simply a straight line, you can or you can take is a curved line also based on the fact that you know you are trying to do the higher order or lower order whatever. But essentially what you need is, you need to discretize the ok let me do it in yellow. So, you need to discretize this free surface by some small small you know lines like this.

So, these are the your weighted surface, because I really do not want anything above the water. So, I can leave the top part of this body and then this one right or you have to discretize only the this body. Now, this is in case of a 2D, but in case of 3D it becomes a surface. So, we can assume that let me draw a surface like this. So, just draw a small 3-dimensional field something like this and then you can draw some kind of a surface like this and we can consider this as your free surface.

So, then you have to discretize or you have to mesh the whole free surface and as well as you need to discretize the body. So, you can see that; though it is a very bad grid, but just try to understand the concept what I say that in this case you need to discretize the not only the body, but also the surrounding free surface, fine. Now, how you are discretizing this body as well as the free surface.

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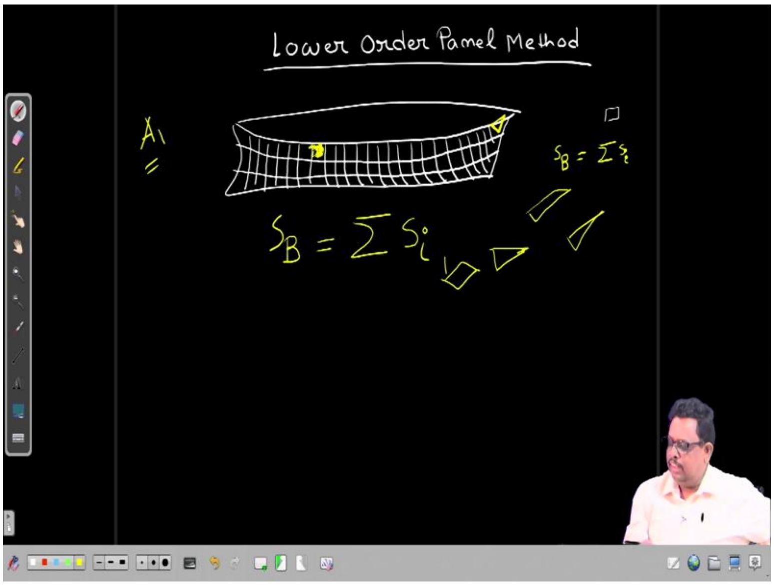

Now, there are lot of options are available. Now, if I try to do this, with lower order panel method which is more popular, if you try to do it with the lower order panel method then actually you are discretizing this whole domain using some sort of quadrilateral. So, we cannot say it should be typically the rectangle, it is not possible right because you can see this part actually you really cannot do some rectangular stuff right. So, maybe you can you have to do.

So, now, I if I even if I discretize like this way some sectional lines these are some sectional lines as I mentions. So, I can draw some section lines, so you know this since it is symmetric. So, you can only one part you can mesh and you can define that is symmetric. So, other part can automatically one can take care and then this is you can see it is a water line.

Now, you see here what I did is as follows, I am making some small small quadrilateral right and therefore, I discretize. So, this S_B the whole discretization let us take this method A_1 only, where you need to discretize only the body let us take that, let us go with this. So, then what you need to do is this S_B we can see it is sum of this small small you know areas S_i. So, finally, I understand what is the meaning of the discretization, it is this domain this computational domain I discretize by some small small areas, S_i or quadrilaterals.

Sometimes in let us say in this situation you might have some rectangular panels also. Because this corner part you cannot handle nicely, with this quadrilateral. Suppose if you try to do the quadrilateral its become so thin like this. So, instead of doing this you can assume that to be an triangle. So, therefore, this S_i should be the combination of any type of quadrilateral, not any type of certainly like we have to take care many aspect that we are going to discuss in coming days or it could be some kind of triangular.

