Numerical Ship and Offshore Hydrodynamics
Prof. Ranadev Datta
Department of Ocean Engineering and Naval Architecture
Indian Institute of Technology, Kharagpur

Lecture - 22
Frequency Domain Panel Method Part 2

Hello, welcome to my Numerical Ship and Offshore Hydrodynamics.

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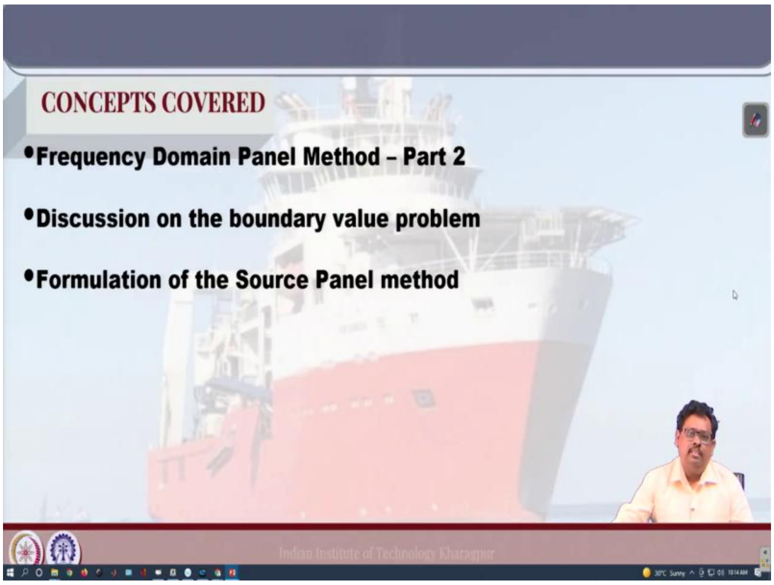

So, today we are going to discuss the following things like, we are going to continue the Frequency Domain Panel Method and today we are going to discuss that how I can change the boundary value problem that general boundary value problem we have discussed in the last class.

Today, for attack this problem particularly using this discretization technique that we have discussed in the last class that phi you know divided into three parts phi I plus, phi d plus summation i equal to 1 to 6 then, so all these things taking all everything together how to solve this that we are going to discuss and how I how the boundary value problem changes because of this that also we are going to discuss. And then, we are going to discuss something called the source panel method.

Normally, you remember the last class, not last class the previous classes we have discussed the source dipole method; however, the classical software like Vermont and other thing they prefer to use source panel method. So therefore, we are going to discuss on that also.

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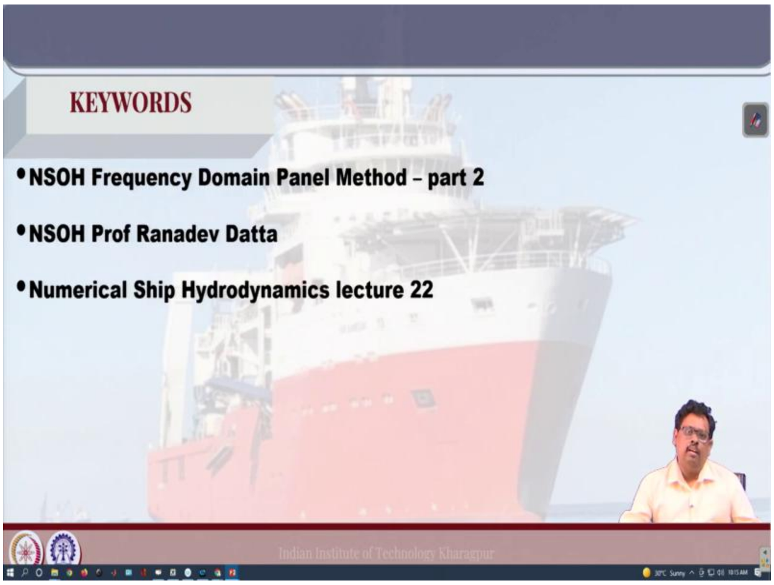

So, this is the keywords that you are going to use to get this lecture, ok.

