Numerical Ship and Offshore Hydrodynamics
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Lecture - 23
Frequency Domain Panel Method Part 3

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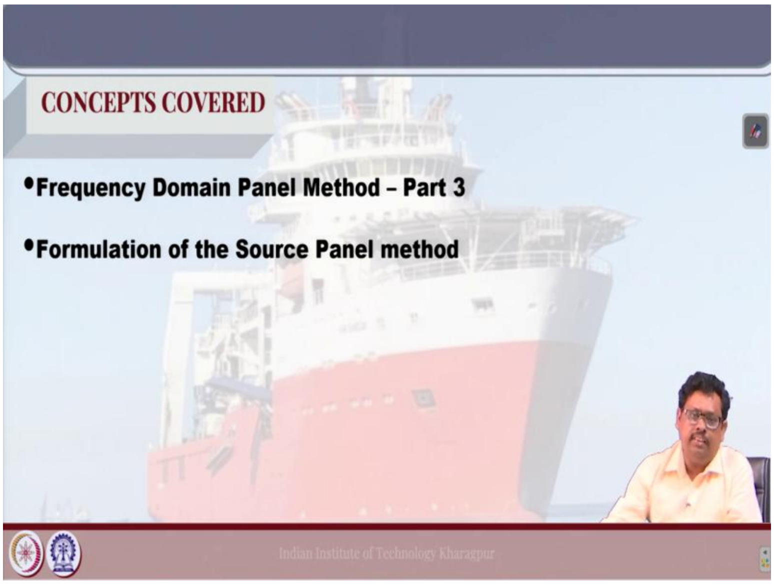

Hello. Welcome to Numerical Ship and Offshore Hydrodynamics. So, today we are going to discuss this following topic that Formulation of the Source Panel Method. Now, if you remember in the last class, I said there are three different kind of panel method based on the discretization- one is source dipole that we have solved for the infinite fluid domain and then the source panel method another one is the dipole panel. Let us see that what the other methods are here.

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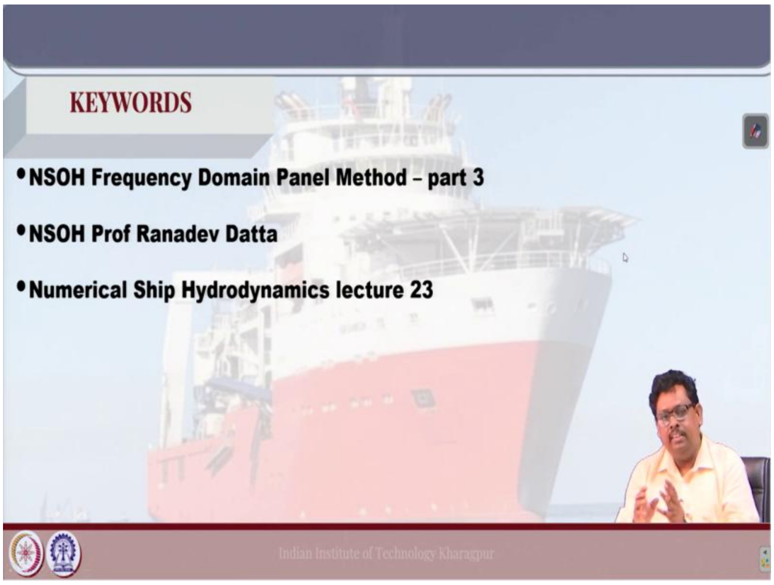

And, this is the keyword that you have to use to get this lecture, ok.

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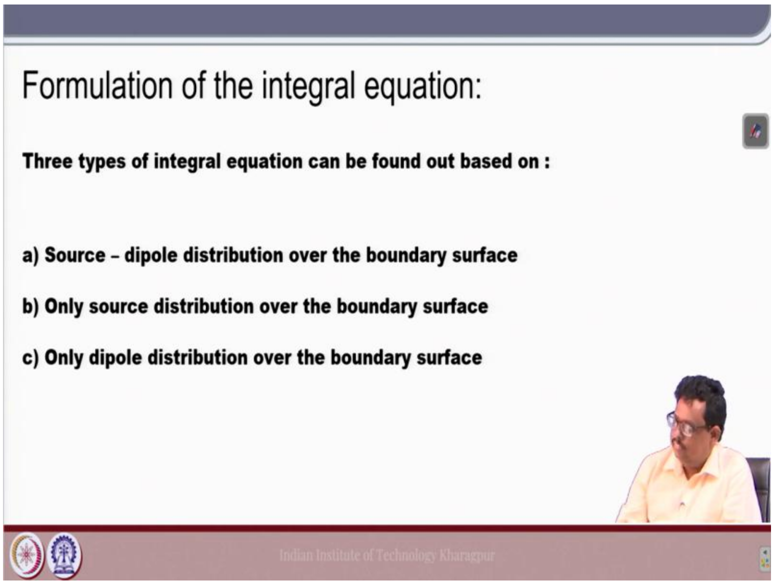

So, let us go back to this. Now, formulation of the integral equation there is a three type of integral equation that we can think of. One is the source-dipole distribution over the boundary surface and that we have already discussed, in when we solve the infinite domain the fluid problem and get the infinite frequency added mass that time we have discussed this.

