Numerical Ship and Offshore Hydrodynamics
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Lecture - 25
Frequency Domain Panel Method Part 5

Hello, welcome to Numerical Ship and Offshore Hydrodynamics. So, today we are having the lecture 25.

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Today, we are going to discuss about the integration of the Rankine part of the Green's functions. So, if you remember in the last class we have discussed about the Green's function and we have two part – one part is $\frac{1}{R}$ which is called the Rankine part of the Green's functions.

So, let us first discuss that how we can integrate this $\frac{1}{R}$ part and after that we can discuss about how to numerically how to integrate the second part which is the free surface Green's function, ok.

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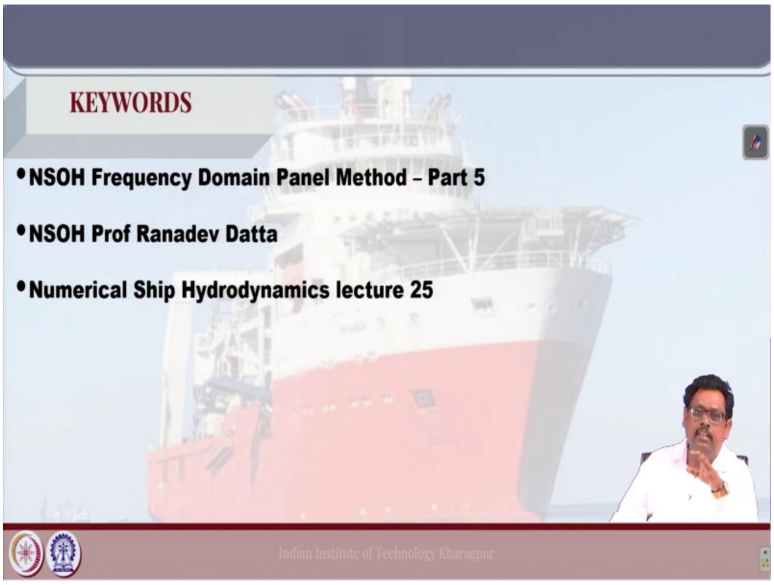

And, this is the keyword that we are going to use to get this lecture, ok.

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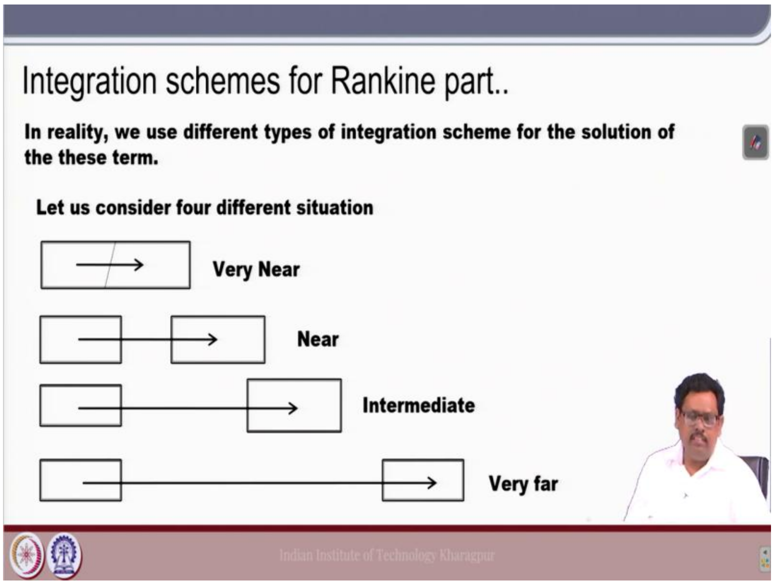

So, let us start. Now, in reality if you remember in our numerical ship initial days class, when we are going to discuss about the infinite domain radiation problem we are using the 1-point Gauss quadrature rule.

