Numerical Ship and Offshore Hydrodynamics
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Lecture - 24
Frequency Domain Panel Method Part 4

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Hello welcome to Numerical Ship and Offshore Hydrodynamics. Today is the lecture 24. Today we are going to discuss this following topic. We have to discuss about the how we can make the influence matrix when you are solving this source panel method, right. Where we know that how we can make this influence matrix, when we solve that problem infinite fluid domain and where you can use the source dipole panel method.

But here, we are going to do with the source panel method. And then again we need to find out how to convert this integral equation into the algebraic equation.

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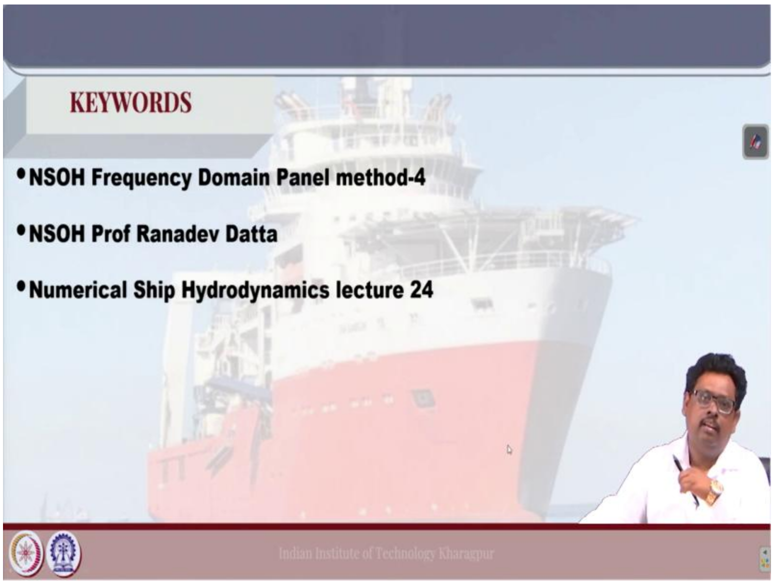

So, these are the keyword that we are going to use to get this lecture, ok.

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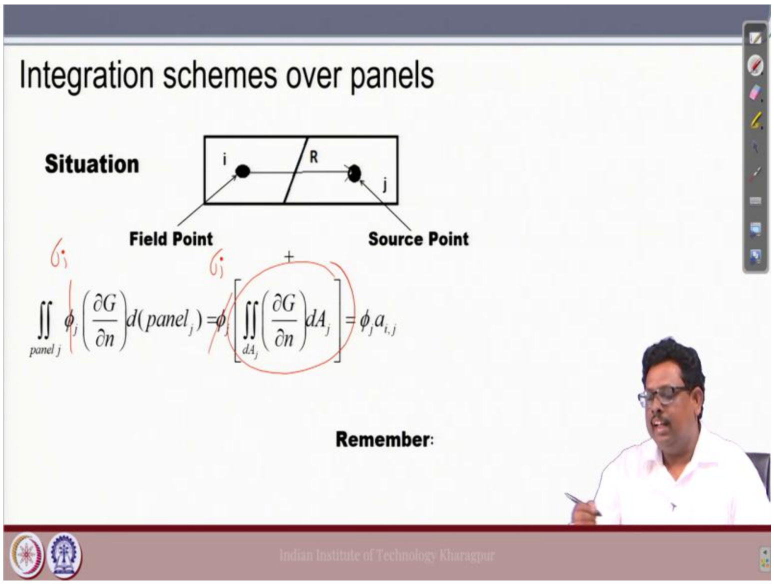

So now, let us start. Now, as you know that when we are doing the integration that integral equation that ϕ that any point in the you know this p point p let us say called the field point.

Now, at this stage you should not have any confusion about what is called the field point, what is called the source point. So, this field point p could be any point in the domain and then we placed it the on the panel. Now, so as you know that we place this field point at the panel i and then, we need to integrate that all the panels in the that inside the boundary I mean in case of a Rankine panel method then the whole free surface plus body.

And in case of this vortex type solution in our case it is the body. So, it means that I discretize the body, right. And then, we need to perform the integration over the body. So, I just take one arbitrary panel j, then j should be runs from one to n like, right, fine.

Now if it is so, then this is how actually we are going to do it the form the influence matrix, ok. Now, here we have we know that how we can do this for the one-point Gauss quadrature rule, but here really we are not going to use any kind of this one-point Gauss quadrature rule.

Here we are going to use sometimes the two-point Gauss quadrature rule, sometimes three-point Gauss quadrature rule. Sometimes of course, one-point Gauss quadrature rule also we are going to use, but now we have to understand very clearly how actually we can convert this.

Now remember, in our case if you if I let us go back to our this just let us just for the for your reference. If you remember that in case of this one-point Gauss quadrature rule. So, and now I am just going dealing with let us take the source panel method and try to figure it out.