Now, unlike like CFD, normally they can do some hexagonal mesh also, but here normally we do not do that. Either we go with the quadrilateral or we go with the triangle ok. So, when I talked about the first part, so now, it is clear when I say that we divide the domain of definition or boundary either by ok, now big patches I missed. So, let us go back again to see that what is the meaning of the big patches.

Now, instead of this for example, we try to draw a circle. Now, how we can draw circle numerically, though it is not a perfect circle, but how we can do that numerically? What we can do is we can take very very small small segment, right.

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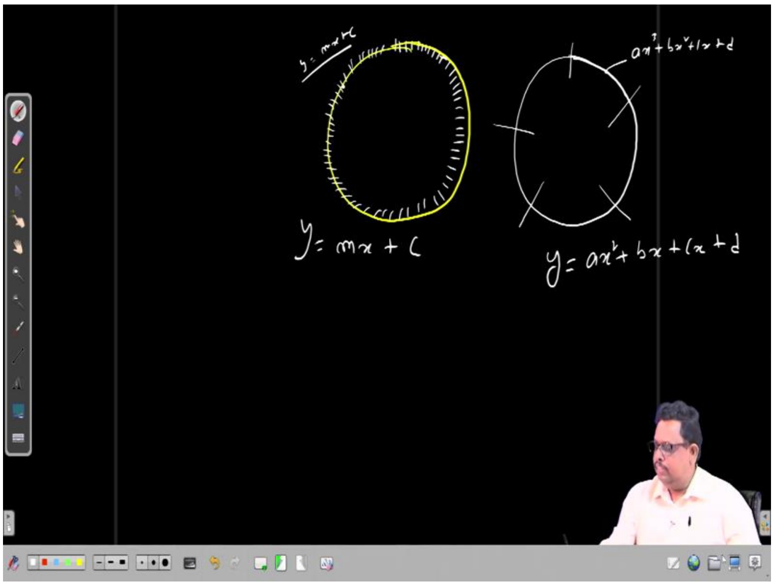

You can take very small small segment here and then you can approximate this joining these two segment by a straight line. Now, if this segment are small enough. So, at least visually you cannot make out that whether it is a circle or it is a polygon. However, you can still you can draw this circle numerically using some you know quadratic function or cubic function anything with a big patches.

So, this big patch may be some, let us say some cubic function it is let us say $ax^3 + bx^2 + cx + d$ something like this. So, similarly this one, this one, this one, this one. So, instead of straight line which is $y = mx + c$ right, just let me write in bigger way. Instead of $y = mx + c$ you are using some higher order function $y = ax^2 + b x + cx + d$ something like this.

So, this is the difference between the big patches and the small segment. So, when you approximate by a straight line, then you can you have to take this segment is really small so that visually you cannot make out that whether this one is circular or polynomial. However, in if you asks for the higher order functions then the same thing you can draw with the lower number of panels. So, segment may be lower because you are right you know already take care about the curvature part right. So, this is what we say when we talked about the big patches, ok.

Now, second part is we assume the velocity potentials are either constant or linearly varying or any higher boundary, higher order functions over the panels. Now, what is the meaning of this? So, now, let us try to understand when we say that the velocity potentials are either constant or linearly varying function or any other higher functions ok. So, again let us go back here and let us try to find out what is the meaning of the when I say that, that ϕ we assume the distribution of the ϕ either constant or linearly varying function or higher order function; what is the meaning of that?