Now, if we use the 1-point Gauss quadrature rule that point we are taking the centroid of the panel is the so field point as well as the source point. And then we are performing the integration like we get the functional value at the middle point and then we integrate like integrate by multiplying the area, right. So, this is how we did that, but this is ok for just to understand about the theorems and just get hang on it I think this is well enough.

But, however, in reality we really do not want to do that because it is a lot of under estimation is possible because if that R is very small, then really it is not good to get only the functional value either it is overestimating or under estimating ok. So, it is not a very good choice to do that. So, then what we do is as you see in my last class we are actually integrating right over the panel.

So, now, we have a panel and then we integrate the functional value over the panel one by one, right. So, now, when you do that for $\frac{1}{R}$ then actually that we have to consider the different situation. Now, if we look at the first picture it is when we say that field point and the source point are really very close and we can call this a let us say very near to each other.

Now, in the second picture what we can see that field point and the source point have some distance, but that we consider the distance is not that much. So, we can call it is a near, and then we can call if it is little this far then we can call the intermediate. And, then it is the source point and the field point is very far and then that time we can call the very far.

And, then based on that what is the distance between this field point and the source point based on that actually we are going to discuss the integration scheme, ok. Let us see that what are the I mean, but then the question is that it is very qualitatively I said it is very near, it is near. It is intermediate it is very far now we need to assign some number, right.

So, in some number that number will tell me when I call this is very near, when I call this is near when I call this is intermediate and I when and when I call this very far.

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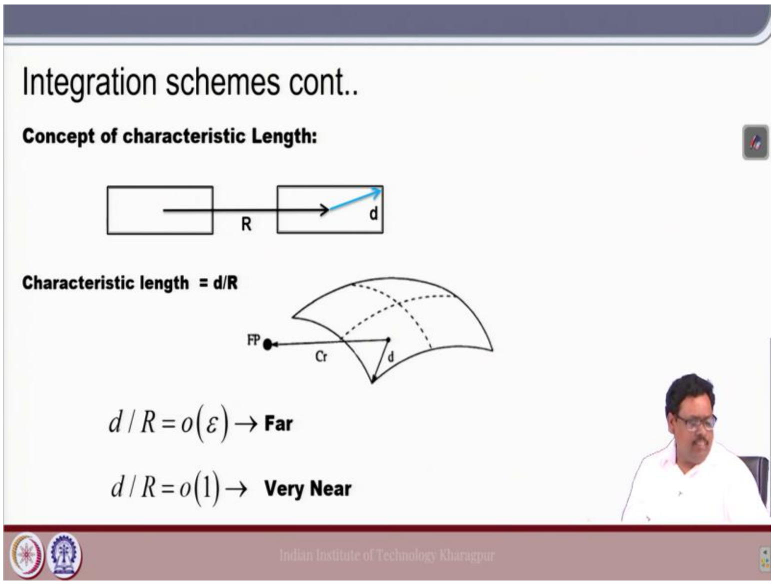

So, now this is how actually we are going to do this it is called the concept of the characteristic length. So, let us see that with respect to this characteristic length when you call it is far and when you call it is very near. Now, you can see that R the capital R basically the distance between the centroid of the source panel I mean point to the centroid of the field point. I mean like if you do the integration if you remember that we need to put place the field point at the centroid of each panel, right.

So, therefore, the distance of the centroid we can call that is R and then the d is nothing, but the diagonal from that I mean you can take any diagonal you can take the maximum value right you know from the. So, now, it is very nicely rectangular thing. So, all the distance may be equal, but if you consider this panel is you know like irregular shape, then this d may not be the same for each corner. So, therefore, you can take the maximum of it whatever ok.

So, then what we do is that we are going to find out the ratio between the d by R. So, now, and this is this we called the characteristic length, ok. Now, we have to find out that for the far like when this d by R is order of epsilon you can see that when is very far R become. So, big compared to the d and then we can call that is a far field or the point is far.