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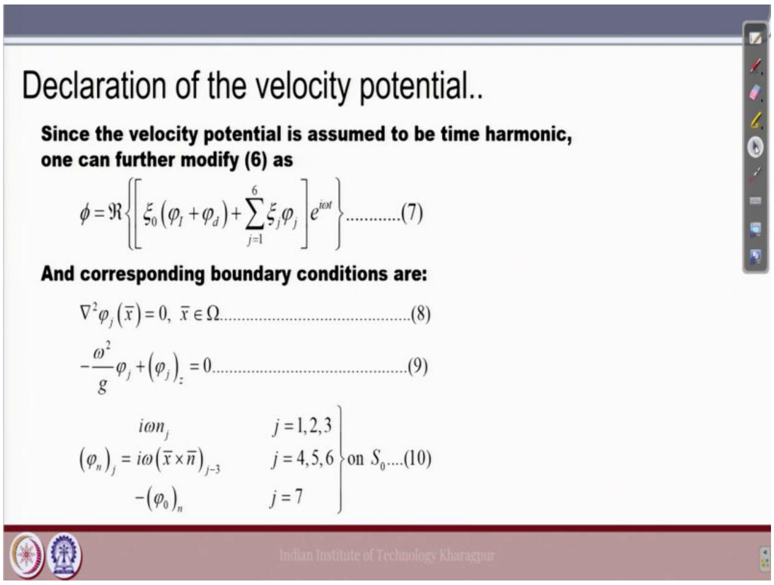

So now, let us jump into this equation. Now, here you can see that phi that we discussed the summation of that $\phi_I + \phi_d$ and then j = 1 to 6 ϕ_j. Now, here the interesting part is I am going to do this for you know in complex domain. So of course, $e^{i\omega t}$.

So, there is nothing new on this, but here you can see that I multiply $\xi_0\left(\varphi_i+\varphi_d\right)$ and I multiply ξ_j in front of the ϕ_j. So then, it is very normal questions like, why these things comes here. Because, if you remember in my last class that what we discussed over there as follows.

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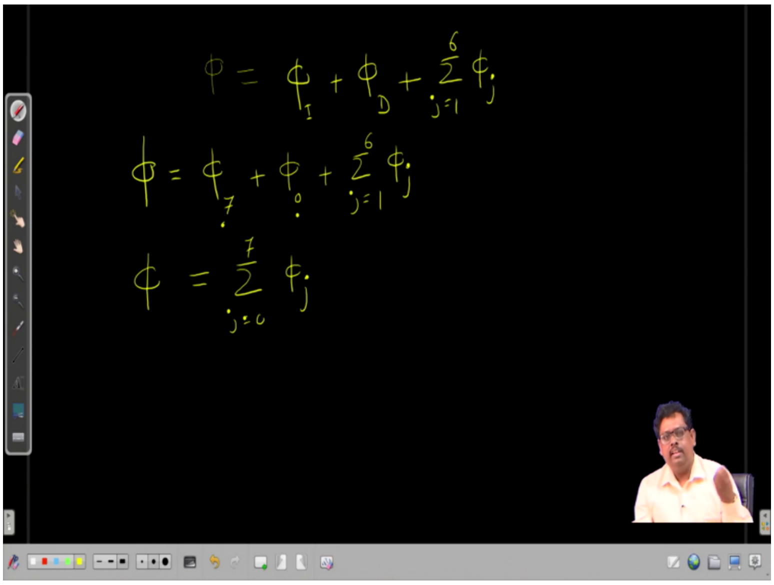

So, we discussed that our ϕ total the total ϕ is equal to that we discuss it is ϕ_I and then, we discuss as ϕ_d and then, it is $\sum_{j=1}^{6}\phi_j$. So, that is what we have discussed in the last class. So, however if you come back here; now I can see that it is there of course but with this ϕ_0 and ϕ_j is actually we are multiplying, right. So, what is the need for that? So, first let us understand these things, ok.

Now here, now just in sake of you know convenience let us put some number to I and D also. So, normally what I do is we can write ϕ equal to now instead of ϕ_I we call as ϕ_7, ok. And an instead of ϕ_d we call as ϕ_0 and then, I can add some j = 1 to 6 into ϕ_j ok or sometimes we can call this as 0 this as 7 does not matter.

