Numerical Ship and Offshore Hydrodynamics
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Lecture - 16
Lower Order Panel Method

Hello, welcome to Numerical Ship and Offshore Hydrodynamics.

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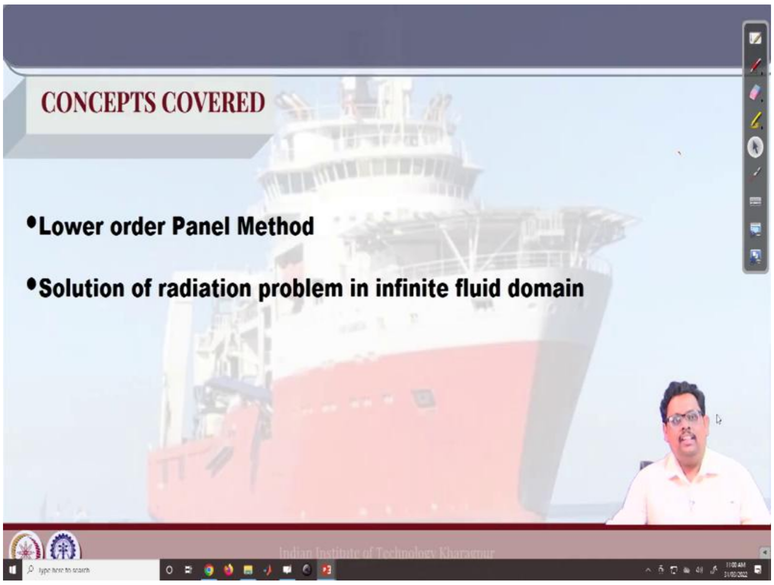

So, today, we are going to discuss how to solve a radiation problem in infinite fluid domain, considering Lower order Panel Method.

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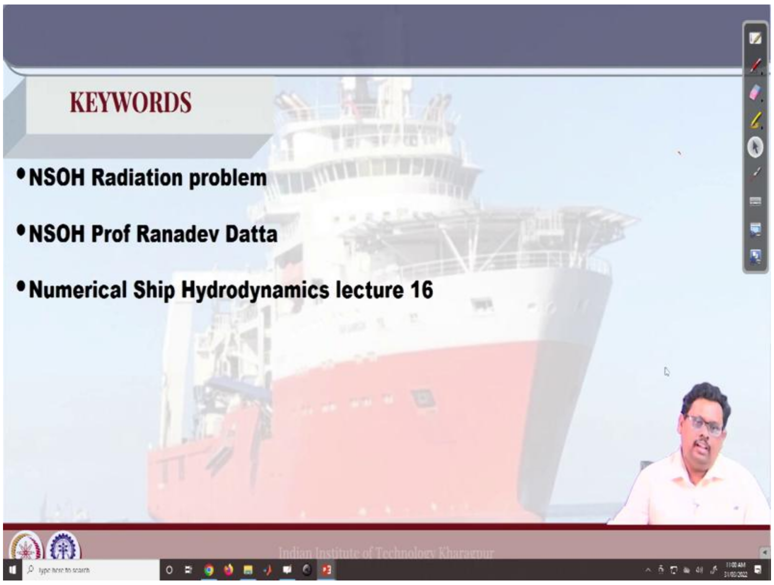

And these are the key words that you have to use to get this lecture ok.

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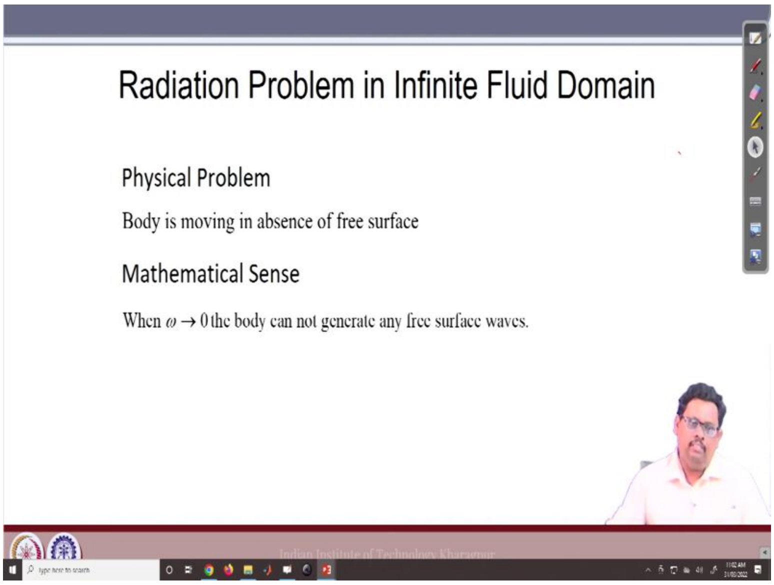

So, let us start. Now, what is the physical problem, we are going to discuss. We are going to discuss that body is moving, of course with acceleration in absence of free surface. So, we assume that the whole domain, there is no waves. So, you can consider may be not exactly the same; but you can consider that moving of a submarine inside the ocean, like with some acceleration of course. So, this is the physical problem.

Now, mathematical sense, what you can say that when Ω tending to 0, so we can approximate very long waves and this can be approximated as there is actually no free surface and therefore, the entire domain, we can call as a infinite fluid domain ok. So, let us see how we can solve this problem with the help of lower order panel method ok.