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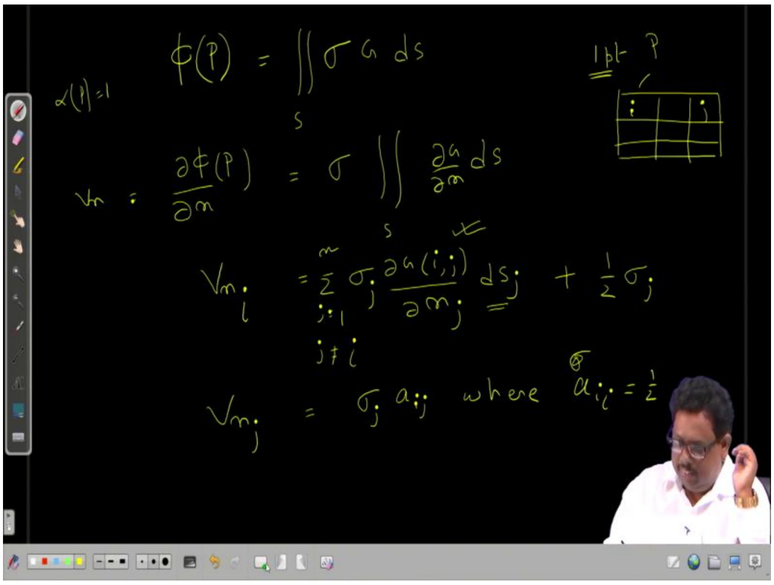

So, now in case of a source panel method it is that this is nothing but this $\alpha(p)$ now, in case let us not take $\alpha(p)$ is let us take $\alpha(p)=1$ and this 4π let us bring back into the right hand side of the equation. Now if it is so, so then $\alpha(p)$ is nothing but for the source panel limit is sigma $\sigma G ds$. So, when you have this the surface s. And now, if you differentiate with respect to n, then the left hand side become $\dfrac{\partial \phi(p)}{\partial n}$.

And which is nothing but, here σ will come out and over the s it is $\dfrac{\partial G}{\partial n} ds$. Now, if you remember, if I try to do this with our lower order panel method of course lower order panel method and then, if I take this 1-point Gauss quadrature rule. If I take this 1-point Gauss quadrature rule, then what is our strategy? Our strategy is supposing this is my panel. So, this is my panel right and then, in this panel let us take this is my i^{th} panel, right.

This is my the source the field point P and these all are the j_{th} panel where you can think of the source point j, right. Now, here $\dfrac{\partial \phi(p)}{\partial n}$ is equal to of course Vn. So here, I just I just write it is V_n for the i^{th} panel. So, I called this is Vn_i. This is what is I am taking in my left-hand side. Now then, if you remember in the right hand side I should write it is now it is runs over j = 1 to n, right. Where j $\neq$ i, that is the same thing that is what I said this equals to $\sigma_j \times \delta_j$ it is i,j by δ_n for the j^{th} panel and we need to multiply by ds_j. This is how I should write down my influence matrix and this is also of course the σ_j.

And when i = j this is we have to take plus half of you know sigma j, that is what is that is what actually we have seen from the last class. So, this is how we are going to discretize the whole thing, right. So then, it is very simple for in case of a 1-point Gauss quadrature rule. I am taking the functional value at the centroid of the panel and then I multiply it by the area, right.

So, it is very easily it is sigma j and then, it is and in fact call it is a_{ij} where we can show where I can write that when a ii that should be is equal to half. So, this is how I actually very easily we can do this, right. This is how we are going to do it for our 1-point Gauss quadrature rule. But now, this is not 1-point Gauss quadrature rule. So, now here, we really we are not doing this way.

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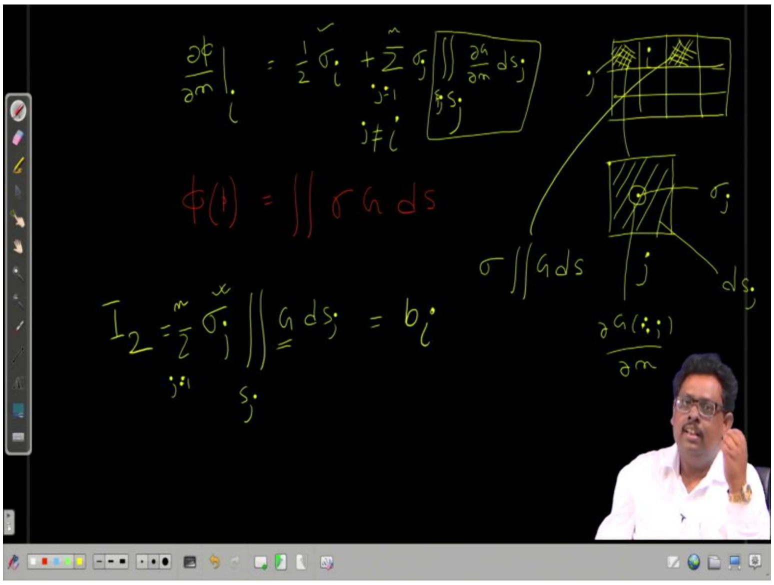

Now, if you remember this what in the left hand side let us start from here. It is $\frac{\partial \phi}{\partial n}$ this is for the of course for the i^{th} panel and this should be as you know it is half of σ_i that you know. Then, plus now what I am doing over here this panel; now here let us see this is your i^{th} panel where I can get this half of σ_i .