So finally, that I can write in summation form $\phi=\sum_{j=1}^{7}\phi_j$ equal to some j equal to you know 0 to 7 into some ϕ_j. So in that way, actually this is how I can write in coding. So, instead of I, we writing ϕ_7 instead of D we can writing ϕ_0, but that is not the issue. The issue is that why I am multiplying this ϕ with you know this ϕ_I plus you know ϕ_d why I multiply this is something called ϕ_0.

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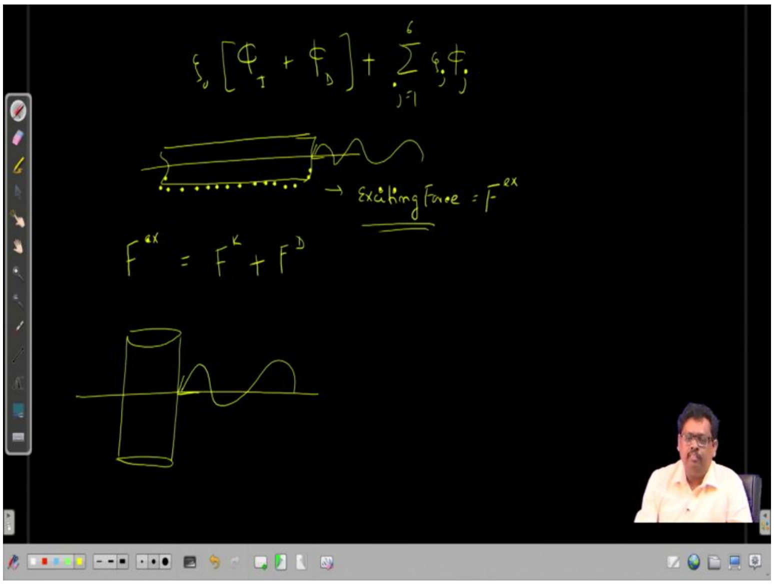

And why I am multiplying this j equal to 1 to 6 in ϕ_j some x_{ij}. So, the question is this, right. Now, let us try to answer this question first. Now, what we do? Now if you look at this physical phenomena, what is happening. Now, I have a ship over here and then this incident wave potential actually hitting this object and because of you know this wave hit this ship, then we have some pressure variation along the hull, right.

If we integrate the pressure you can get a force. Of course and the name of the force is exciting force and normally we define this as F^{ex}. Now, if you know that this F^{ex} this exciting force is summation of the force is equal to force we call is a F^{k} plus the diffraction force we call F^{D}. Now, it is similar to if you remember in my previous class it is similar to that picture where it is this is still it is not moving; however, the wave is hitting to this object, right.

Now, what is happening when this exciting force actually we apply over the ship? Then ship start oscillating. Now, when ship start oscillating we can have different set of waves. What is that?

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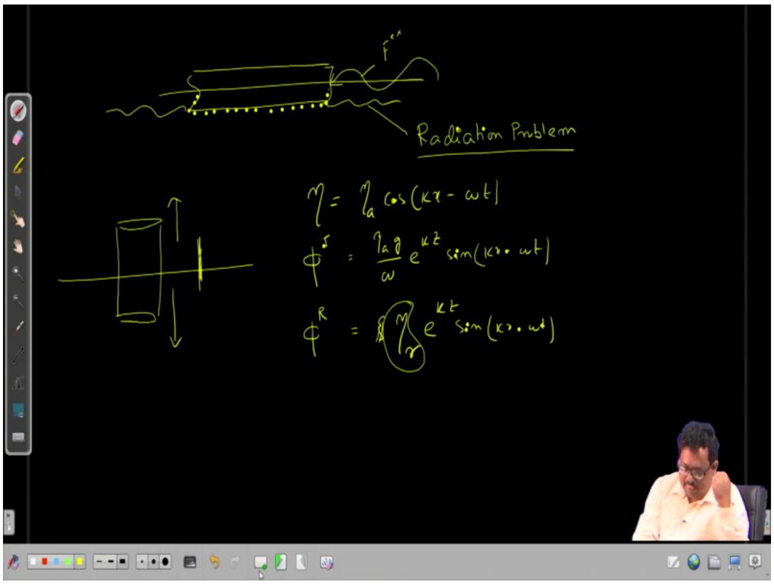

Now, assume that initially this ship is very much statically stable weight is balanced by the buoyancy and everything is fine. Suddenly, you know this the ship hit is this I mean the wave hit the ship and then the ship start oscillating, right. So, actually this force why is called the exciting force because, this excite the ship to oscillate. Now, the moment the ship start oscillating, then again we have some different wave field altogether and these waves we can call as a radiated wave; and this problem we call the radiation problem, right.