And, now today we are going to discuss, what is the only source distribution over the boundary surface. And, what is the only dipole distribution over the boundary surface, ok.

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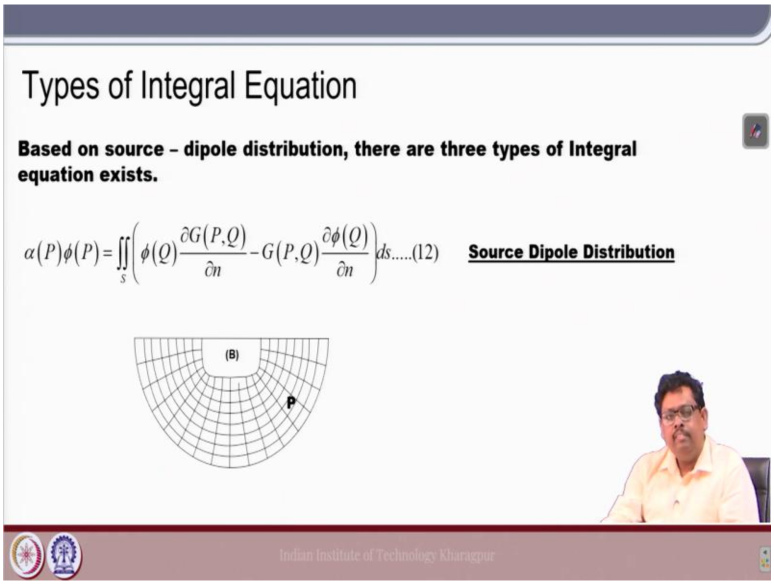

So, now this is the typical source-dipole distribution. And, where we know very well what is this equation, right. Here you have this left-hand side, this $\alpha(p)$ if you remember this $\alpha(p)$ is nothing but, 4ϕ inside the fluid domain, right. And then $2\phi - 4\phi$ inside the fluid domain -2ϕ on the boundary and 0 outside the boundary. Sometimes, people actually take this $\alpha(p)$, the value is 0 half and 1 based on that position is outside on the body or it is the inside the domain, ok. So, you can take $\alpha(p)$, 4ϕ, 2ϕ, 0 or you can take 1, half and 0 anyway.

Now, in inside this now let us consider that if there is a point here, inside the boundary, ok. So, then this is the equation. So, where ϕ is the here inside the integral equation. This ϕ is distribution of the velocity potential on the body. And then, in the left-hand side, this ϕ indicate the velocity potential anywhere in the fluid domain, right. And then this is the equation, I can call this is a source dipole distribution.

Now, think of a hypothetical situation. So, what is that hypothetical situation, like, in order to place this arbitrary point inside the fluid domain; let us place inside the body.

Now, inside the body there is no fluid, right. So, it you can think of that as a imaginary domain. Now, if you write the same integral equation for this imaginary domain, ok. So, then actually you know we can write this equation as follows.

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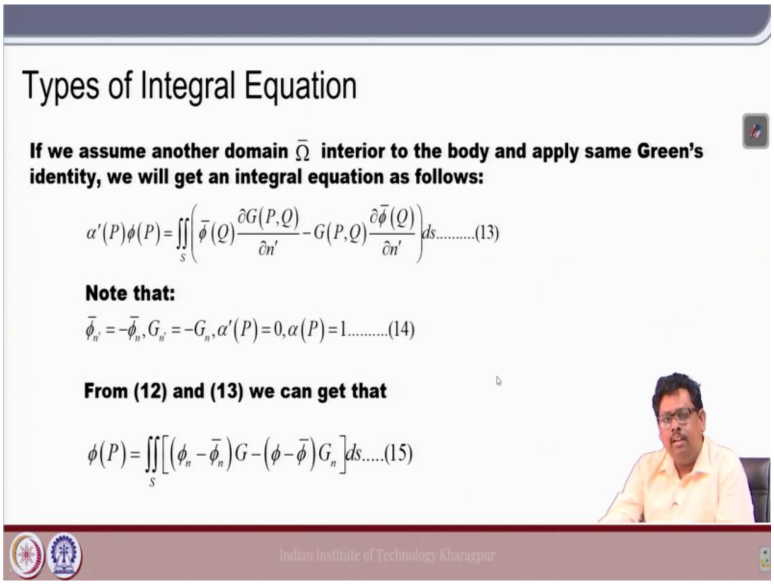

Now, here what I said that, this now this P, now inside that if I go back here, this P is now in inside this B, ok. Now, if this P inside this the B then, this is let us assume that this is the boundary integral equation. now, here one must note that, the normal the normal is definitely the opposite. Now, see always we take a convention, that normal should be the inward to the fluid.

Now, if we look at this the previous this one, here, this normal now you can see it is along this P here in this direction this is the inward to the fluid. Now, if you take inward to the domain betters better to say. Now, if it is here inside the middle of the B, ok, somewhere here. So, then the normal should be the inward to this. So, definitely the direction is normal should be opposite, you have to understand this very well.