Then, what is doing. Now, here in this panel let us say this is a particular j^{th} panel. So, in this j^{th} panel what I am doing as follows. I assume in this j^{th} panel my sigma is constant.

That is what we are doing for the for lower order panel method. We are assuming that for each panel, your velocity potential phi as well as the strain sigma is constant. For a single panel different for the different panel.

Now, while doing this integration over this j^{th} panel since we are assuming, now if you take this let us say I am just taking out this sigma it is a big this is my j^{th} panel, ok. Now, in this j^{th} panel what is happening that sigma is at the center of this panel and you call this as your σ_j, right.

And then, but we need to integrate this. Earlier what we are doing is that, at this point only we try to find out the functional value that is $\frac{\partial G(i,j)}{\partial n_j}$. That is what I am doing at the centroid and now, I multiply this by dsj that is what the strategy for 1 point Gauss quadrature rule.

But now, here we are not doing this. What we are doing is as follows I am writing this $\sum_{j=1}^{n}$ and of course, this j $\neq$ i this till this point everything is same. But then, what I am doing I am taking σ_j out of this integration sign. Because, σ_j is constant over that panel; however, we need to perform this integration, ok. So, it is let me write in bigger way. We need to perform this integration over this panel then, it is like this. Now, it is clear.

Now, the difference is earlier I am using the functional value multiplied by the area and we are getting it, but now, we are performing the integration we are performing the integration over this panel. And this would be the numerical integration; it could be if it is a 1 point Gaussian quadrature rule, then it is similar to this. How otherwise it will be different. So, that is why now let us go back to our main slide.

So, here that is why I am writing that this is this in panel j that is it is actually it should be σ_j , ok. So, let me just correct this one here, here it is actually it is σ_j , right. So now, this actually what I am doing that I am taking out this sigma from the out of this integration sign, right. And then, I am performing this whole integration, ok. So, this is actually we are going to do, right.

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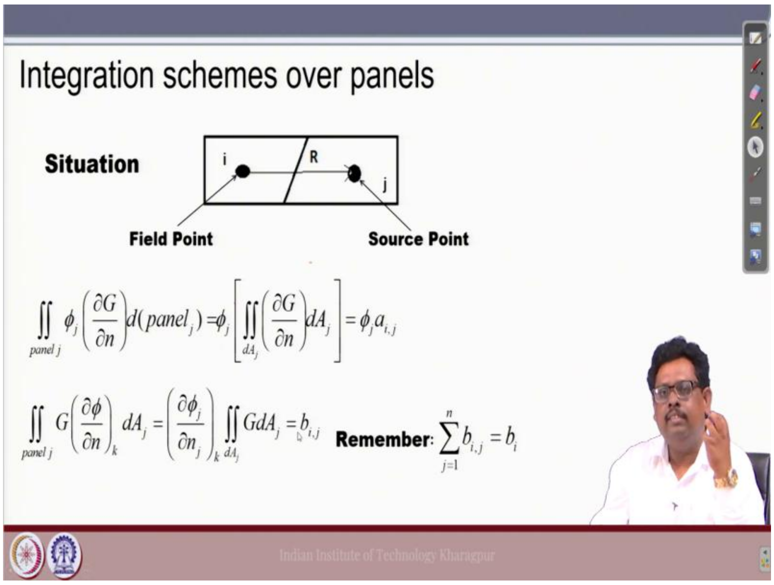

Similarly, if we consider this other part actually this one, where we just integration over the G; where the $\frac{\partial \phi}{\partial n}$ is the $\frac{\partial \phi}{\partial n_j}$ basically that is again is a constant over a panel. So, it is coming out of the integration sign and then, this integration is only G into over this panel A_j and then, we are getting this right-hand side this one.

Let us see, that how the other part we are doing. Now, if you look at this second part then, once we know this sigma, right. If I solve this equation, I get the equation for the sigma. Now, remember this equation I need to replace this over this sigma into G. ds. So, then these things we need to replace over here. So, after solving this equation I am getting the value for σ .

Now, once I get this value for σ , but then again I need to perform this integration also, right. Now, here is σ is known to me, right. So therefore, essentially my integration becomes here now this let us say for the second part. So, now my integration is become only in this panel it is integration of G into ds, right.