So, now, let us see the picture. The picture is that ship is very much stable here with there is no waves in and then suddenly the wave's hits. So, that excite the ship and then, ship start oscillating and before ship start because of the ship start oscillating then we got some radiated wave and this is called the radiation problem.

Now, because of this phenomena that ship is you know start oscillating and this is the; this is the picture that you refer for when the body we this start oscillating in calm water, right. This is the second picture if you remember and if you add these two picture you will get the original phenomena. So, we have actually this phenomena is splitted into two component, right. And because actually this happens this oscillates, because of this oscillates what is happening again we have some pressure field along this body.

And then if you integrate the pressure field we will get you know the radiation force, now. Now, what is the connection between these two? The connection is this then that this exciting force excite the body and because of this excitation. The ship start oscillating and because the ship start oscillating we can have some force. Now, the question is what would be the amplitude of that force? Right. Because, this is unknown to me.

But, what is known to me. I know what is the wave height that actually hitting the body, right. Now, you know that you can so this is my input that η equal to some amplitude $\eta_a \cos$ let us say in real line $\cos(kx-\omega t)$ if this is hitting and then, we know that ϕ_I the expression is also it is $\frac{\eta_{ag}}{\omega}$ then in case of a deep water e^{kz} into let us say $\sin\left(kx-\omega t\right)$.

So I know this, this is known to me, right. So, I know that what is the amplitude is hitting the this structure, but I really do not know. Because now, this the radiations if I try to write this the potential for the radiation potential this must be some amplitude let us call it η_r. So, let us call is eta r into a similar manner it should be some e^{kz} into either sin or cos $\left(kx-\omega t\right)$definitely it is sin because, this phi R also has to satisfy the linear free surface boundary condition.

So, if the eta equal to cos k x minus omega definitely phi should be sin whatever. So, let us leave that part. Question is, how do I know this amplitude this? Right. Now, here is the trick. We use the property of the linearity. You know it is a very nice property in fact from the class 9 we are doing this maybe you can maybe before that I do not remember right now but, the day you started the unitary method we are actually into the linearity.

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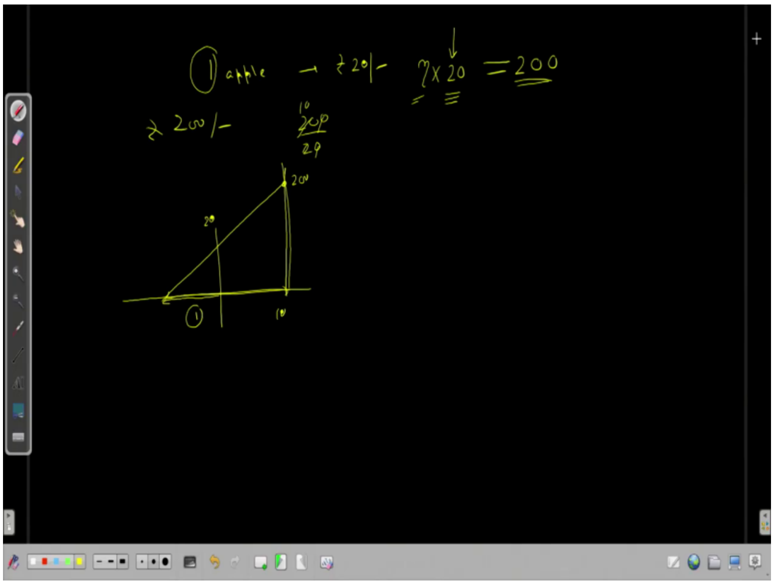

Now, what you say is very simple. It says that let us say that let us take 1 apple if it is cost let us say rupees 20 let us say then, in 200 rupees in rupees 200 I know how many apple you could buy. So, you know that it you know so, it is very easy for this is very simple because, I know that price for 1 apple. So, it is simply 200 divided by 20. So, this is my answer.