And, now if it is coming to the order of one like when you can see that when the source point and the field point are very close by then actually you can have this it is order of one. Even for the self influence panel it is in fact, that time you know d is basically more than the R sometimes, right. I mean if you do not use the 1-point if you use the you know 2-point or 3- point gauss quadrature then this distance is anyway.

So, now based on that actually we are going to discuss whether it is a very far or intermediate or near. So, it is order of one we can call definitely it is a very near and then we have to go for some different scheme and now, if it is let us say order of 0.01 or 001 then we can say that it is this far. So, we can use some other type of integral technique.

Now, this is I cannot tell directly that what is the value for it is your choice that it depends on the coder who doing the computer programming, he will decide that which one is he can think of very near which can think the very far or something like this. I can say that the epsilon in some abstract sense, but when you do the coding you have to put some judicial value for each scheme.

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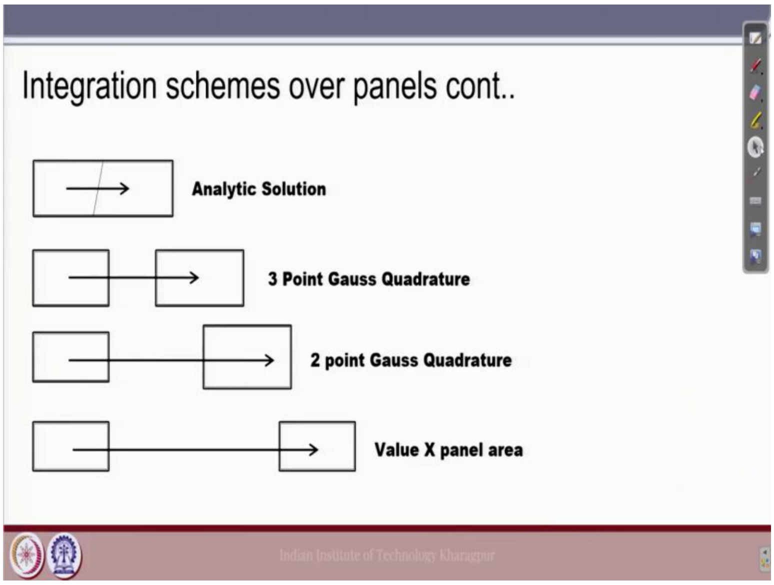

Now, let us see that you know based on that this what are the scheme that is that we are having. Now, if it is very near I mean which is the order of one, we can go for the analytic solution ok and otherwise if it is you can say the near we are going for a 3-point Gauss quadrature rule and if it is intermediate we can call a 2-point Gauss quadrature rule and then normally if it is very far we can call this as a you know 1-point Gauss quadrature rule.

Now, if you remember that actually we are talking about the integration of 1 upon R. So, now, why you are saying like 3 point and then 2 point analytic, etcetera? So, why it is so? Now, let us see that why actually like you need to understand that nature of the curve to find out that you know what would be the region for taking it 3-point Gauss quadrature or 2 point or you know the 1-point Gauss quadrature rule ok.

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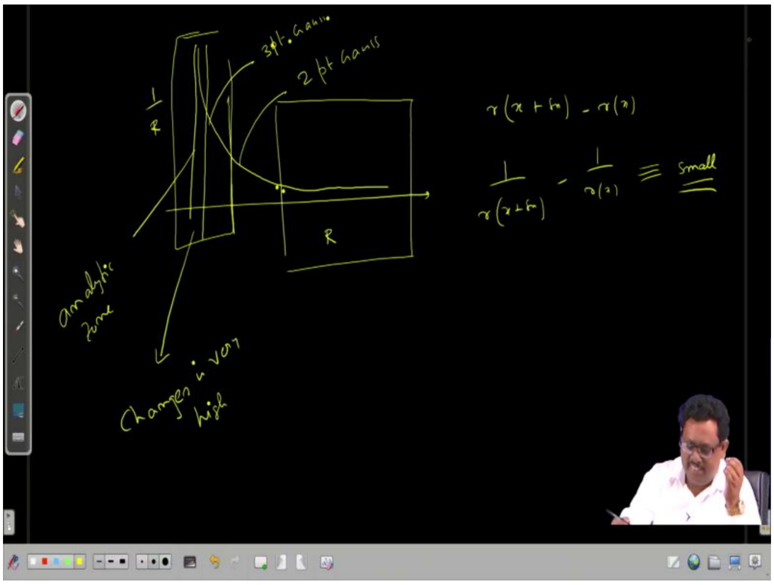