Because, sigma is again constant and then I take the σ outside the integration sign. So, essentially this integral I can call is I_2 is nothing but, my $\sigma_j \iint_{s_j} G ds_j$. And then, we can now we have to repeat this for all this. So, I just write that j = 1 to n and this now everything is known to me right here because, my sigma is already known from my previous equation I solve it I get it. And then, here this in integration this G is known to me. So, everything is known to me. So, essentially I will get a number so, I can call this is for b_i for the i^{th} you know field point, right.

So now here, let us go back again and let us see that if we understand this equation correctly or not. If we integrate this the first equation, this equation if I integrate. So, then I have it is of course the sigma j into A_{ij} right and now, once I know the value for sigma over here. So, then I substitute this sigma here.

So, once I substitute the sigma over here and then, I have this the second part the integration. Now, everything is known to me, right. So, therefore I can get a number and this number is called b_i. So, this is actually this integration scheme for this frequency domain panel method, right. Earlier we have only single equation, where that sigma and phi I mean that means, G and ∂n are both are there.

But here, first I do the integration for $\frac{\partial G}{\partial n}$ and then I can get the influence matrix ok, which is σ_j i j. And then, once I get this value for this σ then, I can use this σ to get the value for ϕ, right. So, this is the scheme earlier in our source dipole method in single equation we are solving everything. Now here, for the only source method first we are solving for sigma and once we know the value for sigma, then I use this value and I solve for phi. So, this is the only difference, ok.

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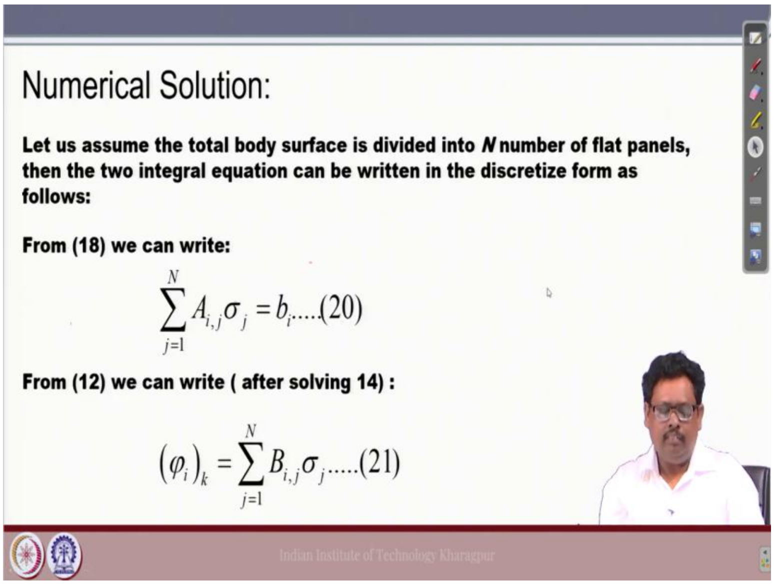

So, as I said that so, once I solve this equation then, actually I am getting this equation $A_{i,j}\sigma_j = 20$ and I know how i this $A_{i,j}$ is basically the influence matrix and once I have this influence matrix I can get the value for sigma; and once I know the value for σ then I substitute this in the $B_{i,j}$ right and $B_{i.j}$ we know that how we got this $B_{i.j}$, right. In this equation, we are going to get this $B_{i.j}$ value. So, once we have this $B_{i.j}$ we can solve and we can get the value for φ ok, fine.

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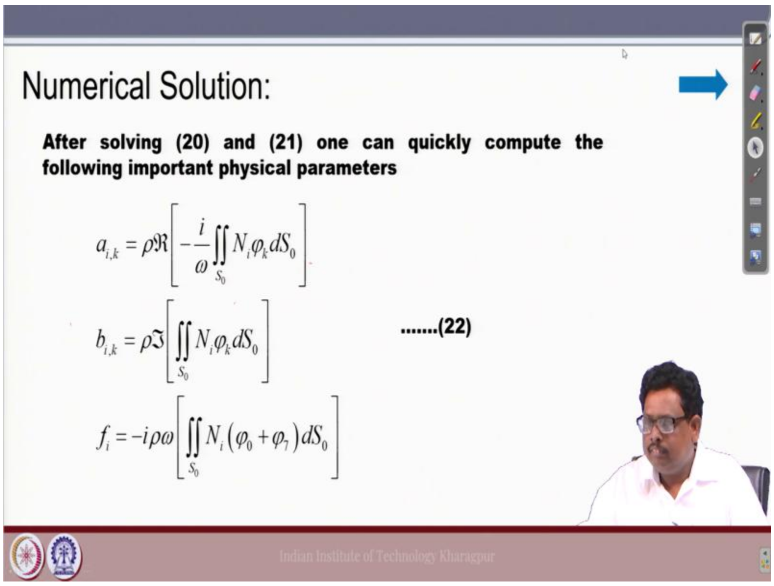

Now, once we know the value for psi then, how actually I can write the added mass and the damping and of course the exciting force. Now here, in this equation 22 I can I write three expressions, right; the first expression for the added mass, the second expression for the damping and third expression for the exciting force. Now, let us see that how actually I get these values.