That is why I said, here definitely you can see the $\bar{\phi}_n$ is nothing but, $\bar{\phi}_{n'}$. It should be n dash should be equal to minus of n so. So, now, so, that is why when you called this as n dash remember, this n dash it should be the inward to the body B, right. Like in case of the previous one, it should be the inward to the fluid. Now here it is inward to the body, that is the positive direction of the normal.

So, therefore, I understand very well that, n dash equal to minus of n, right. So, with this understanding now if you look at this equation 13, its the same equation instead of α I can call this α dash, ok. And instead of ϕ_I call this as a $\bar{\phi}$. And, ϕ is because is the same ϕ, right. So, now, if we look at my equation 12, this one and then if I take this equation 13 this one and from these two equation actually I can get the equation 15.

It is simply by adding the equation 12 and equation 13. Why? Because, $\alpha'(p)$ now it is the outside the domain. So, the value is should be equal to 0. Because, the hypothetical domain. So, therefore, the value should be equal to 0. So, now, if we add this 12 and 13 then definitely, we are going to get this equation, right. Now, what to do with this equation?

So, here it says $\phi(p)$ equal to over the S, it is $\left[\left(\phi_n-\overline{\phi_n}\right)G-\left(\phi-\bar{\phi}\right)G_n\right]ds$, right. Now, from this equation how we can obtain the source panel method or the dipole panel method.

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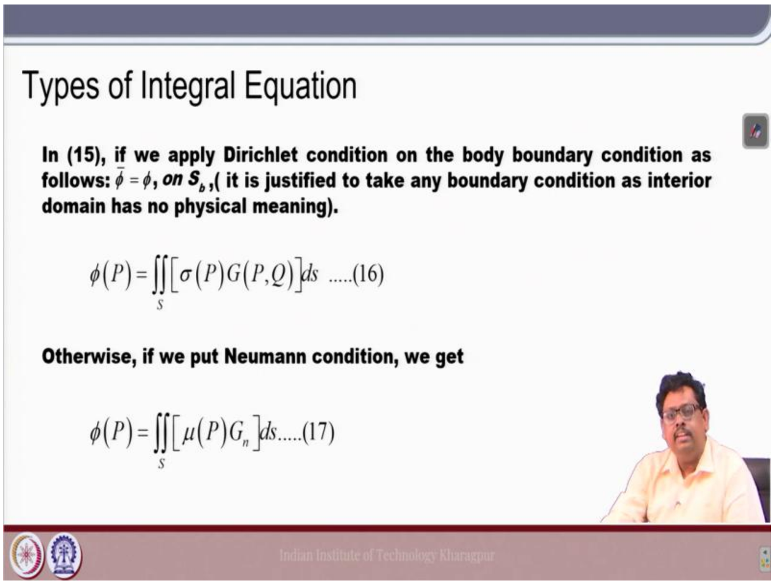

So, now, if I look at this equation 15 and if we apply the Dirichlet boundary condition. So, it says that at the boundary that $\bar{\phi}=\phi$ because you know, it is a hypothetical boundary that second one that based on that actually I am getting this equation 13. That equation is a hypothetical boundary. So, I can take at boundary any value for this $\bar{\phi}$.

So, I said that, if I take $\phi = \bar{\phi}$ and then if I take $\partial\phi$, I mean this is the second one that $(\phi - \phi_n) - (\phi_n - \overline{\phi_n})$, if I take as σ. Now, what I did is here, I use the Dirichlet boundary condition. I said that I am taking $\phi = \bar{\phi}$. So, this part goes to 0, right. If I do that, then actually and I am writing that $\phi_n - \bar{\phi}_n = \sigma$.

So, if I do that then I can get the equation 16, right. Now, if I do the opposite, now if I use the Neumann boundary condition and if I write that $\phi_n = \bar{\phi}_n$ and $\phi - \bar{\phi} = \mu$. So, then I can get the equation 17. So, this equation 16 is the source panel method. I mean this is the boundary integral for the source distribution. And, then equation 17 we can call this is the integral equation. It is the distribution of the dipole.

Why, because, if you look at this equation 16, it is the distribution where the G is only there, right. Only the source is there, but dipole is absent. And, in equation 17 the only dipole that G_n is there, but the source is absent, right. So, that is why we called this is the only source distribution method where, we have the I mean there we have only the G, but we do not have the G_n.