Now, if you look at this now, if you look at this the nature of this function you can say that if it is the functional value is now, if distance is the R the distance and if it is the functional value which is $\frac{1}{R}$, now you can think that it is actually like a rectangular hyperbola.

So, from this nature I understand that you know R very quickly go to this range. Now, when actually R is in this range, then you know the distance r let us say some distance x $(x+\partial x)$ and minus you know $r(x)$ I mean if in it is very far then $\frac{1}{r(x+\partial x)}-\frac{1}{r(x)}$ you know this functional value is may not be you know it is may be some small value may be some small values right in this region.

However, at abruptly in this region you know it is changes is very high. Changes is very high and here in some time here it is little bit more and here is. So, as long as in this region you can call this analytic solution this is you can say it is a analytic zone and may be this zone you can call is a 3 point Gauss quadrature rule and may be this region is you can call as a 2 point Gauss quadrature rule.

Now, you can understand it is the only very small area where actually this solution is abruptly changing, right. So, therefore, we have to only take care of certain number of panels not all. So, therefore, 2 point Gaussian quadrature rule you know we have to we can apply most of the situations right. Only very few cases we can go for 3 point or analytic ok.

Now, let us coming back over here so, now, we understand that what are the solution scheme.

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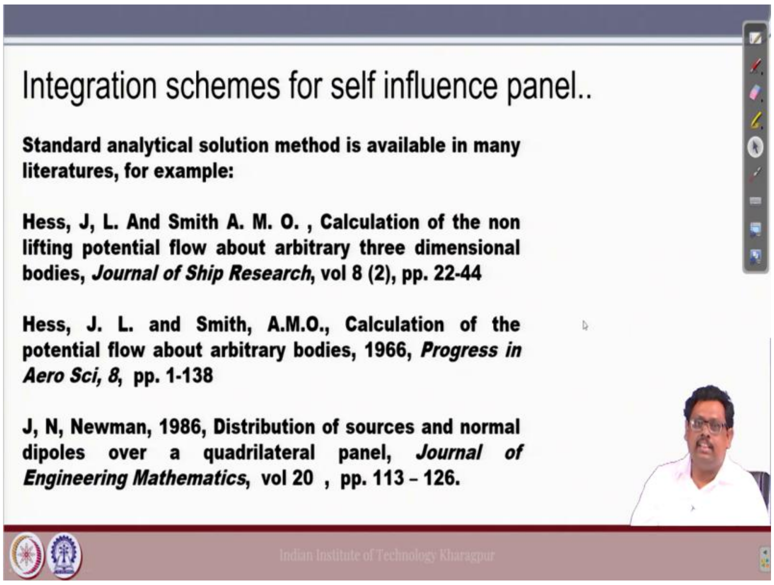

Now, today we are going to discuss about the you know analytic solution. Now, really we do not discuss about the analytic solution the theory because it is complicated and these are the reference that actually. In fact, we can you can get it very easily, it is all available in internet. So, these are the pioneer paper based on that actually this we are finding out what is the scheme for the analytic solution, ok.

Now, the theory is here. So, it is really a complicated theory based on some results from the differential geometry and also we do not discuss those things only. What we are going to discuss over here how what will be the strategy to go for this self I mean integration the self influence panel ok.