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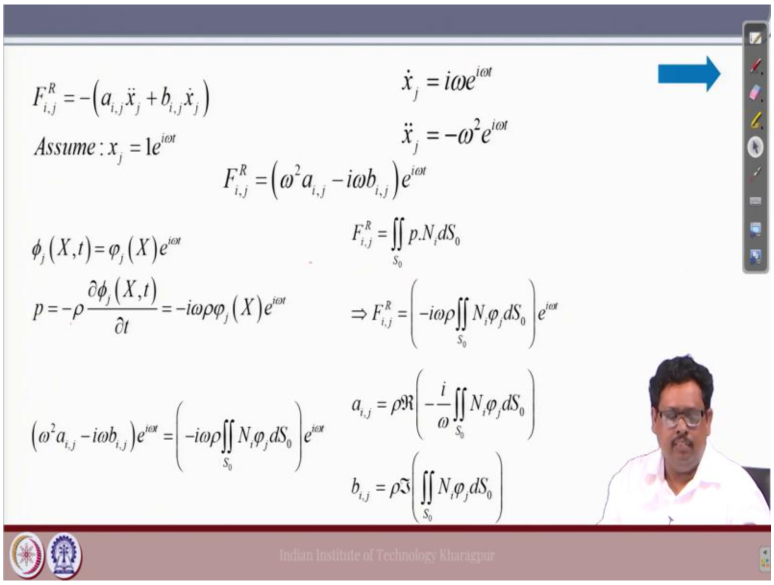

Now, if I go here, I know the radiation force is nothing but $\left(a_{i,j}\ddot{x}_j + b_{i,j}\ddot{x}_j\right)$, right. We know that right; it is added mass multiplied by the acceleration and then the damping multiplied by the velocity. And also if you remember when we write this the boundary value I mean the boundary conditions that time we are taking one assumptions that I am for the radiation force of course, you are solving for the radiation force I am assuming that I am oscillating the body in the direction of j.

And amplitude of the oscillation is 1. So therefore, here I am taking that since I am oscillating the body with a unit amplitude then I can write the equation as x_j = 1 into $e^{i\omega t}$. Now once I do that, then easily I can get the velocity and the acceleration. So, velocity will be $i\omega e^{i\omega t}$ and if i differentiate two times it will be $-\omega^2 e^{i\omega t}$, right.

Now, I substitute this into the radiation R. So, once we substitute this i just sub $\dot{x} = -\omega^2$. So, I just substitute over here and I substitute x_j = $i\omega$. So therefore, I can get my radiation force in terms of omega square a_{ij} minus this, right. Now, here this is how actually I can get the force, how I get the force. So now, so solving that equation right; solving that equation I mean solving this equation I get the value for ϕ k, right.

So now, once I get the value for phi k the next job is I am going to get the pressure; that means the radiation pressure field, right. So therefore, if I do that, now I assume my phi is harmonic. So, I assume the phi equal to $\phi = X \times e^{i\omega t}$. Now, if I make the you know the pressure is $-\frac{\partial \phi}{\partial n_t}$. So, definitely I am getting $-i\omega\rho.\phi_j.e^{i\omega t}$, right.

So then, how I get the pressure. So now, this pressure if I integrate in the direction of i so, I get the force in the direction of i, right. So, I oscillate the body in the j direction, right. If you have confused this I mean refer to my previous lecture, right. When I say that, I can oscillate a body in some arbitrary mode j, but still I can expect some force along the direction of some i; that means, I am oscillating the body in the heap mode, but still I can exit some moment along the pitch or I can oscillate the body in the pitch and still I can expect some kind of force in the direction of heap. So, let us take I am oscillating the body in the pitch.

So therefore, I have to solve this equation where ϕ 5 right and I solved this equation for ϕ 5 and I get the solution for ϕ 5. But then, if I need to find out the force along the heap mode then normal has to be 3. So therefore, I am getting F of 3 5. So, the second index refers to the mode I am oscillating the body and the first index refers to at which mode I am going to get my force.

Now here, so since it is the i^{th} mode so, I have to multiply it by the i. So therefore, my force becomes $-i\omega\int N_i\phi_j.ds$. Now, remember this is definitely a complex number when you do that you are definitely going to get a complex number. So now, I just equate this two. So, I equate that $\left(\omega^2 a_{i,j} - ib\omega b_{i,j}\right)e^{i\omega t}$ into the right side where I am going to get the force, right.

So, now if you now relate the real part and imaginary part of it; so then, you understand that the real part goes with the added mass and the imaginary part go with the damping, right. So, that is why that I am getting the added mass equal to rho into real part of minus i by omega because, if you divided it by ω^2.