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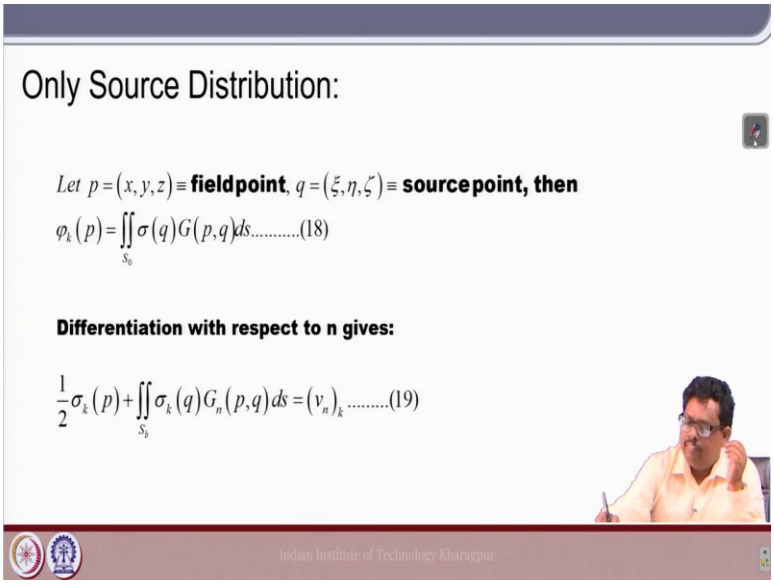

Now, taking this only source distribution, how do I solve this equation? Now, here, if you look at this equation now here you can see there is a two unknown, right. Now, let us go back to little bit in the let us go back to our board work.

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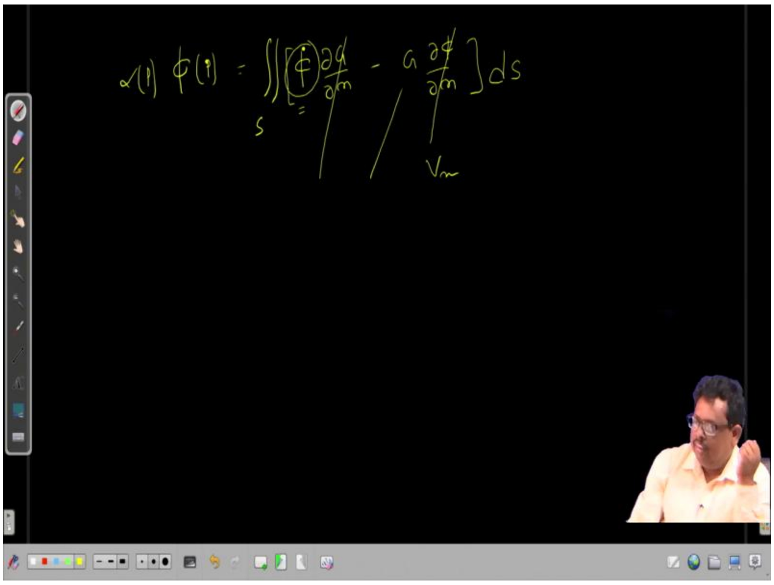

If you remember, that in our source-dipole distribution this is nothing but, let us take $\alpha(p)$ of course, $\alpha(p)\phi(p)$ equal to here you have over this body S and then we can call this let us take $\iint\left[\phi\frac{\partial G}{\partial n}-G\frac{\partial\phi}{\partial n}\right]ds$, right. Now, if you do that if you do that then here if you look at this equation, everything is known to you apart from this ϕ, because $\frac{\partial\phi}{\partial n}$ is nothing but, your v_n that is known to you. And then G is also known to you and then $\frac{\partial G}{\partial n}$ also known to you. So, only thing that is unknown to you is nothing but the ϕ.

Now, if you look at this equation if you look at this equation, here you can see that you have actually two unknown. One is that ϕ of course, this is what we are going to obtain. And, second one is the σ, the strength of the source, right so, but then we need two equation, right. So, what I do is, we differentiate that equation 18, with respect to n. So, if I do this then in the, right-hand side we have the $\frac{\partial\phi}{\partial n}$ which is nothing but my V_n, the boundary condition if I apply over here, it is V_n. And, then, if we integrate this we will get this expression.

Now, here you know this equation 19 it is strictly for the lower order panel method. However, in the higher order panel method you really do not have this the term half of σ_{kp}. So, now, how this equation is coming, let us try to see, ok.

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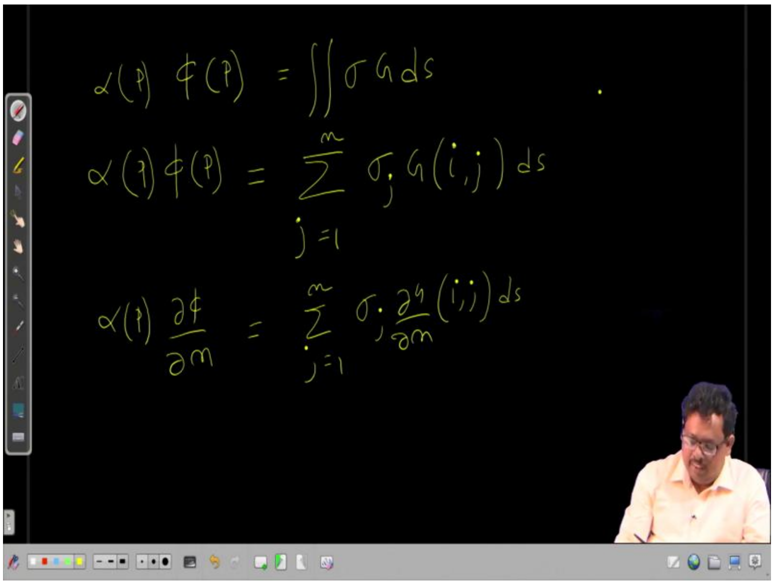

Now, in this equation, what I am getting right now is $\phi(p) = \iint \sigma G ds$. So, this is I call this is the only source distribution. Now, if you discretize, ok, I just miss the $\alpha(p)$, sorry. So, if you discretize this one. So, in lower order panel method what you are getting is $\alpha(p)\phi(p)$ and that should be equal to. Now, if I do the summation. So, we can take summation j=1 to number of panel n then it is ϕ_j and if you remember it is $G(i, j)ds$.