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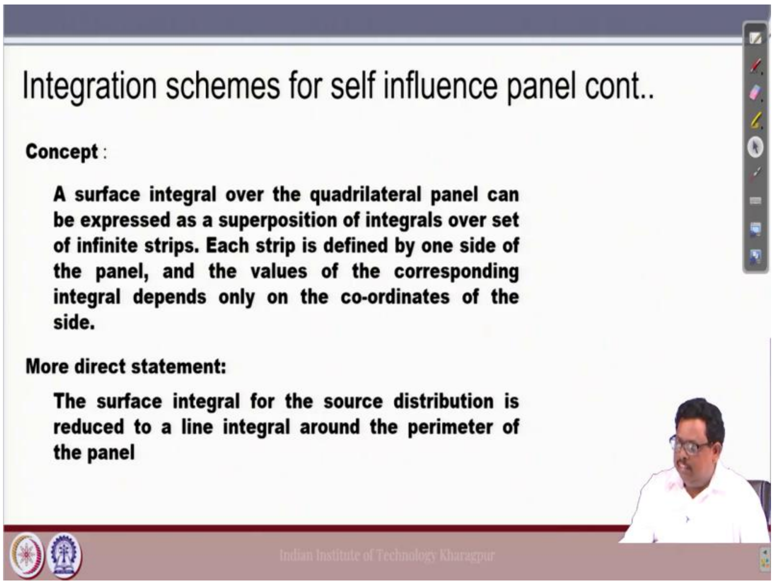

Now, the concept I mean the basic concept is as follows: the surface integral over the quadrilateral panel can be expressed as a super position of integral over the set of infinite strip. So, that means, you know this is true if you look at this trapezoidal scheme and all that the surface integral actually what you are doing is you are draw infinite number of strips and then you integrate that thing and then actually you can get the value, ok.

So, you know the more direct statement is as follows that you can say that the surface integral for the source distribution is reduced to the line integral around the perimeter of the panel. So, that means, in analytic scheme we really do not need to you know integrate over the surface that is the that is that if you integrate over the panel actually we can get the thing.

Now, interestingly this actually very funny, but you know if you try to calculate the area of a you know arbitrary surface 2-dimensional surface still we can use the same technique to do that. So, before we go into the main theory not main theory that main the strategy just for fun let us see that it is actually very interesting.

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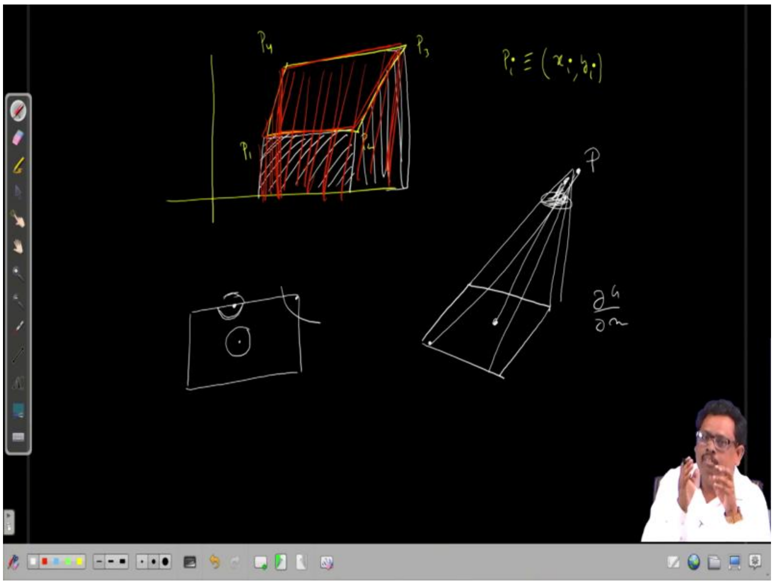

Now, what I meant to say that suppose you have a arbitrary ship shape arbitrary shaped a panel like this and if you are try to find out the integration over the panel. Now, in general if you know that we make it a vector and then we do lot of things like.

But, now if it is a 2-dimensional body let us say so, if it is let us say P_1, P_2, P_3 and P_4 and all this P_i it is let us say 2-dimensional body (x_i, y_i) then very actually very easily we can do that by using the line integral. Now, the simple line integral we know what is that trapezoidal rule. So, now, if you apply the trapezoidal rule when the area is positive I am using the white shade when the area is negative I am using the red.