So, it is become $-\frac{i}{\omega}aN_i\varphi_j ds_o$. Now, here the dS_0 refers to the linear linearity; that means that it is z = 0, right. When you do the non-linearity that time we need to integrate over the S_b that exact weighted surface, but here I am doing the integration on the mean weighted surface, right.

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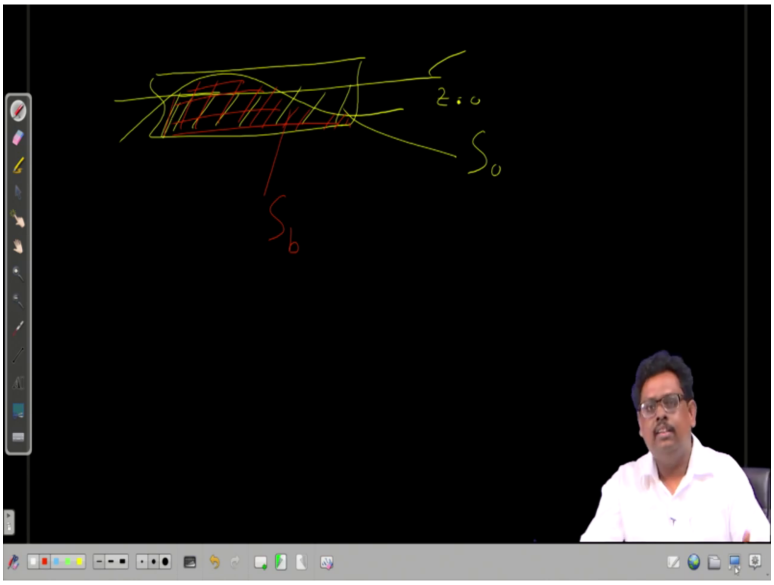

Now, if you confuse with what is the mean weighted surface and what is the exact weighted surface just again let me explain you. Here the mean weighted surface, here like if it is the let us take the body like this then, the mean weighted surface is basically the this the line z = 0.

And then exact so this I refer for this under this main weighted mean line this weighted surface I refer to S 0, but then if I consider this wave and if I take let us do this in the red, if I do this exact. So, this is called the exact weighted surface and if I do this integration normally we define by S of b, ok. So, let us coming back to the slide again now, here if you see that I am doing with the mean weighted surface that is why I am multiplying by the S_0, ok fine.

So now, similarly this a_i also now it is well understood that it is the similarly it is also the same way I am getting the sigma. Now, σ_0 is nothing but you are the Froude Krylov pressure the Froude Krylov force and 7 1 is nothing but your diffraction force. So, once you solve this σ_0 and σ_7 you multiply it by the normal N_i you will get the value for f_i, ok.

So, this is how I am getting this added mass $a_{i\,j}$, the damping or $a_{i\,k}$ if it is so here the notation is k and then the damping $b_{i\,k}$. So, here in this case actually in kth mode I am oscillating the body and then that exciting force the right hand side f. So, this is the overall the process to get the value for this quantity added mass damping and then the exciting force.

Now, once I get this, then I substitute this value in the equation of motion right and then I can get the value for the displacement, ok. How we are going to get this; how I mean that putting this value into this system of equation and getting the value for displacement and velocity those things definitely we are going to discuss, but not today.

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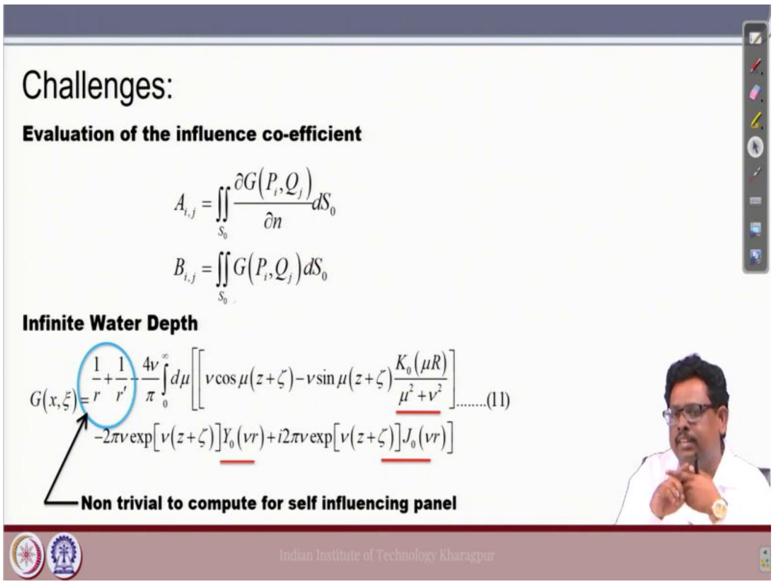

Now today, what I am going to discuss is that what is the complexity or the challenges. I mean, to compute this $A_{i\,j}$ and then $B_{i\,j}$. Now, if you look at here there is a two challenge. The first challenge is to get the value for $\frac{1}{r}+\frac{1}{r'}$.