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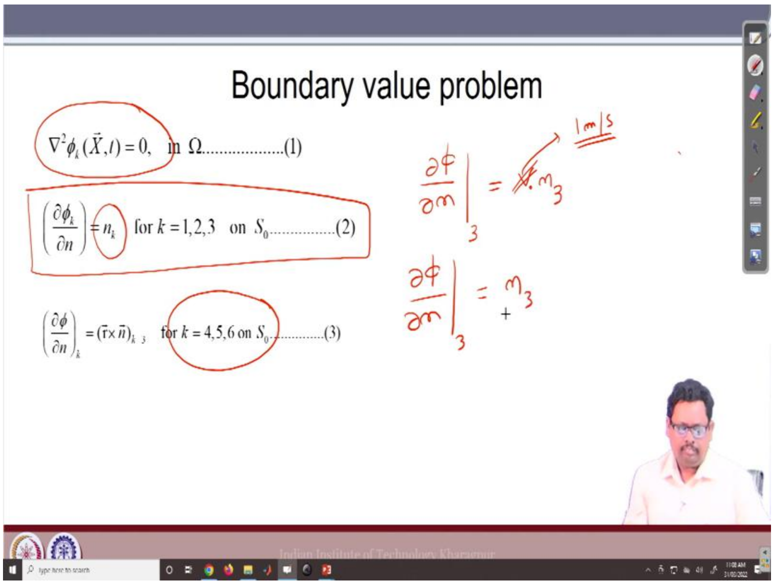

Now, if you remember that first point is to find out the appropriate boundary value problem. Now, here this is the boundary value problem for this infinite radiation problem. Here of course, that governing equation is nothing but this Laplace equation. So, this is my, the governing equation right?, Everywhere.

Now, what are the other boundary conditions? Remember that this is an infinite fluid domain, so here the free surface is absent. So, therefore, we do not have to apply the free surface boundary condition and also, we can assume that the bottom of infinity ok. So, there is actually no bottom, we can approximate. So, we can omit that bottom boundary conditions.

So, also the radiation also is infinity and if you remember, our previous videos, we showed you like we are dropping something if I drop something on the water, we can see that in the free surface, we can have waves. However, if the body is oscillating inside the domain, when there is no free surface; when the body oscillate, the surrounded fluid particle oscillates with the body or moves with the body. The moment the body stopped, therefore, the water particle also stops. So, therefore, there is no possibility of the radiated wave, right? So, there is no point of the radiation condition.

So, no wave is going till that, I mean till the infinity and die down right. That is what is happening in case of a free surface, if you drop a stone and then this wave got created and slowly slowly, it radiated at the infinity. However, in case of infinite fluid domain, the moment the body stopped, similarly the water particle stops. So, really we cannot see any kind of such wave. So, we do not have the radiation condition also. So, therefore, the only condition that we have is the body boundary condition; this.

Now, what is the body boundary condition? So, if you remember that our kinematic body boundary condition, we said that the normal velocity of the body should be equal to the normal velocity of the water particle; otherwise, what will happen? This water particle will coming out of the body right.

I mean if you remember that I explained that with the simple explanation of this, this pen with in my hand. If this has different normal velocity, now if the normal velocity of the pen is more than the normal velocity of the hand; what is happening? As long as I move this hand up and this pen is coming out of the surface of my hand.

So, therefore, this is something called the body boundary condition and we can say it is a kinematic body boundary condition and if you carefully look into this, we can see that now why it is n_k ok. So, let me explain it. Now, the radiation problem also, you know what is the radiation problem? Radiation problem is that you have a body, you are oscillating the body in a calm water. Now, how many ways you can oscillate your body? Now, if you consider this body to be rigid, so let me take this mobile and I can show you that how many ways you can oscillate this body.

So, now, this body can be oscillate this mode which is called the heave, you know it can move this way which is surge, also this way this sway and also we have this pitch, this roll, yaw everything. So, it can actually oscillate; you can see here, it can actually oscillate in six different way right. Now, suppose it oscillate in some mode; let us take heave. So, what would be the body boundary condition in that case?

So, in case of a heave mode, the normal velocity $\frac{\partial \phi}{\partial n}$ in the direction of heave, so I can write in 3 because if you remember that is the notation that we are going to use; 1 for surge, 2 for sway, 3 for heave and so on. And this should be is equal to the velocity of the body in the direction of the normal, direction of n_3.

Now, for sake of simplicity, I assume that velocity is unit. So, you can assume that velocity is nothing but let us take 1 meter per second. If we assume this, so definitely you know this equation 2 automatically comes right. Because in this case you assume that velocity equals to 1 right. So, that is why you can see here this equation 2, the body boundary condition is $\frac{\partial \phi_k}{\partial n}$.

Now, what is *k*? Now, this *k* is defined is 1, 2, 3 right. It is its *k* is 1 equal to surge; k equal to 2 equal to let us say. So, a 3 equal to heave, it could be anything right. So, therefore, this let us take let *k* = 3. So, that means, I am oscillating the body in the heave mode or I am moving the body in direction of the heave. So, therefore, in that case, I can write this $(\frac{\partial \phi}{\partial n})_3 = v.n_3$.