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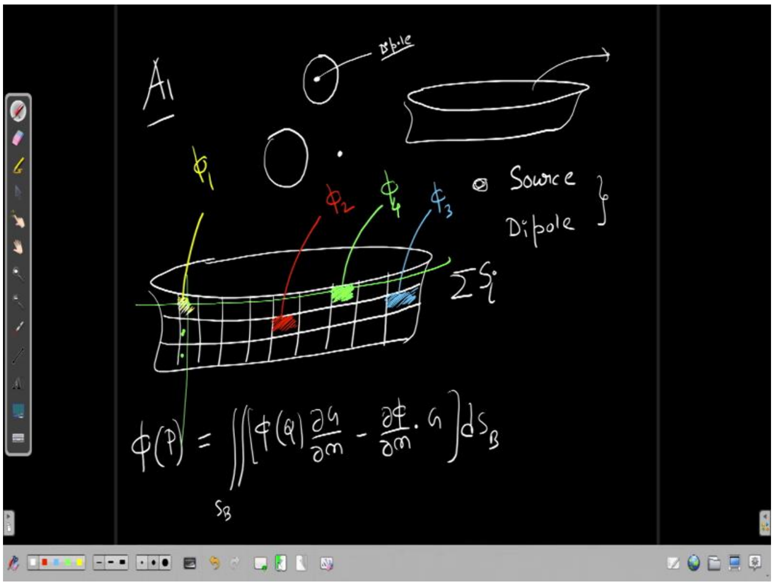

Now, if you remember that last class when I talked about that to find out the flow past cylinder and we put here a dipole right. And that means, that we claim that if we put a singular function instead of the body. So, you can have the same effect as if there is a body right, because we replace see we really do not put any cylinder when we talked about the flow past cylinder. We replace the cylinder and then place a dipole at in there right.

So, this is the idea, at then we argue that in case of a ship also if I try to find out the velocity potential over a ship, then instead of the ship I replace the ship with the with some either some source function. So, let me write in this way either some source or some dipole or maybe the combination of both. So, that was the argument.

Now, let us go with this method A_1 in method A_1 we need to find out this problem and then we discretize my body right. And then how do I get the ϕ over here? Now, if you look at this integral equation, let me write over again. The integral equation this $\phi(p)$ is equals to this over this S_B right, and then you have $\phi(Q) \times \partial G/\partial n$ - $\partial\phi/\partial n \times G$ and then we have to integrate over s in dS_B. Now, see here I discretize the body definitely, I discretize the body. So, I understand that how I discretize the body is the summation of S_i. So, I discretize the body.

Now, we need to find out then in this discretization how ϕ is varying, right. Now, you see here the value of ϕ cannot be same, the value of the ϕ here. Similarly, the ϕ is varying different value in this panel also, this ϕ has different value in this panel also. So, then how we can understand that how this ϕ is varying and actually how I can you know approximate the value of ϕ over here ok.

Now, the popular way of doing it, we assume that for each panel the value of ϕ is different. Now, if you assume in this yellow one, the value of ϕ let us say ϕ_1 and may be the red one the value of ϕ may be ϕ_2 and again in the blue one that value of ϕ may be some ϕ_3 and let us the shaded something in green this one, this green then then value of ϕ here may be you know ϕ_4.

Now, remembering this this choice of ϕ_1, ϕ_2, ϕ_3, ϕ_4 it is arbitrary can be arbitrary ok, not necessarily this ϕ_1, ϕ_2, ϕ_3, ϕ_4 should be in order, when I call it should be in order I mean that not necessarily that ϕ_1 may be here, then this may be ϕ_2 this may be ϕ_3. So, I mean this all this ϕ are arranged in column fashion or all the ϕ may be arranged in the row fashion, no not required.

So, if the choice of ϕ may be arbitrary, but what I say is this ϕ is constant over a panel. So, what I say here as follows I say this value of ϕ_i is constant for this panel S_i ok.

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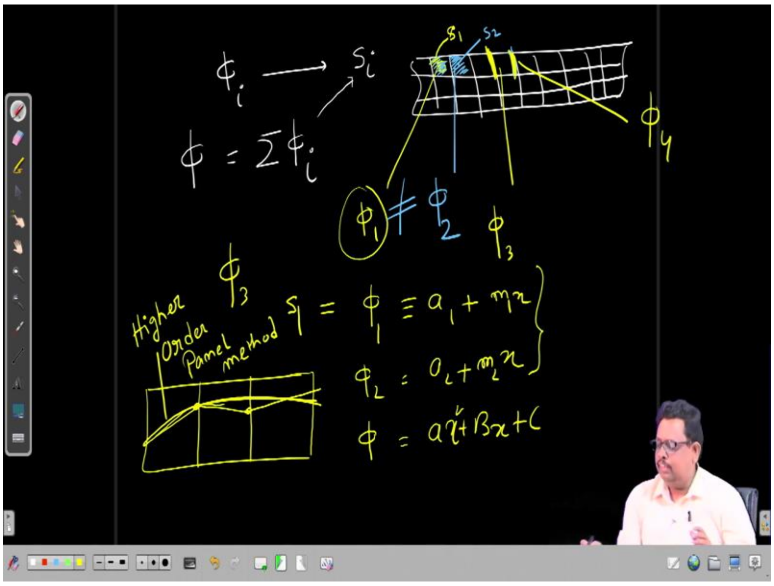