So, now if you use the trapezoidal rule which is the line integral between here and here I mean then you can get this area. I am just only showing the intuitively like how this integration along the line helps you. Now, if you do this integration you get this I mean along the line P_2 to P_3 if you integrate it and then if you along the line P_3 to P_4 if you do that then actually you can have lot of red thing.

So, after doing this what you are getting is you know that this will you know this will cancelled out all this white part till this point and you can have only this area in this part. Now, if you do the P_4 to P_1 the farther so, then it will have cancelled everything and you can get the area only P_1, P_2, P_3, P_4. It is hold for any polygon also, ok.

Now, let us coming back here. So, we are doing it for doing the line integral to get this the coefficient the integration the surface integration. Now, here it comes some geometric you know knowledge.

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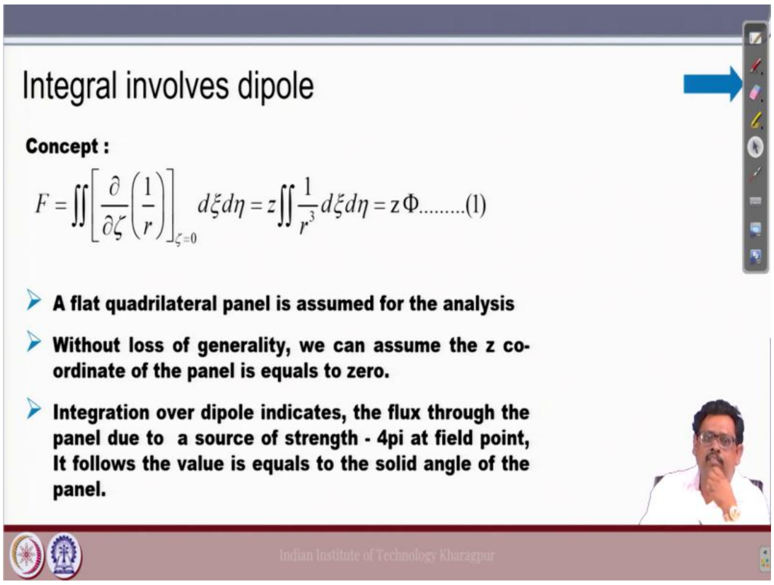

And, with the help of you know the concept of the dipole and then differential geometry we get some nice results actually, ok. Now, those things as I said it is very elaborately given in those paper here we are only going to discuss how actually we apply those results over here, but even if we try to do that still we need some basic concepts.

Now, here we can take a flat quadrant panel is assumed for this analysis. Of course, now here is the question now that we are going to discuss in the in next lecture not today. Here we are assuming that we have a quadrilateral panel where we have the local coordinate system the z coordinate is 0. So, therefore, we are having let us say x_1 y_1 $_0$, x_2 y_2 $_0$, x_3 y_3 $_0$ and x_4 y_4 $_0$. So, it is a local coordinate system I am talking about, ok.

And, then without loss of generality as I said that we can said z coordinate to be zero so, it becomes a you know local coordinate or local reference frame we are actually discussing, ok. So, now this is a; this is a result that you know we discussed a lot. It is F is nothing, but $\frac{\partial}{\partial \zeta}\left(\frac{1}{r}\right)$. So, let us take one a quantity and if you look at this quantity, now what is zeta is nothing but the point on the local point on the panel itself, right it is in the source panel.

If you remember that x, y, z refers for the field point, however, zi eta zeta refer for the you know the source point. Now, here in this case not z basically zeta equal to 0. So, if I do $\frac{\partial}{\partial \zeta}$ it is nothing, but the component of the under normal component, right.