Now, if you consider $v = 1$, so definitely you can have del $(\frac{\partial \phi}{\partial n})_3 = n_3$ right. So, we understand the equation number 2. Now, in case of the moment how we can write the body boundary condition? Definitely it is $(r \times n)$ right. So, therefore, in case of let us say that when the mode is 4, 5 or 6; when you consider this mode is 4, 5 and 6, then how I could write the body boundary condition?

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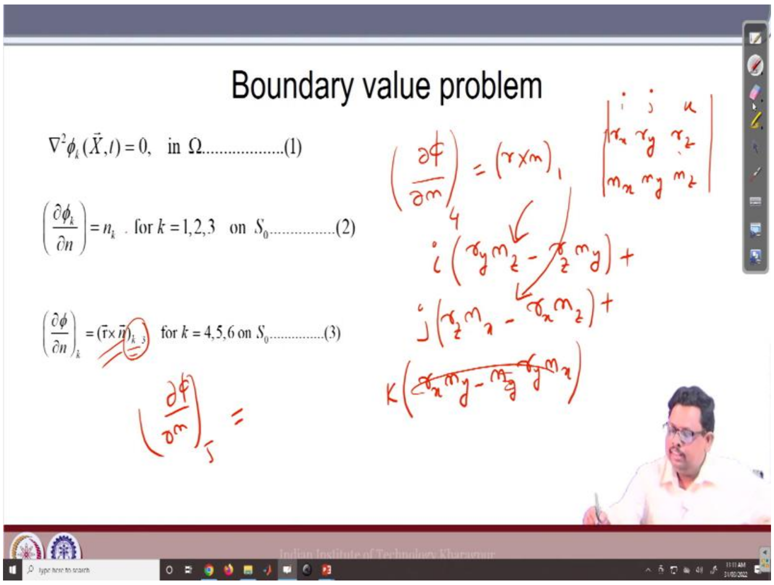

Now, inside the water particle, now see it is now I can say let us say $\frac{\partial\phi}{\partial n}$. Now, let us take it is in the fourth which is the roll. How we can write it? Now, to get this, we have to find out that this $(r \times n)$ right. Now, you know that it is basically i j k and you can call it is r_x r_y and r_z and then, it is let us take n_x n_y and n_z.

If it is this is so, so then you can take the component of i is nothing but $(r_y n_z - r_z n_y)$ and then, plus you can take the component of j, this nothing but $(r_z n_x - r_x n_z)$. And then, you can take the component of k which is equals to $r_x n_y - n_y$ into sorry $r_y n_x$.

So, this is how we can get. Now, this i^{th} component is nothing but in case of a roll. So, now, you can see that it is the first component in this equation. So, basically a (k-3) is nothing but the 1. So, it is nothing but $(r \times n)$ in the first component which is the i^{th} component. Now, if you take for the pitch, so then this $(\frac{\partial\phi}{\partial n})_5$ should be is equal to it is (k-3) which is (5-3) which is the second component; that means, this the j^{th} component and then, (i-3) for the yaw.

So, in this way actually this is moving right ok. So, now, this is also clear. So, now, I understand fully what equation 1 is; what equation 2 is and what equation 3 is.

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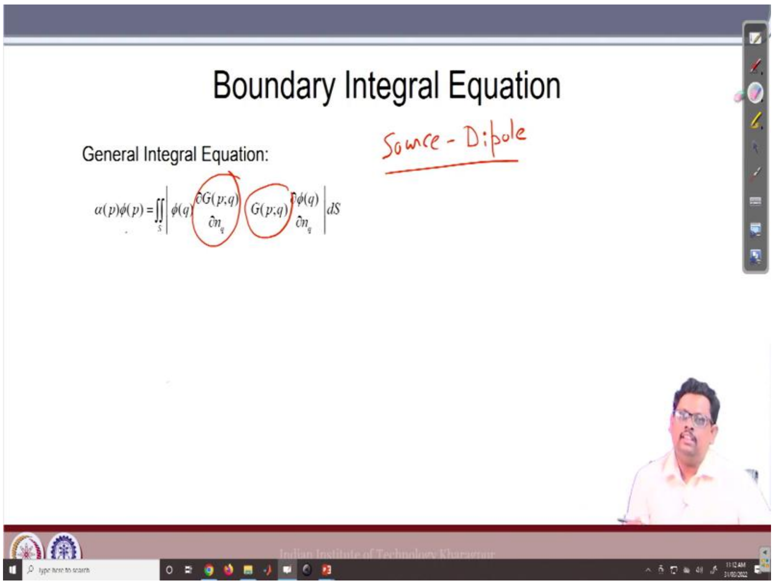

Now, the second thing if you remember, the second part of that you know that I listed in the last class or you know repeatedly where one by one you were describing this. So, in the second part is that you have to choice your Green's function. Yeah, it is a third point; second point is the find out the integral equation.

Now, you know either you can choice your Green's function that could be also second that is what I said. So, it is not very strictly you have to follow the thing; you can first choose the Green's function, then you can choose the integral equation. Now, at this moment, we are dealing with the source dipole integral equation right.