Now, here what I am doing, in each panel, I am doing it. Let us say take the constant panel method and if you take the - you know if you take the - what we call that one-point Gauss quadrature. Normally, we do not do here. But, for the understanding this- the second equation let us try to do that. Now, if you differentiate with respect to n, then what you get? You get $\alpha(p)\dfrac{\partial \phi}{\partial n}$.

Of course, and then it is $\sum_{j=1}^{n} \sigma_j \dfrac{\partial G}{\partial n}(i, j)ds$, ok.

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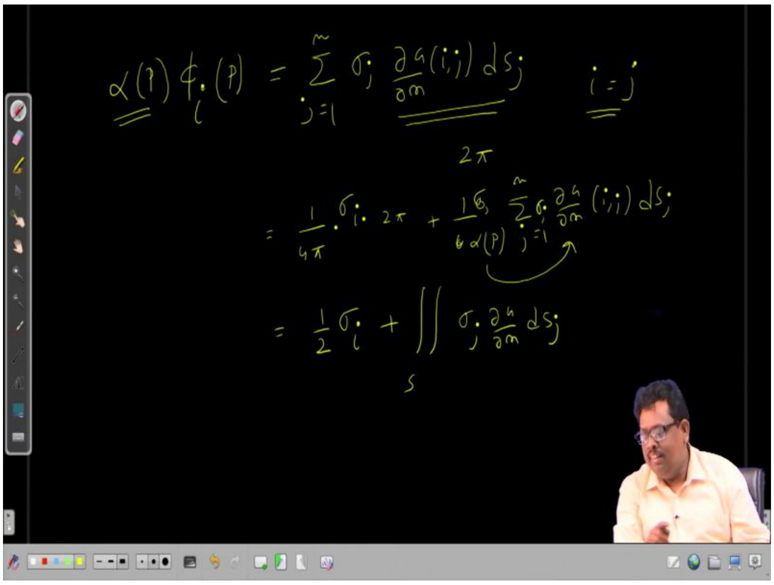

So, now $\alpha(p)$ is there. Now, if I replace this as $\phi(p)$, if you remember correctly, the last time, when we solved with this you know if you take i, then here I have the $\sum_{j=1}^{n}$ and then we have $\sigma_j \frac{\partial G}{\partial n}(i,j)ds_j$, let us say. Now, here, if I take i = j, ok, then I know this integral of this value is become you know 2 π.

Now, if I break it i = j and $\frac{1}{\alpha(p)}$ the whole thing, ok. So, then what you get as follows, when i = j then, I have here it is $\frac{1}{4\pi}$ and then I have the σ_i and multiply by the 2 π, right, plus of course, $\frac{1}{\alpha(p)}$ is there, we can take it is $\frac{1}{\alpha(p)}\sum_{j=1}^{n}\frac{\partial G}{\partial n}(i,j)ds_{i,j}$.

Now, if you take $\alpha(p)$ inside this Green's function, so here I can write it is, half of σ_i , because this will canceled out. And, then I just write this same as the integral from S into we can write here at, ok, σ is here, right σ_j is here. So, it is $\sigma_j \frac{\partial G}{\partial n}ds_j$. now, you see that is why this half of σ_i is coming. Now, it is only happening when you have this lower order panel method, right, fine.

So, now so, this is now I understand the equation 19 correctly, right. This is how this equation 18 and 19 should be. Now, I solved this equation 18, right, I mean so, first you need to solve the equation 19, right, because in equation 19 now, all the parameters is now known to you. You know the, right hand side is nothing but your the body boundary condition, right, that you know.

You know the Green's function, $\frac{\partial G}{\partial n}$ and also. So, this is some that is the only thing that you I mean sigma that you need to know. So, here the only unknown is nothing but your, the sigma, right. So, therefore, first we solve this equation 19, I get the expression for sigma and then that expression I substitute in the equation 18 and then I get the value for phi, ok. So, now, this is how actually we can discretize the surface.