So that means, this unitary method you know teach you if everything is under the linearity and if you know the cost for unit then, then you can find out the total thing that you are spending, right. So, this teach you that the total cost so under the linearity. It teach you this 200 is equivalent to some amplitude eta multiplied by the 20. That means, I know the price for a unit apple and then I multiply by the you know in that case the number of apple in our case is the basically the amplitude that mode the mod.

If you so, the underlying principle of the linearity is as follows that, if you know the thing for the unit amplitude and if you multiply this unit amplitude with the modulus then definitely you are going to get the total force. So, this is the idea, ok. So, see you understand and it is only possible in the approximation of the linearity. Because, linearity you know gives you a nice property right and it is called the - this is the ratio and proportional. So, it means that if 1 this cost 20.

So, definitely this much so it is 10 this will definitely cost you 200. So, this by this should be equal to this by this. So, this is what we learned unitary method so, this same logic actually in applying here also. So, let us see how we are applying this principle here. So, again let us go back to the slide. Now, if you look at this slide, here now I understand that if you look at this part. If you look at this part this is now I understand this psi I plus psi d is nothing but the thus the velocity potential for unit amplitude wave, ok. So, let us so here.

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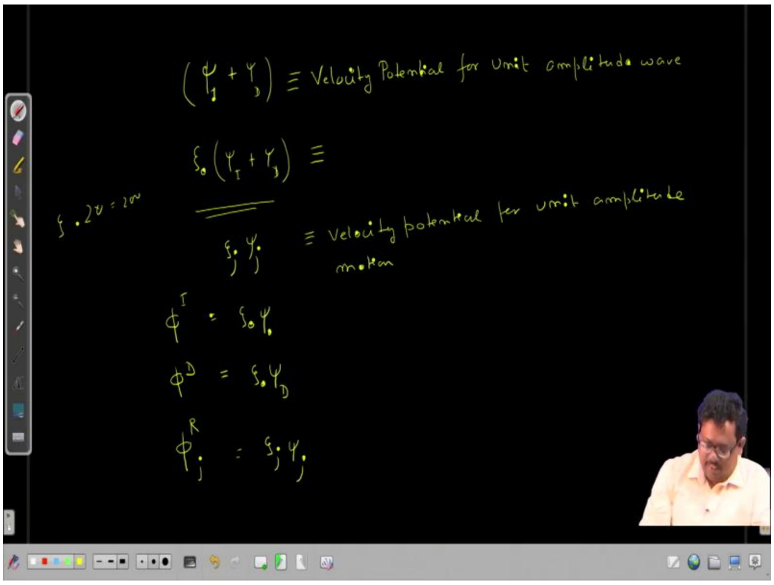

So now, we understand in that slide, in that slide this $\psi_i + \psi_D$ is nothing but velocity potential for unit amplitude wave. So, if it is so then if you multiply the wave amplitude ξ_o. So, definitely you know with this same linearity analogy I can say that this is nothing but the you know that you know the force I am getting is basically the velocity potential for our problem.

Similarly, if we look at this the right hand side; now here I have the ψ_j, right. So now, this ψ_j is nothing but now this ψ_j is nothing but the velocity potential for unit amplitude motion. You know that this for radiation probably in steeled water this body has to oscillate. Now, that magnitude of this oscillation is 1, then it is ψ_j.

Now, if you multiply this ψ_j as I said the similarly that the remember this 2^{nt} multiplied by some ψ or ξ this equals to 200. So therefore, this ψ_j must be multiplied by the amplitude to get the total velocity potential so, right. So, I am writing here that so therefore, I can write my ϕ_i is nothing but $\xi_0.\psi_0$, where ψ_0 is the velocity potential for unit amplitude wave.