Now, you see here this part we can call the source; this part we can call the dipole. So, you can call is a source dipole; source dipole. Now, there are other types of Green's I mean integral equation also exists, something called the only-source distribution, something called the only-dipole distribution. So, what is only-source distribution, what is only-dipole distribution that we are going to discuss in the in a coming days.

Now, let us go with the source dipole distribution ok.

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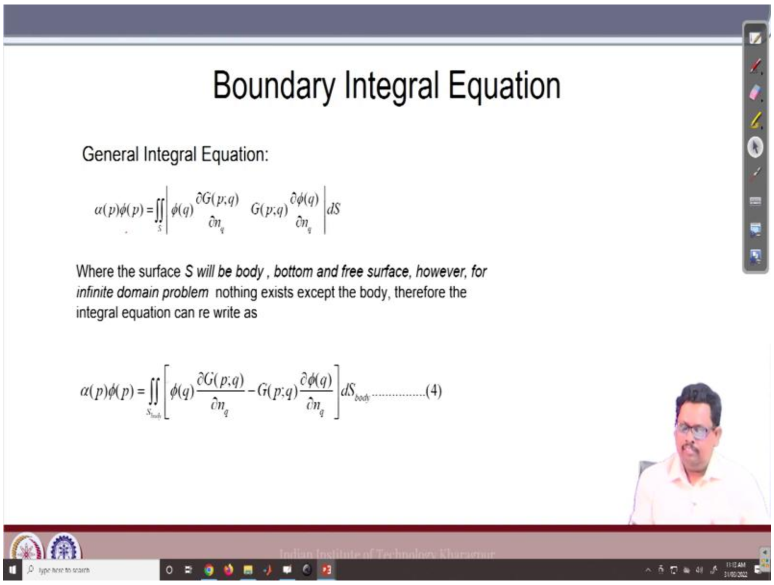

So, this is my integral equation. Now, what about the S. Now, S, now in this particular case, the since there is no free surface, there is no bottom surface, there is no radiation surface. So, only surface here is basically our body. So, therefore, this S now you know considered as the S body. So, I know I have this integral equation also with me right. I have this boundary value problem and I know what is the body boundary condition and now, I know my integral equation also right.

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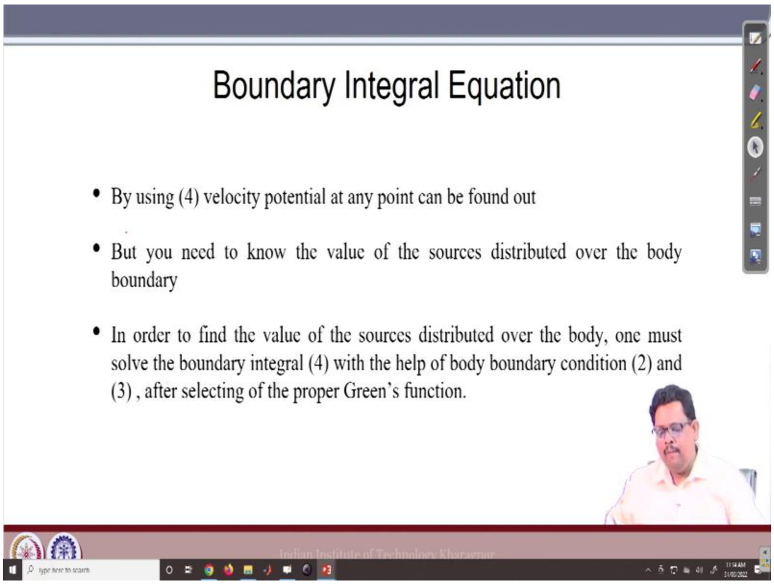

So, if it is so, then you know this equation 4 can be you know if you use this equation 4, you can get the velocity potential at any point on the fluid domain right. However, you know to know this, you need to know the velocity potential on the body because if you remember this integral equation, the right hand side the integral sign is over the body. So, unless you know the velocity potential on the body, you really do not know how to get the velocity potential at any point on the fluid domain.

So, therefore, in order to find the value of the source distributed over the body, one must solve the boundary integral equation 4 with the help of body boundary condition 2 or the body boundary condition 3 and that is after the selection of the Green's function ok.

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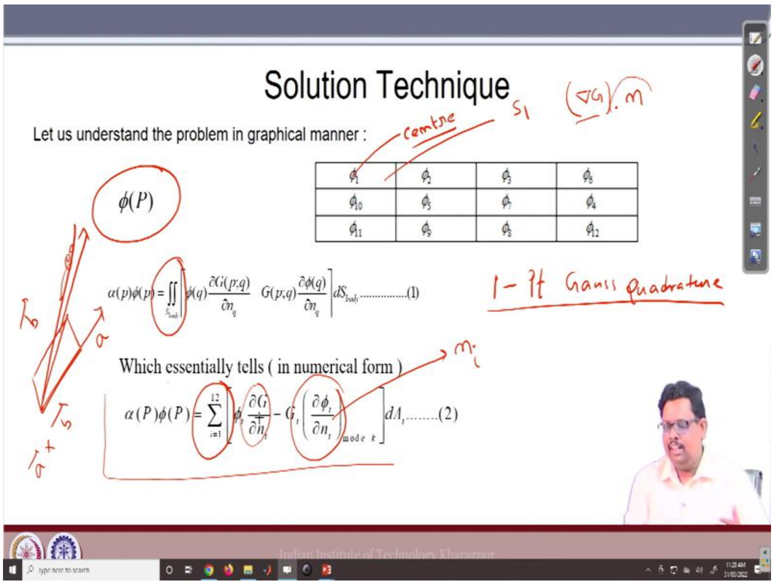

Fine so, now, let us we take that the choice of the Green's function. Let us leave for this moment and let us try to understand what I said graphically. Now, this is $\phi(p)$ is the point anywhere in the fluid domain; this one and suppose this one is the body or the ship, now you try to find out the pressure of any point over this fluid domain. Now, this whole this white board is your fluid domain ok. So, then you need to know the value of all this ϕ; this ϕ_1, ϕ_2 etcetera.