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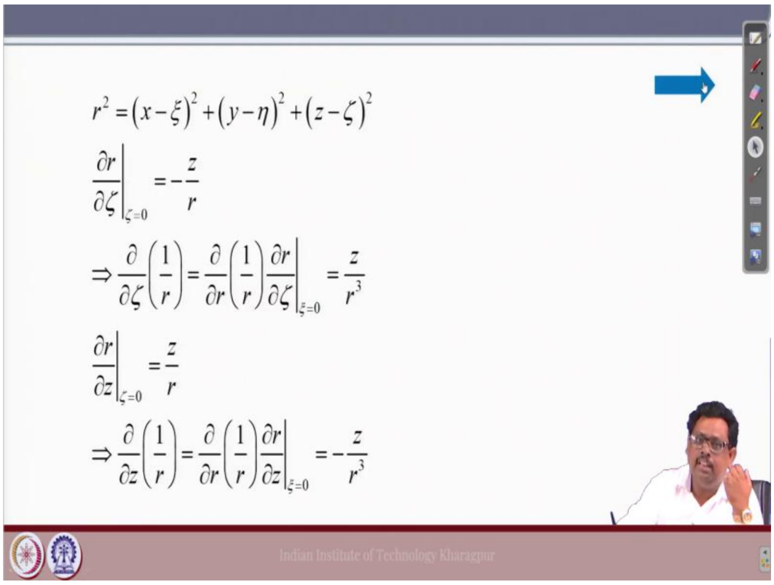

Now, let us see now you know that this definition for r is r square equal to $r^2 = (x-\xi)^2 + (y-\eta)^2 + (z-\zeta)^2$. Now, if you $\frac{\partial r}{\partial \xi}$ it is $-\frac{z}{r}$ definitely $\partial \zeta$ it is $-\frac{z}{r}$ definitely and then if you substitute the value $\frac{\partial}{\partial \zeta}\left(\frac{1}{r}\right)$ which is $\frac{z}{r^3}$ coming.

So, this already we have done normally we have done the later on right that del $\left.\frac{\partial r}{\partial z}\right|_{\zeta=0}$. Normally, we have done that, but now it is with respect to the ζ , right. Now, what is $\frac{\partial \xi}{\partial \zeta}$? It is nothing but the vertical normal quantity. So, it is so, what actually you know this F? Now if you look at the F if I go back if I look at this F it is nothing, but the dipole, right. So, here I can see that the integration of the dipole you know.

Now, here comes the that is that classical concept that is discussed in this Newman paper, they said that this dipole is nothing, but the normal flask and we also discussed before. Like the meaning is as follows. Now, if you have a flat panel let us say here if you have a flat panel over here and then if you have this here the source sorry the field point P, then the integral as I said that is nothing but the solid angle.

So, this is actually it is how it is so? That it is nothing but the normal flask and then geometrically it is proved that it is comes out to be a solid angle and that is the reason now as I said that if for the self influence panel when you do this $\frac{\partial \xi}{\partial \zeta}$, I told you we have to take the 2 π because if this point if you put it here, then solid angle become 2 π.

Now, if the point on the corner then it is this is the angle is $\pi/2$. If it is here, then not this side it is the this is the angle which is the π. Now, so, that is what we understand from the you know the discussion based on the differential. So, when you do this line integral and it is turns out to be the solid angle and if you connect that to the differential geometry. So, we are really not going to discuss in depth that part, but we take that result for us.

So, we are taking it is it should be 2 π when it is here and it is the solid angle here, ok. So, we use that result to find out the value of that integral F. So, that is the idea. So, with the help of this geometry what we are going to do is we are getting this value for this integral F which is nothing, but the dipole right, fine.

So, this is what I am trying to say that integration over the dipole indicate the flux through the panel due to the source strength of minus 4π at the field point. So, it follows the value equal to the solid angle of the panel. So, this is this results we are going to use like. We are really do not know why it is so.