Similarly, my ϕ_d is nothing but $\xi_0.\psi_D$, right. Where the ψ_D is the velocity potential for the unit I mean diffracted potential for the unit amplitude wave. And similarly, I can write my radiation potential ϕ_r in jth mode is equals to that $\xi_j.\psi_j$ where ψ_j is nothing but the radiation potential for unit amplitude motion, ok. So, with this understanding let us go back to this slide, ok.

So now, you can see that now this equation seven is absolutely clear to you right; it is nothing but the I mean $\xi_0(\psi_i + \psi_D)$ and $\xi = \psi_j$. Now, here what I am going to do is as I said in the last class that each of this ξ should satisfy the Laplace equation. So, ϕ satisfy the Laplace equation so, definitely this $\xi\psi_j$ also satisfy the Laplace equation. Now, since $x(i, j) \neq 0$ the other part should be 0.

So therefore, $\nabla^2\phi$ must be equal to 0 right and that is my governing equation. Now, if I know my governing equation then, then you can see the equation number 9 which is nothing but the free I mean combination of kinematic free surface condition or dynamic and dynamic free surface condition, right. So, how it is coming now let us see that how this second equation has come this is also very easy.

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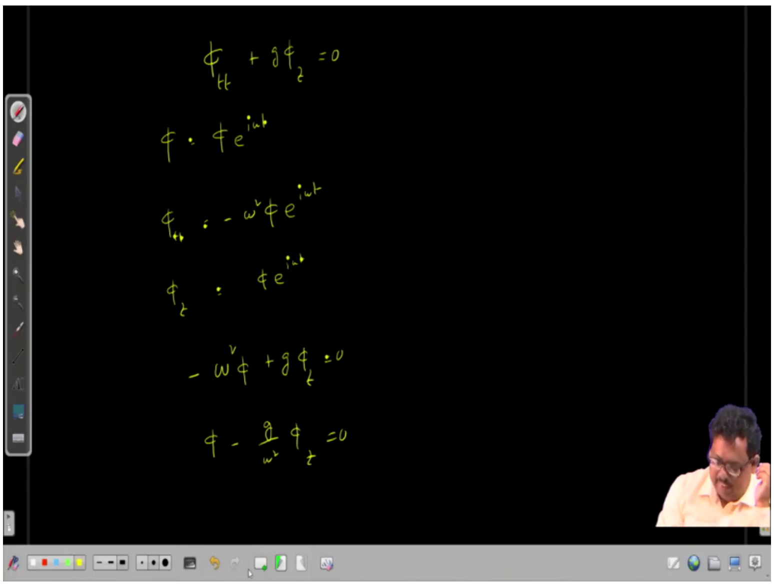

Now, here you can you can you can write that you know that in time domain it is $\phi_{tt} + g\phi_z = 0$ that is in the time domain case. And so therefore, if you take $\phi = \phi.e^{i\omega t}$ now if you so, if you differentiate this two time. So, ϕ_{tt} is nothing but $-\omega^2 \phi e^{i\omega t}$ and of course that ϕ_z is nothing but $\phi.e^{i\omega t}$.

Now, if you substitute over here what you get is minus omega square into $-\omega^2\phi + g\phi_z = 0$ and then it is nothing but $\phi - \frac{g}{\omega^2}\phi_z = 0$, right. So, this is the alternative expression for ϕ. So, you can see it is here, ok. Now, I did here the opposite it is I just divided g right that does not matter. And then, we need to again understand what is the third equation which is $i\omega n$ I mean, why it is this let us we need to understand that also.

I mean that equation number 10 for j = 1 to 3 for j = 4, 5, 6 the boundary condition. Now, here, ok. So, in this slide actually we did the indexing is the opposite does not matter ok like, but let us see that where the indexing here is here.