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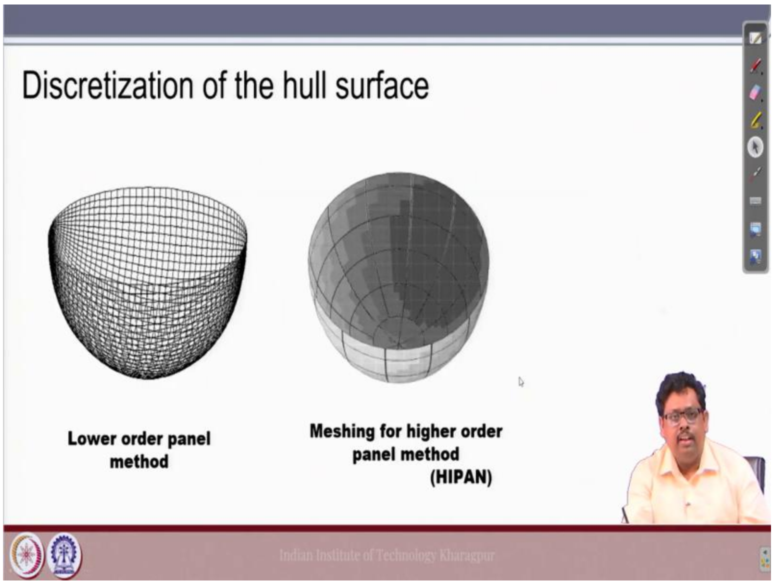

Now, as I said, that in higher order panel method, this is how we discretize the surface, the patches are big and we really do not assume that phi to be constant. And, in that case you know remember in this equation 19 you do not have this half into σ, that you do not have, right. Now, in here in lower order panel method is a fine discretization we do. We assume everything is a quadrilateral panel and then we can solve this problem. Solve the thing and here in that case you have to take this as half σ_i, right.

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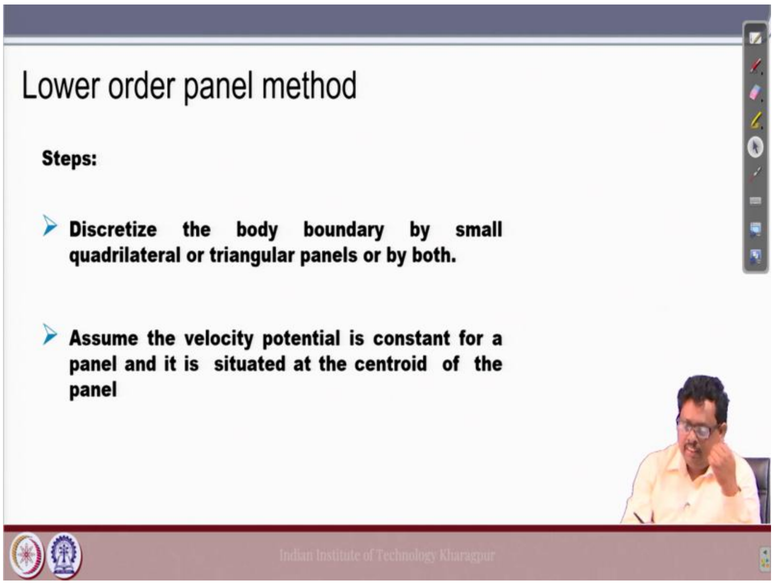

So, this is how I mean again I am repeating the same thing again and again just so, that you will be more comfortable with these things. I understand that, it is you know in order to understand it fully the repeatedly you have to see the thing. So, again many times I said the same thing here also the idea is discretize the body that boundary that this small small quadrilateral patch or triangular patch or both quadrilateral or triangular.

And after that, we have to assume this velocity potential to be constant for a single panel and it should be the you know different for the different panel, right.

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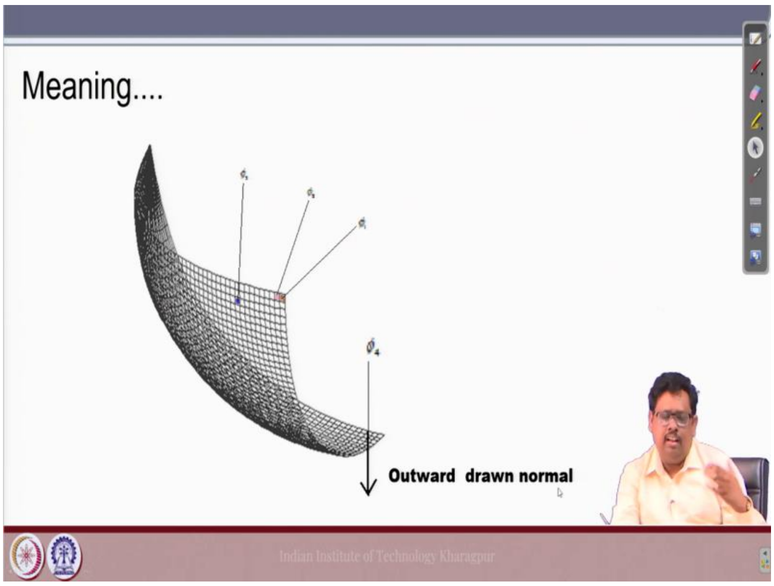

So, once we do that, then we can solve the equation. Now, here this is some understanding of the what I said. Now, I just take a one quadrant of the you know the this sphere now here you know sometimes we are using the symmetric property. Now, this is the geometrical symmetry in all other axis, right. So, I think we will going to discuss later on like, if we model if you if the body's has a symmetricity then you do not need to model the whole thing. So, you can model the one part of it and then you can use the property of the symmetric property and you can get the all other things anyways.