Now, when r equal to 0; that means, when the source panel or field point you know coincide together. So, that time you can have a difficulty to get and also you have a you know nevertheless that we have the difficulty to get the value one upon r anywhere, right. Forget about is the close like, you have to set that what integration scheme one should follow to get a convergence solution anyway.

But, again if you look at the second part then you can see there are three terms K_0, Y_0, J_0 that means the there is a Bessel function is there. Already we have discussed that already this integration range is 0 to infinity is the improper integral in the mathematical. Let us forget the math's part, but at least we understand this integration that means, it is a infinite sum. If I do that in numerically I have to do a infinite sum.

Now, once we do this infinite sum then, it is a time consuming first of all and second thing it may converge it may not converge, right. And if we get a non converging sum, then the solution completely wrong. Second one, again we have this Bessel functions of different kind. All these functions are again is a infinite sum. So, again to compute these values also sometimes you have to make sure this the range is always in a convergence like, you have value for K_0, Y_0, J_0 all are convergent, right.

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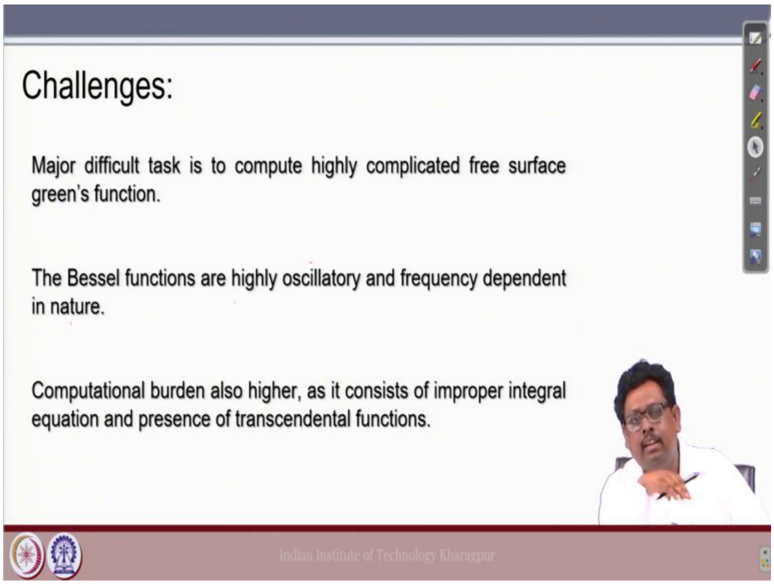

Now, I will show it is not always the case and let us see. Now here, so as I said the major difficult task to compare highly complicated free surface greens function right and then, it is highly oscillatory and frequency dependent in nature, right. And also as I said this is the improper integral that whole integral is 0 to infinity so, it is an improper. So, we have to look for the convergence, right.

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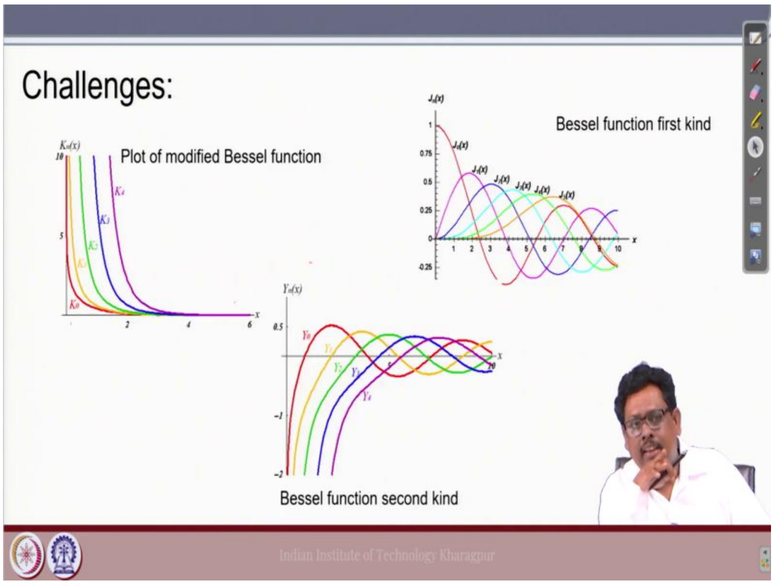

Now here, I say that not always it converges. If you look at this graph for K_0, now when you can see that when it is K_0 going towards 0, then you can see that the functions becomes infinity and similarly, same for Y_0 also, right.

Now, the thing is that it is asymptotically go to infinity. Now, thing is that these are the complex things that we need to deal with right, but we definitely we are so say we need a strategy to integrate the greens functions. So, what are the strategy to get the greens functions like this the whole part 1 by r as well as the free greens function that we are going to cover in the coming lectures, right.