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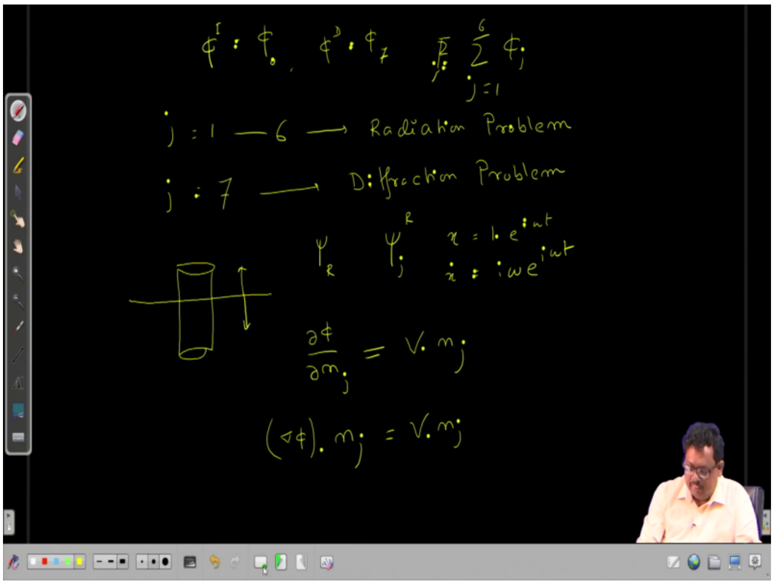

We can we use that ϕ I is equal to we can call this as a ϕ_0 and then the ϕ_d we call as let us say ϕ_7, ok. So, let us take this ok and of course the radiation potential definite it is j equal to; j = 1 to 6 into ϕ_j. So, let us go with this notation fine. Now, if you remember that from j = 1 to 6 is the boundary condition for radiation problem, ok. And then, from j = 7 this is the boundary condition for the diffraction problem, ok.

Now, let us try to understand what is the condition for the radiation problem? ok. Now, if you remember the radiation problem is nothing but there is a calm water and then the body is oscillating and then, you can have some radiated wave and you have a pressure field if you integrate it you will get the radiation force.

Now, here if you remember I said that ψ represent for the velocity $\psi_{\text{R or I}}$ can just if I make index j. So, radiation potential for the jth mode is nothing but I oscillate the body in unit amplitude motion. So, I am giving here the oscillation of this body. So, I oscillate this with unit amplitude of the motion. So, definitely my oscillation this x is nothing but $i.e^{i\omega t}$, right.

So therefore, my velocity x dot should be $i\omega.e^{i\omega t}$ right, fine. So now, if you remember my radiation condition is nothing but that is $\frac{\partial \phi}{\partial n_j} = Vn_j$ mode, right. Now, this $\frac{\partial \phi}{\partial n}$ is nothing but $\nabla(\phi).nj$ mode should be is equal to $V.n_j$ mode. So, this is quite simple that we understand this, right.

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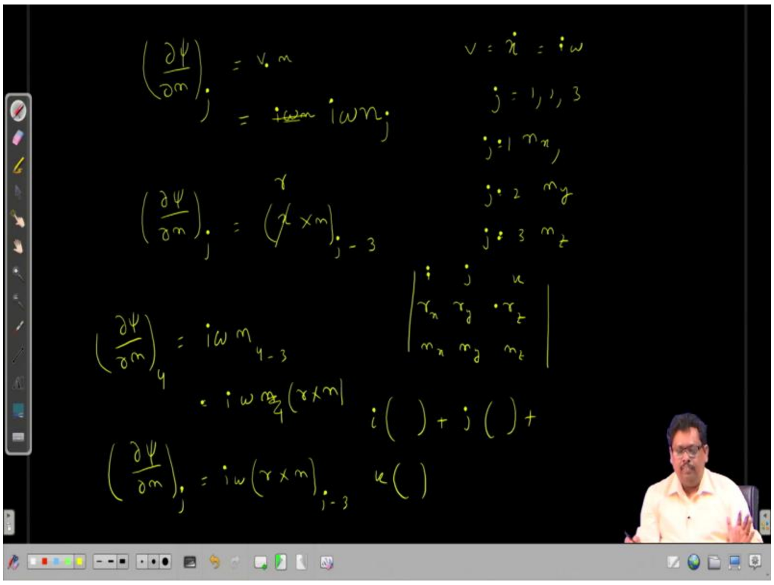

So therefore, my velocity I mean that body boundary condition from the left hand side it is nothing but my $\frac{\partial \psi}{\partial n}$ right that is in the jth mode of course, that should be equals so now, now what is my V_n it is nothing but V_n. Now, I just find out my $V = \dot{x} = i.\omega$. So therefore, I replace this, equal to $i\omega_n$; whether it is not ω_n.