So, it means that this ϕ also I can say that this ϕ also the summation of some small small ϕ_i and this ϕ basically, this ϕ is constant over some area, but the different from the different area, what is the meaning of that very simple, this is my ship. Now, I just do this for the 2-dimensional like, you understand right this is the patch just do the profile view let us say. So, what I call this, this value of this ϕ_1 it is only associate with the area S 1. So, over this ϕ, I mean sorry over this area this ϕ_1 is constant.

So, you understand this right; however, just next area let us take let us take now let us make it in order, ok. So, now, maybe the this one is basically that is second area S_2 and then then this value for here may be it is called as ϕ_2. Now, this ϕ_1 is not = ϕ_2. However, here in whole area S_1 everywhere the velocity potential is ϕ_1 and this is called constant panel method.

It means that for a particular area ϕ is constant, for the different area ϕ is different area ok; that means, for one panel the throughout the panel the ϕ is same for the different panel ϕ is different. Then one can ask I mean normally people ask at this particular point, then what happened in the boundary what is happening in the boundary which is very valid question right. Because in this boundary let us say this side I have ϕ_3 and let us say this side I have ϕ_4, then what happen to the boundary?

We accept that in the boundary we have a discontinuity because in the left-hand side we have ϕ_3 in the right-hand side we have ϕ_4. So, in this boundary definitely there will be a discontinuity again second level of approximation is as follows. We claim that even if there is a discontinuity it is marginal or it is ignorable. So, we ignore this discontinuity.

But if somebody is very sincerely and if he says that no I do not allow the discontinuity in the this boundary, then they can assume let us take in S_1 area, this definition of ϕ may be definition of ϕ_1 that may be some a 1 some linear function. Some $a_1 + m_1 x$ kind of thing, some linear functions.

So, and then maybe the ϕ_2 = some a_2. So, it is m_1 x it is some linear function m_2 x, something like this. So, then what by doing this you can avoid the discontinuity at the you know at this boundary, because this is also linear this is also linear. So, if I see closely let us say there are 3 panel and if you assume this variation of ϕ is linear. So, you can say that it maybe you know something like this and then you can make sure that you know this part would be the continuous ok.

So, this also consider as lower order panel method, when either we can assume ϕ is constant or we can assume ϕ is linear. Now, suppose still you are not happy and then you can say that no I want this ϕ to be the quadratic function. So, it should be some a x^2 + b x + c something like this ok. So, therefore, at this point you know there is you know it is absolutely a nice thing, absolutely there is no question of the discontinuity at the boundaries at this particular in this case.

So, this called as higher order panel method ok. So, now, it is very clear in case of a lower order panel method we assume that ϕ to be constant and at least linearly varying, but if it is a higher order function some cubic function. Some quadratic function you know many people takes this as bist line function also in that case we called this panel method is higher order panel method. But in this particular course we are doing with the lower order panel method and therefore, we assume this ϕ to be constant of a panel, I mean same panel and different for the different panel.

So, now let us coming in the third point, now this third point again we need some more time to understand this based on the above assumptions one can convert the integral equation into algebraic equation in the form of A_ϕ = b. So, this needs some more time to realize ok.