Now, you can see here, I make this ϕ in order; but it is really not necessary ok. A ϕ_1 could be here and there ϕ_2 could be anywhere in this surface; anywhere. So, then however, if you want to find out you need to know the all this ϕ_1 to ϕ_{12}. Let us say that body at discretizing, let us say that 12 areas and we can call this 12 panels right.

Then, how we can do this? So, we need to apply this integral equation of course, to get this right. Now, you see here I have $\phi(q)$. So, I do not know what is the solution for $\phi(q)$ right. So, in this integral equation, how I can use this integral equation to get this ϕ that is ϕ_1, ϕ_2, ϕ_3 that is the actually that is our main aim or goal right.

Now, before further we understand how we do that, let us try to find out how we can get this value at any point at any point $\phi(p)$ with the help of I mean for this moment, let us assume that I know all this value ϕ_1 to ϕ_{12}.

Then, how can I get the solution for this particular problem so that I can get the velocity potential ϕ at any point in the surface. Now, you see what I do is I replace this integral sign with a summation sign right and here, we are using the 1-point Gauss Quadrature rule. So, we can say that we are using 1-point Gauss Quadrature. Now, this is the simplest one we are using to understand the concept. So, what I do here actually? I split the whole surface into 12 small surface and then and each surface, I place this ϕ_1, ϕ_2, ϕ_3 at the centre ok or you can say at a centroid.

So, I know the location of the centroid also because I know the exactly geometry of this of this, let us say surface S_1. So, in this surface S_1, I know exactly the location of the ϕ_1 which is basically center of this S_1 and then, at this particular x y z location, I know the value of ϕ_1; let us say I know the value of ϕ_1 and also, at this point, I know my this value also because G is known to me; G is any Green's function.

So, if I know the Green's function G, so definitely I know what is the value of grad G dot n; where, I know because if I know my geometry, I know the 4 point and I know how to find out the normal right. We discussed a lot you know. So, if I have a quadrilateral panel I take 2 vector, I take a cross product, I get the normal right. Again, if you forget this, let me again explain this. So, if this is my panel. So, I define a vector over here, let us say a. I define another vector, not here another vector over here with passing through this corner point, I can call it as a b and if I take a cross b, I get the normal vector to this surface. So, I know my n.

So, entirely I know what is my $\frac{\partial G}{\partial n_i}$ and also, I know my G because this is that is what I select; either I can select the Rankine panel method or Rankine that 1 by r or I can take 1 by (r + h) or I can take 1 by (r + h) which is p, q and t anything right. So, if I do that, then the G is known to me.

Once G is known to me, this grad G known to me. When the grad G is known to me, I know the grad G dot n. So, $\frac{\partial G}{\partial n}$ is known to me. Similarly, $\frac{\partial \phi}{\partial n}$ also known to me; how? Because I use the boundary condition $\frac{\partial \phi}{\partial n_i}$ is nothing but my n_i right. So, that is the boundary condition. So, I use this boundary condition. So, this part also known to me.

So, you can now understand that everything in this equation 2 is known to me right; provided I know the value of ϕ_i right. So, so, this is the idea. This is how I discretized the whole thing and that is why I called this a 1-point Gauss quadrature rule. Because I assume that all these values actually lies at on the centroid of the panel.

So, I am really do not use any Gauss point inside the surface right and to understand the concept, I think this is the best way to understand. We really do not go into the numerical complexity rather like that how I integrate using Gauss quadrature rule etcetera etcetera.

Let us not go into this. Let us try to understand that very simple way assuming everything situated at the center, so value can be get that at the value at center multiplied by the area. So, I can get the value over that panel. So, this is the simplest way of going with this ok.

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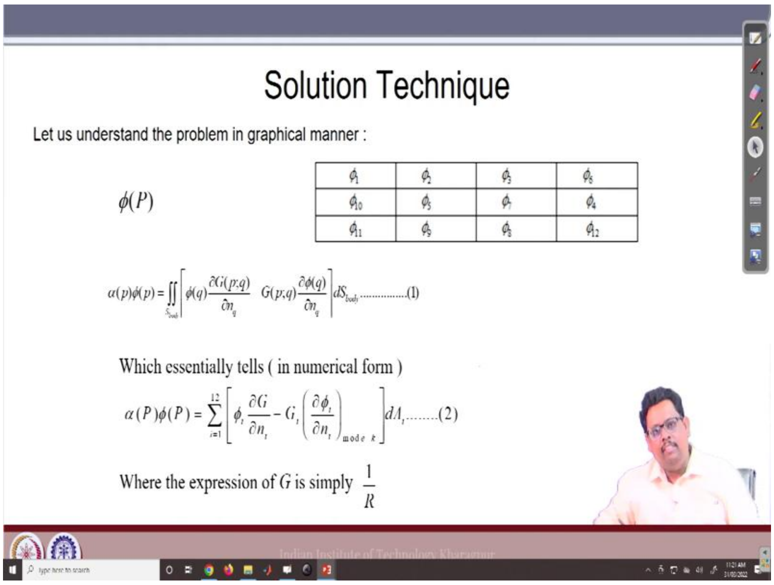