So, in case of a sphere only one fourth is good enough for modeling. Now, if I do that now, the idea is to tell you that here you can see this phi 1, like it is may not be visible properly, I just write ϕ_1 this panel ϕ_2 next panel and ϕ_3 some panel is very away from this first two panel. And, you can see the ϕ_4 is further away. So, here unlike the finite element where you need this matrix in a specific format here we really do not need that way, ok.

And, if you remember the finite element is the-this is the diagonally dominant matrix, but here is a you know this matrix sparse matrix. I mean; that means, it has the value everywhere, right, its not diagonally dominant, ok. So, therefore, here it is not essential that you take all the panel in sequential order or some order not necessarily, ok. So, this is that actually makes your life little bit easier, why? Because, I will tell you that when you model it or you mesh it using some commercial software.

Sometimes, it maintain, sometime it may not the maintain this the this orientation. So, absolutely there is no problem with this, ok.

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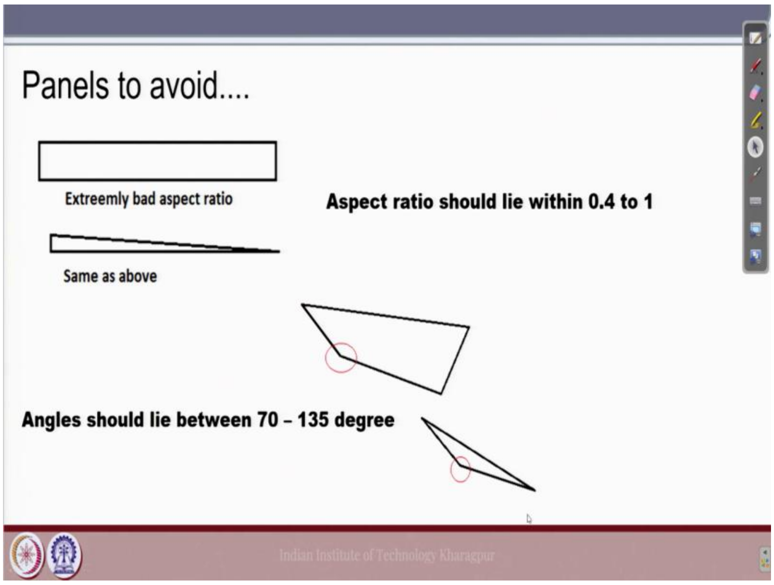

And also, again the same thing that the panel you need to avoid, right. So, these are the panel that extremely bad aspect ratios panel you need to avoid or also you need to avoid that this angle is very high, right. So, it is a thumb rule, I do not say this is some theory associate with this, it is not.

Of course, but, when you using some software like (Refer Time: 21:30) thermoses some other software, so that time you need to take care that your this aspect ratio, I mean you will get the better result, if this aspect ratios if you maintain 0.1 to 1 and for that and also for the quadrilateral panel this angle should be between 70 to 135 degree. I mean if you maintain this, then you know definitely this that what is said that quality of the result would be good.

Like, it is very tough to tell that you know it there is a there is a many many dependencies like this numerical code, writing numerical code never be an easy task because, it is not all about the theory; it is all about that numerical I mean sometimes everything is correct, but you really do not understand, why it is not numerically stable. So, there will be many many reason.

So, you have to be make sure that everywhere that you are at least you are following the basic thumb rule so that the core engine which is the writing the integral part, right. So, that does not take you know taken care of other thing like the normal is define the normal is fine. And, then this area is not very thin, aspect ratio is not that bad.

So, that variation that you are what you are assuming that phi is constant for a one panel different for the different panel. Now, if these two panel is very large so, we can see that you know I will tell you that what I am what I mean like.

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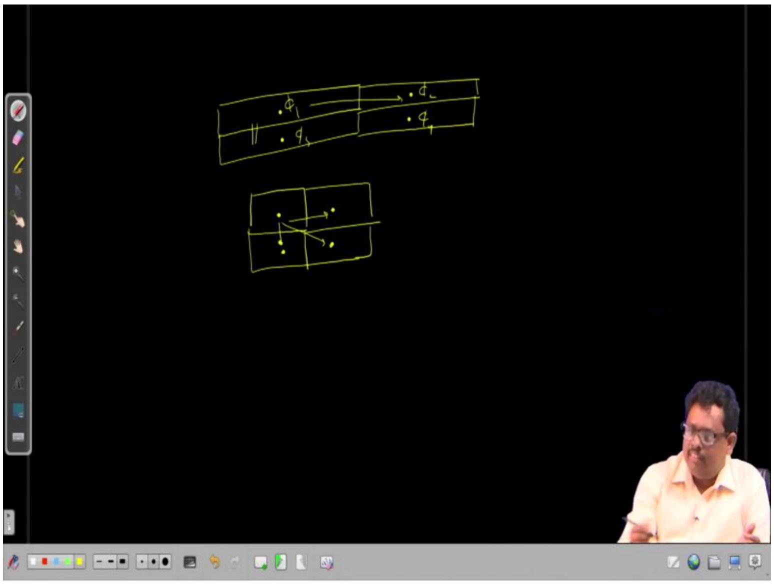

So, now, if the aspect ratio is bad, then you can assume that you know that now this is the two thing. Let us take these are the your panels. Now, here you have this let us say phi 1, let us say phi 2, then phi 3 and you have phi 4. Now, you see that phi 1 and phi 3 is very close by; however, phi 1, phi 2 is far away, because of your elongated panel.