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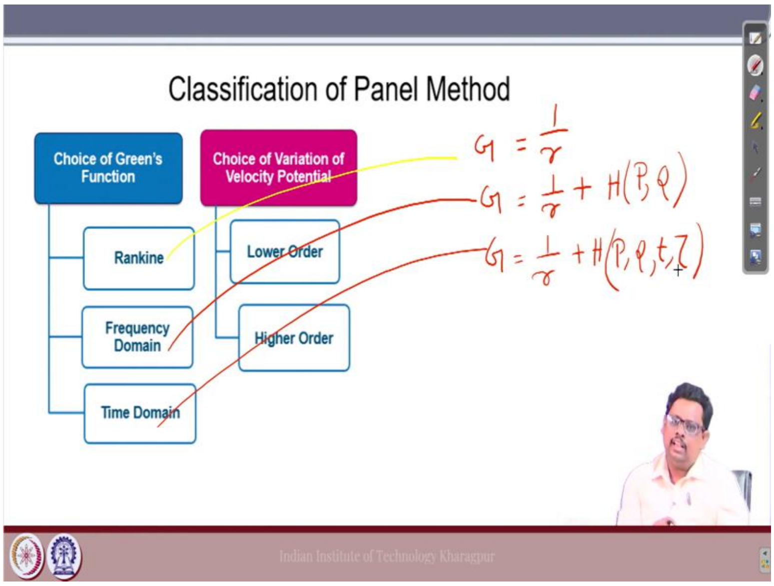

Now, as I mentioned that classification of the panel method, they can divide into two part. Now, the second part we understand what is the called the lower order panel method and what is called the higher order panel method. And then in the right hand side is basically based on your choice of the Green's function, as I said that we can take many type of Green's function to solve the same problem ok.

Now, here very briefly let me explain that when I call that different type of Green's function what is actually this Rankine. If you talked about this Rankine, the Rankine's Green's function, we take this G the greens function this equals to some 1 / r something like this. So, this is called the Rankine.

Now, in frequency domain Green's function what I do is normal I take G = 1 / r + we can take some function H, which is basically depending on the point P and Q, where P is the point any point in the domain where Q is the point on the body. And if you talked about the time domain then Green's function is can be taken as this Rankine part plus.

Now, we can now assume this this second function is not harmonic, in that case it is harmonic. Now, it is P, Q and also it takes some parameter t and τ. Now, what is this parameter (P, Q, t, τ) that definitely we are going to discuss when you discuss the time domain panel method. But you know categorically this Green's function can be divide into three part, one is Rankine, now Rankine Green's function can be solved in frequency domain on time domain.

Already, if you assume the second part is harmonic or you can say the pulsating Green's function, where the Green's function is harmonic. So, you can call it is a frequency domain Green's function and if it is you know impulsive Green's function it is called the time domain Green's function.

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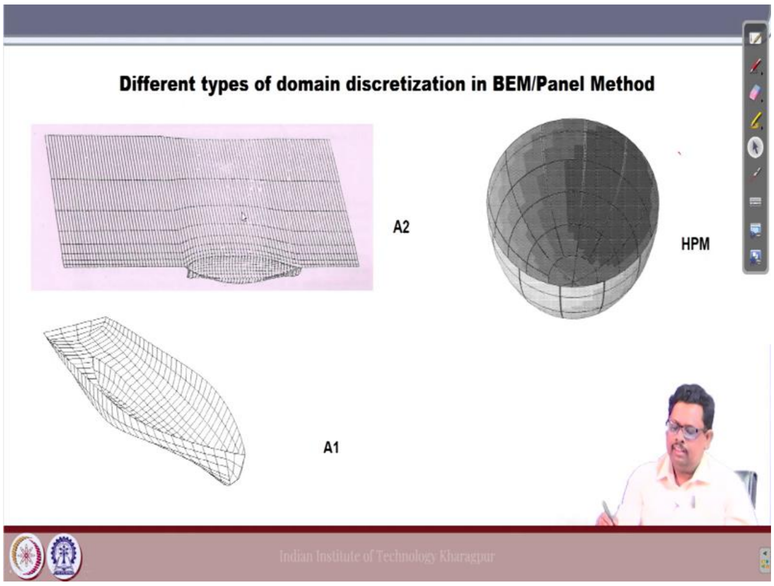

So, now, it is the same now we understand this all. Now, in case of A_2, we have to discretize the body as well as the free surface. In case of A_1 we have to discretize the body only and in case of higher order panel, now we can see that these patches are really big comparison to the other two mesh right, ok.