Now, here in this particular problem, I am taking that Green's function G is $\frac{1}{R}$; understood fine? So, now, I have set everything. I set the value for G which is $\frac{1}{R}$ and also, I discretized the integral equation in summation form. So, now only thing I have to find out the way to get this ϕ_1, ϕ_2, ϕ_3 etcetera ok.

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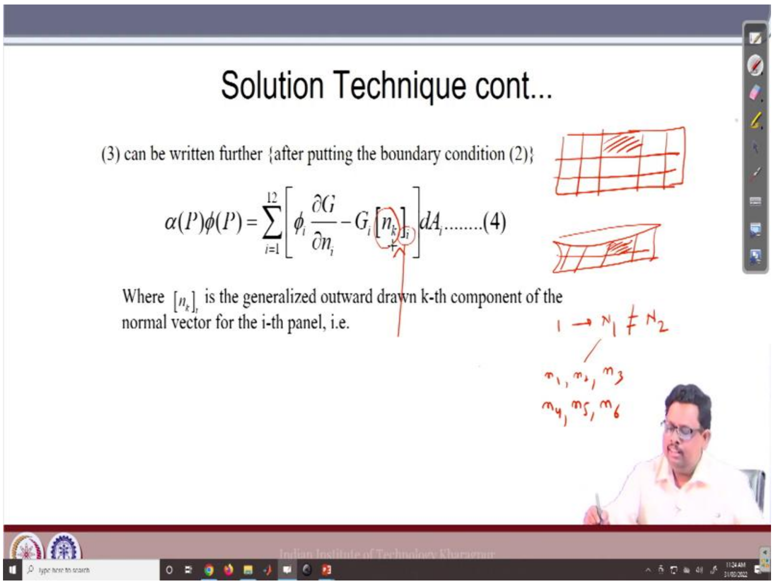

So, therefore, this further I can now once I when I can take that G equal to $\frac{1}{R}$. So, now, I just instead of I can replace everything here in the G and this is basically my integral equation right ok. Now, here actually I replace this $\frac{\partial \phi}{\partial n}$ by n_k. Now, I think this is now you understand right; here do not confuse with the notation *i*. So, here you have $[n_k]_i$ means what? Now, many people confuse that this k is more than what is i right?

Now, this is idea is very simple. Now, as I said that this is let us say the domain of discretization right. See this is a discretized domain. This i is basically my panel. Now, remember that in case of a reality, if you consider the real ship let us say. So, at each point your this mesh is a is not horizontal or vertical, it may be some inclined right. So, in each of this panel, your normal is different; is it not? So, therefore, in panel *i*, let us say panel 1, then your normal let us say n_1 is not similar to the panel at n_2; that means, in second surface that normal is not equal to the first panel.

So, in each panel, you have actually three linear component which is n_1, n_2, n_3 and then you have three angular component for the roll, pitch and yaw which is I can say that n_4, n_5 and n_6. So, when I talked about n_i $[n_k]_i$, when I talked about this $[n_k]_i$, this $[n_k]_i$, so I mean that for panel *i* right and we try to find out that which component of the normal you are actually try to you know find out because if you oscillate the body in heave mode, so n_k should be n_3. If you oscillate or moves the body in direction of the surge, so this n_k should be n_1.

So, this component n_k defines you that which component of a normal you are asking for and this *i* implies that it is for which panel right. So, I think now you understand this and since and please do not confused with this the double notation *k* and *i* because from experience, I know that many people got this doubt and they really do not understand that why what is n and what is *i* ok.

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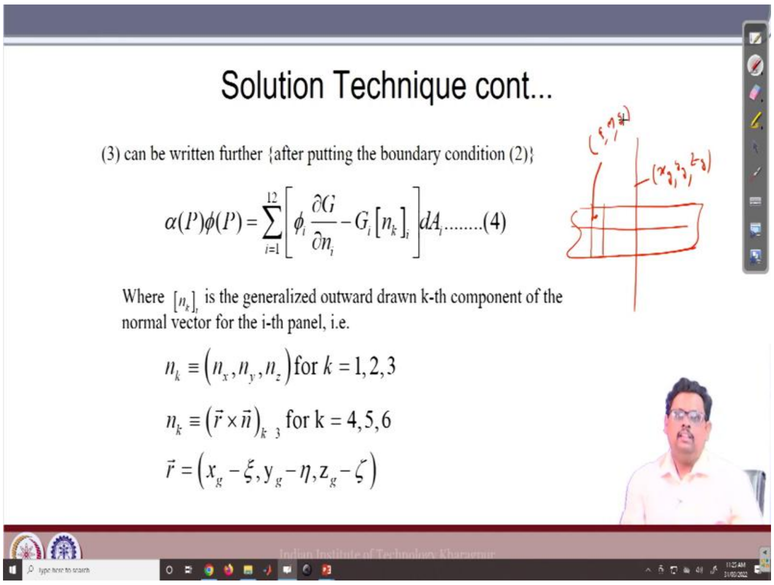

So, now here also I just write that what is the value for n and i and what is of course, what is the value for r right. Now, here this x_g, y_g, z_g right is basically the center of gravity and ξ, η, ζ is basically the panel centroid. So, now, you understand the definition for r also right.