Now, compared to that, if you have this panel, right, some panel like this. And, then you can see that the distance from this and distance from this, I mean this neighboring distances are in kind of almost like a same order. Like this sort of things helps actually in your in your coding. So, that is why you need to take care about all these things, right, to do that, ok.

So, I think today we are going to stop here at this point. And, then, we are going to set up the integral, I mean the conversion of the integral equation to the algebraic equation. How actually do with respect to this lower order panel method. Now, here it is not as same as before, I am not taking that one-point Gauss quadrature rule, we have to take either two-point Gauss quadrature rule or three-point Gauss quadrature rule, right.

And, what is that? And then, how I take care about the normals in very unified way? And, how I do a do the perform the integration over the panel? All such thing will be discuss in the future class.

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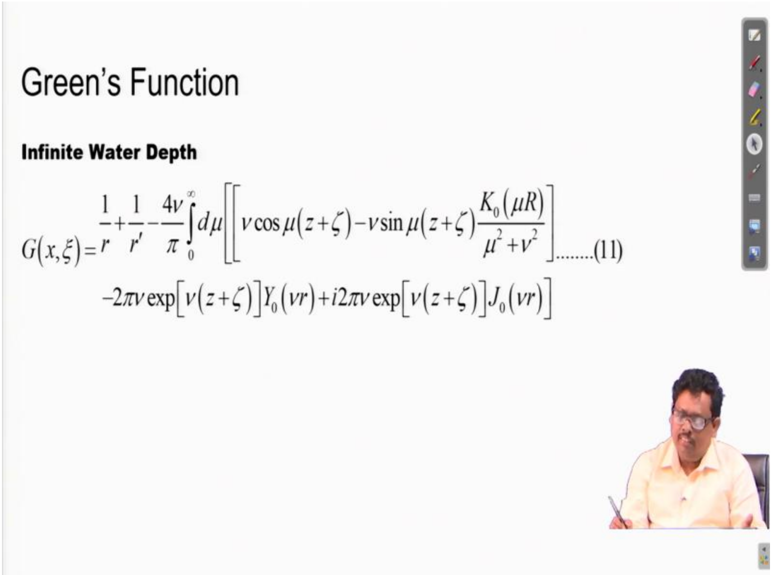

Why, I am discussing all such thing that is one major reason is this computation of this Green's function, ok. Now, here we definitely we are going to discuss later on you know, here that just for to realize you that how complex things are here. Here you can see there is a two component, one is $\frac{1}{r}+\frac{1}{r'}$ the that is we can call the Rankine. And, when this i and j is very close to each other, that time finding out this Green's function this Rankine part is very difficult.

Now, this is one part; forget about this. Let us take the other part of it which is the regular part we can call. Then you can see here we can have in this equation now, you can see the integration is 0 to ∞. So that means, you understand that, if you look in this summation form. So, at that point that summation should be infinity. So, we have to go look for a you know the convergence solution, right; that means, if; that means, your numerical solution should be convergent, right.

And, then, top of that so now, the integration limit is ∞. Top of that you have this all these Bessel function this first kind, second kind, modified Bessel function and all of these functions are also in infinite series as you know. So, now, you can see the complexity of this particular thing. First of all, it is not only, that it is as a singularity, where $i = j$, at that particular point also, you have to you know formulate a - you need to compute a Green's function, which the integration limit it is you know 0 to ∞, right. So, therefore, it must be I mean you have to look for the convergence, right.

So, that is how that sometimes you can say that this results is not coming correctly, you can find out some numerically inconsistents result is coming, why it is so? Because, sometimes when you integrate this not necessarily that you are if you are not writing very tight code, ok. And like if you take care of all aspect of the meshing everything, then this might go infinity. So, then at that point you do not get the convergence solution.

So, therefore, that is why that is the main reason I am keep telling that thing that you can actually these things is not in your hand it is the coding, the converge, non-converge, does not depend on then what is your source point? What is your field point? How are doing the integration? Many things and it is not possible for us to look at each aspect going through the code and make sure that things are going well.

However, what we can control from the very beginning, that the basic things the basic geometric parameters if I set it correctly, like the meshing, the normal should be uniform, it should be the outward drawn normal or inward drawn normal, based on that you have to write the code, you have to; you have to adjust the sign minus, plus everything. If these things if you do correctly, definitely that there is a possibility that the - that you will get the better results at the end, ok.

So, I think today let us stop at this point and in the next class, we are going to form the algebraic equation; that means, that we have this integral equation and then this integral equation, we are going to transfer into the algebraic equations and then we have to see that where we have the challenges. And then how numerically we sort out those challenges. And, finally, how I get this added mass damping exciting force etcetera, ok, we are going to discuss from the next class, ok, till this point.

Thank you.