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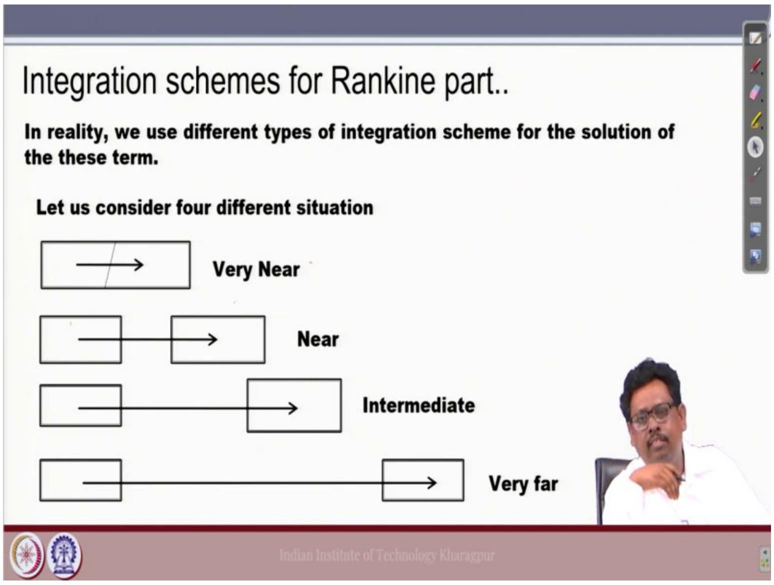

So, just for a brief that what we are going to discuss in the coming classes initially we started with the Rankine method and then, we have to see that how this is this based on the distance between the source panel and the field point we need to find out the different scheme for it, right.

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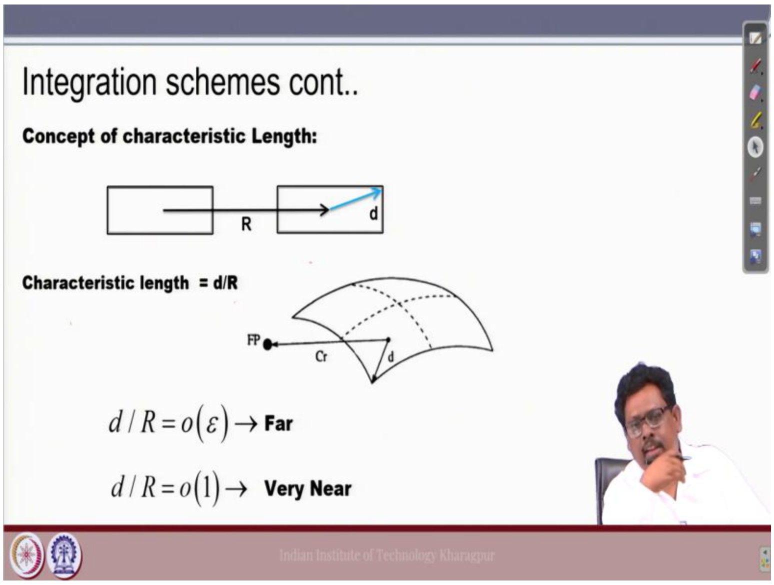

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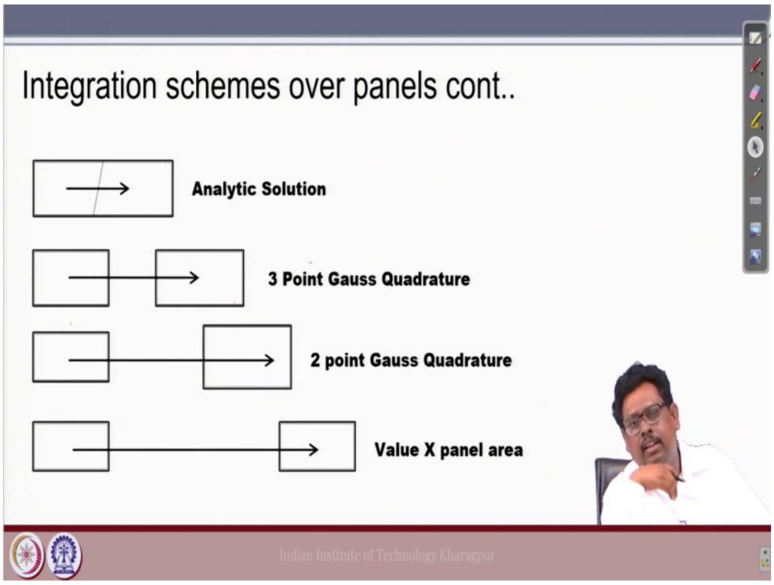

For example, we have to discuss something about this characteristic length what is the meaning for this characteristic length and also based on this characteristic length we are going to discuss, what are the integration scheme right. So, those things we will going to discuss from the next class. So, for today let us stop over here.

Thank you.