So, what I said let me write here again. Suppose, this is your body and this body has a global c g and that global c g is nothing but your x_g, y_g and z_g and if you do the panelling, so this panel this centroid; this centroid it is ξ, η and ζ ok. So, this is the difference.

So, these are the these are the common notation that we are going to use and you know in future, you also understand this very well like because otherwise how we can do the r x and $(r \times n)$ right, if you do not have the information about what is your global cg right ok.

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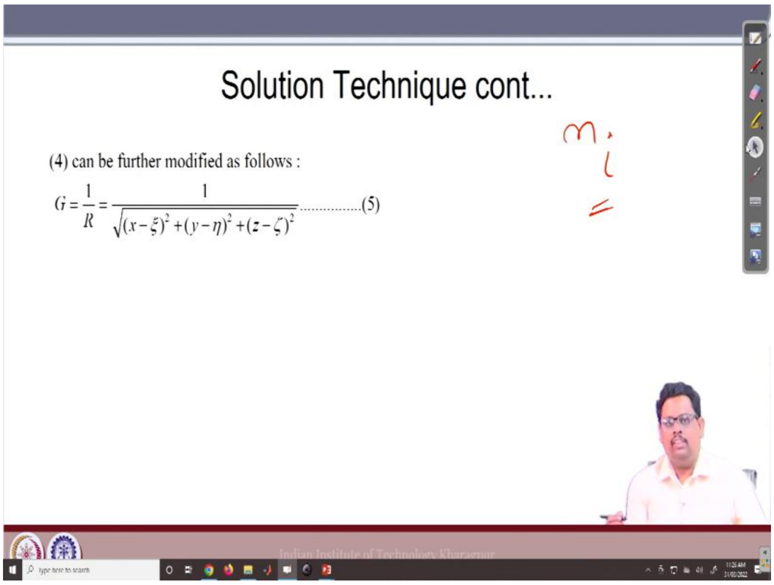

So, let us try to find out that value of $\frac{\partial G}{\partial n}$ and the value of ok, let us find the $\frac{\partial G}{\partial n}$ first and then, you know $\frac{\partial \phi}{\partial n_i}$, we already actually we know. So, we really do not need to discuss about how I can get this value for n_i; we have already discussed value for n_i before. So, we need not discuss now. But we need to discuss about the $\frac{\partial G}{\partial n}$ ok. Now, in fact, that is also we already done it; we solved one problem. From that problem, we have defined many things right.

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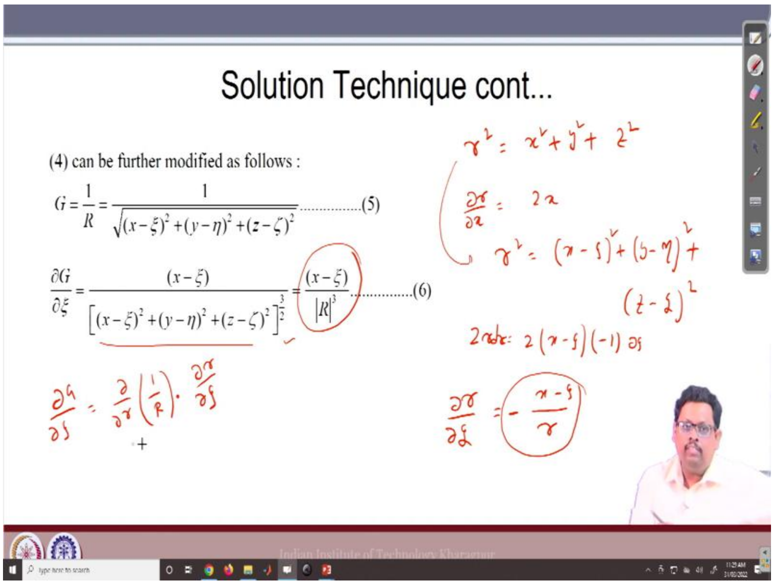

So, now, you know that what is $\frac{\partial G}{\partial \xi}$. Now, if you remember that we already done it in I think last class or last to last class, I; right now, I do not remember. But in we have done this for $r^2 = x^2 + y^2 + z^2$ and then, we find out that you know that let us say that now in this case here again you know that $\frac{\partial r}{\partial x}$ equal to in that case this is $2x$ something like this.

Now, here instead of r, now if the $r^2 = (x-\xi)^2 + (y-\eta)^2 + (z-\zeta)^2$. Now, if you differentiate with respect to r, so we have 2 r = 2 $(x-\xi)$ and then, it is $\frac{\partial \xi}{\partial i}$. So, it is (-1). So, now, I have ok. So, d r I missed; right into $\partial \xi$. So, I know that $\frac{\partial r}{\partial \xi}$ equal to now - $\frac{(x-\xi)}{r}$. So, you know it is and now we have the minus 1 with this. Now, this actually we have already done right.