So, just careful it is $i\omega_n$ or you can say n j or the j^{th} direction right, fine. And so, it is and it is happened for j = 1, 2 and 3, right. So, then that means, if it is j = 1 it is nothing but your n_x if it is for j = 1 for j = 2 it is n_y and j = 3 it is n_z fine. So, similarly I know that other part also you know ξ_{nj} it is definitely we can call $r \times n$ or here it in slide is called $x \times n$ it does not matter and it should be j-3. Now, we discussed already what is this. Because, how I get it, it is also very easy it is if I take i.j.k and then, let us call this r.

So, you have three component r_x r_y r_z and then, you have component n_x n_y n_z and if you do that then you have some i^{th} component, then you have some j^{th} component and you have some k^{th} component. So, here for del $\left(\frac{\partial \psi}{\partial n}\right)_4$ should be v is $i\omega$ right multiplied by the first component of this. So, definitely it is n so, j = 4 - 3. So, it is nothing but $i\omega$ you know it is the n the first component of this the matrix the i^{th} component.

So, that is why I write this as that is why you can in common you can write del $\left(\frac{\partial \psi}{\partial n}\right)$ at jth mode should be $i\omega(r \times n)$ into the not nice it is this $(r \times n)_{j-3}$, right. So, this is all about of the radiation potential. Now, only thing is let what happened for the diffraction problem.

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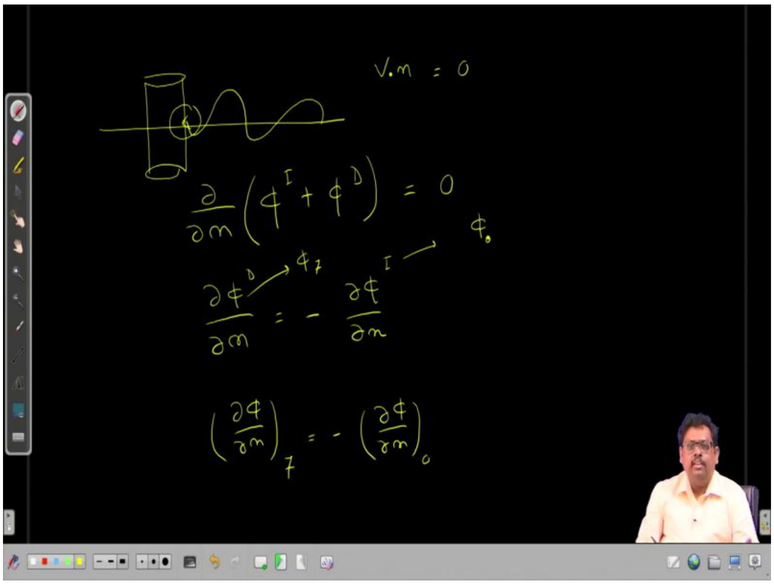

Now, in diffraction problem if you remember that what I said is that this - the body is fixed and then wave is hitting over here, but body does not move. If the body does not move, then $V.n = 0$ and then the exciting force as I know is the combination of the $\phi_I + \phi_d$.

So therefore, the boundary condition become $\frac{\partial}{\partial n}\left(\phi^r + \phi^D\right)$ that should be equal to 0. So therefore, del $\frac{\partial \phi^D}{\partial n}$ should be equal to $-\frac{\partial \phi_I}{\partial n}$. Now, if you remember this I notation this as ϕ_0 and this I noted as ϕ_7. So, then I can write $\frac{\partial \phi}{\partial n}$ in 7th mode should be equal to minus of $\frac{\partial \phi}{\partial n}$ in the 0^{th} mode, $\left(\frac{\partial \phi}{\partial n}\right)_7 = -\left(\frac{\partial \phi}{\partial n}\right)_0$, right.

So, let us go back here and you can see that this is the boundary condition for our problem. So now, we understand this boundary conditions now and then, from the next class from next class actually I am going to discuss about the source panel method, ok. So, till this point.

Thank you.