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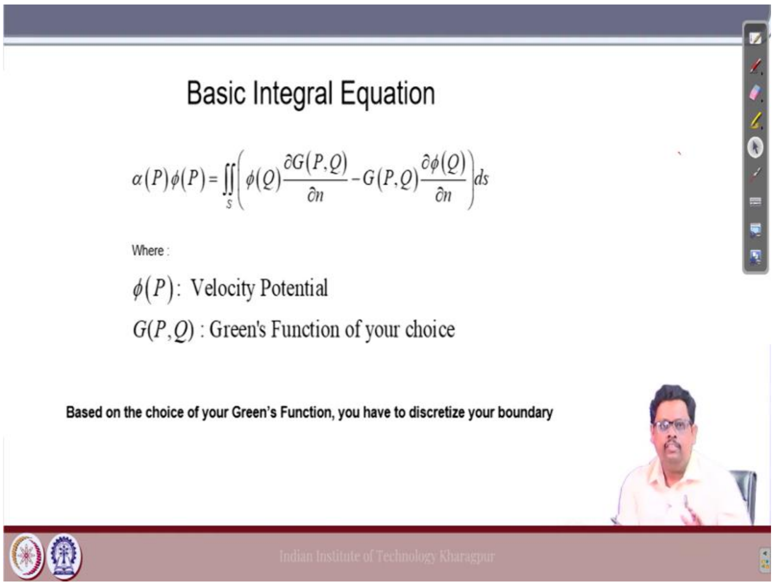

And this is the basic Green's integral equation we are going to discuss from the next class and how we can solve this problem right. And now, I understand what is ϕ(P), what is G (P, Q) and what is the s everything ok.

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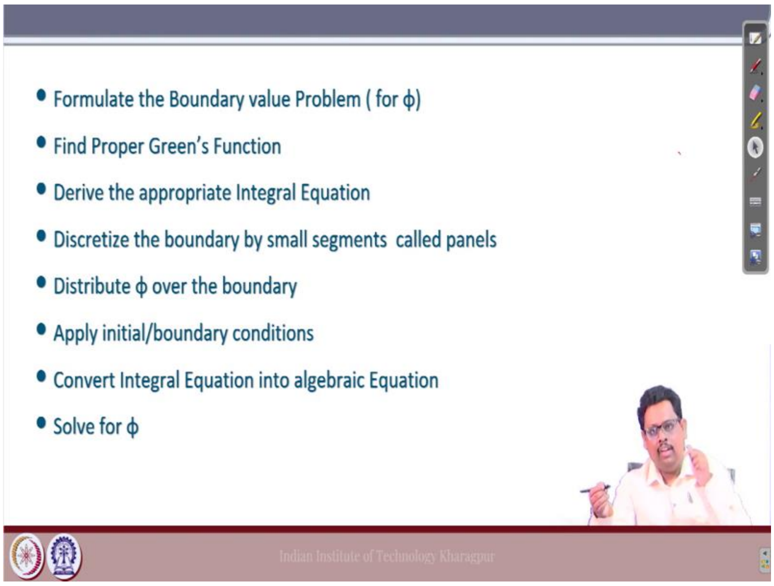

So, the idea is formulate the boundary value problem for ϕ and then we find the proper Green's function this remember this, because we are going to discuss again next class the same thing. Now, these things again need some kind of discussion that we have to going to discuss in the, when you solve the practical problem. In the next class we try to solve a problem for a infinite domain radiation problem, that time one by one we are going to discuss all these things ok.

Thank you.