Now, similarly if you now differentiate $\frac{\partial G}{\partial \xi}$, now $\frac{\partial G}{\partial \xi}$ you have to do it del $\frac{\partial}{\partial r}(\frac{1}{R})$ and then, multiply by del r by del xi. Now, $\frac{\partial r}{\partial \xi}$ it is - $\frac{(x-\xi)}{r}$ and then, you know that $\frac{1}{R}$ if you differentiate, it is coming out to be this one; $\frac{1}{R^2}$ right ok so. So, now, you replace this. So, this is your $\frac{1}{R^2}$ and then you have $\frac{(x-\xi)}{r}$. So, then you have this multiplied by the R and in together you can get this value $\frac{(x-\xi)}{[R]^3}$.

Now, here since I have already done this. So, I really do not do much over here. So, I will request you, you try to do this by your own. It is nothing you have to use this simple thing this capital R^2 $=(x-\xi)^2+(y-\eta)^2+(z-\zeta)^2$ and then, find out $\frac{\partial r}{\partial \xi}$ and then, you use the chain rule and then, you can get this value ok.

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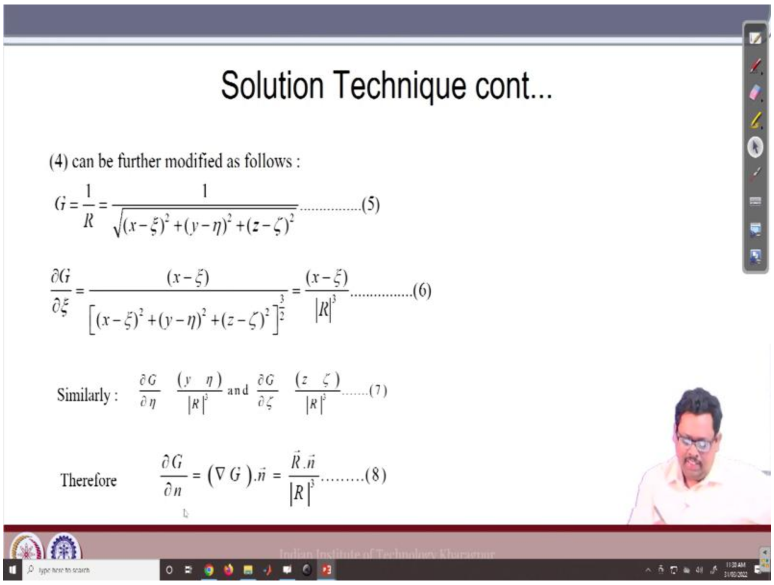

Now, similarly, I can make that del $\frac{\partial G}{\partial \eta}=\frac{(y-\eta)}{[R]^3}$ and $\frac{\partial G}{\partial \zeta}=\frac{(z-\zeta)}{[R]^3}$ right and then, this $\frac{\partial G}{\partial n}$ is nothing but $\frac{R.n}{[R]^3}$ ok so. So, now, I have this value for $\frac{\partial G}{\partial n}$ also.

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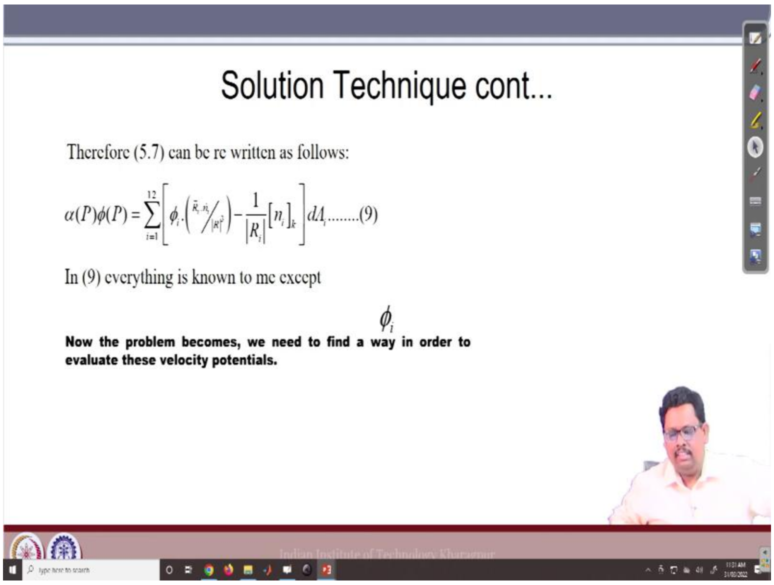

So, let me now again replace over here. So, if I replace over here; I can get $\alpha(P)$ $\phi(P)$ = i = 1 to 12. Now, $[\phi_i(\frac{R.n_i}{[R]^3}) - \frac{1}{[R_i]}[n_i]_k]dA$ straight. So, everything here is known apart from the this value of ϕ_i. So, how to get this value for ϕ_i? So, this is the main problem.

So, today, let us stop at this point and from the next class onwards, we are try to find out how do I get this value for ϕ_i ok.

Thank you.