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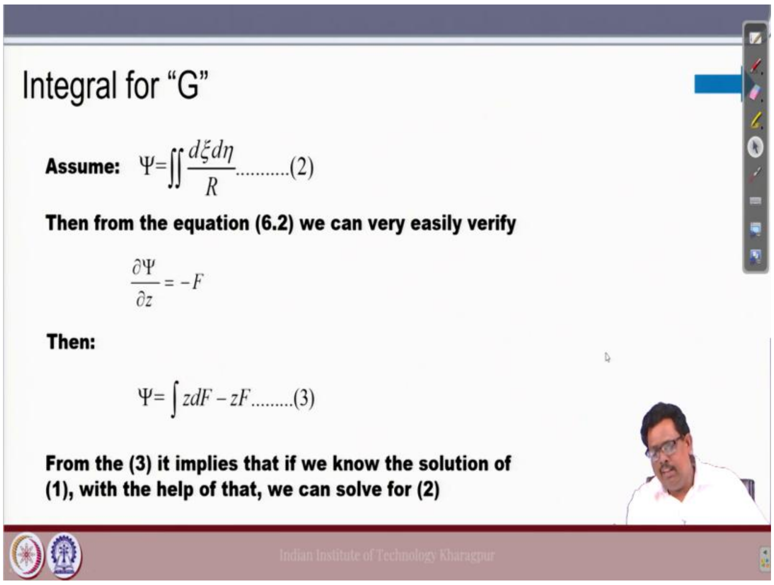

Now, having said that we are taking this value and then we are using some other concept. Now, this is what actually we are going to do which is the integration of $\frac{1}{R}$. So, psi you understand is nothing but the integration over $\frac{1}{R}$ in this local coordinate system $\frac{\partial \zeta}{\partial \eta}$, right fine.

Now, here you know here we are using this result that if you differentiate $\partial \psi$ with respect to δz you are getting is nothing, but -F.

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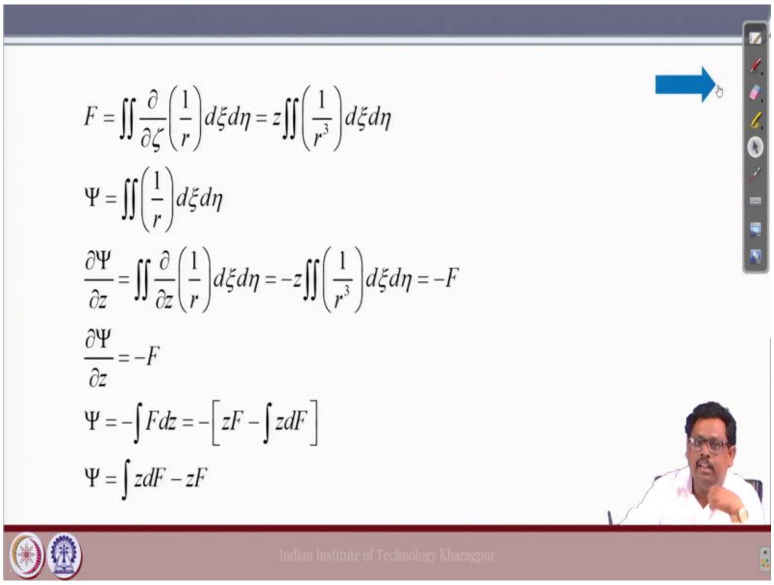

So, let us see that how we are getting it. Now, here that ψ is $\frac{1}{r}$ now if you do $\frac{\partial \psi}{\partial z}$, so, again it is nothing, but $\frac{\partial}{\partial z}\left(\frac{1}{r}\right)$. Now, if you remember that it is as same as you know if you remember that it is $\frac{\partial}{\partial z}\left(\frac{1}{r}\right)$ we did that it is $-z\left(\frac{1}{r^3}\right)$. Now, if we if I do that with respect to xi I am getting z into one integral $\left(\frac{1}{r^3}\right)\partial\xi\partial\eta$ that is that result actually we have already shown.

And, I think by this time you everybody should understand that this is now becomes should be become bread and butter for you. So, I really do not need to discuss much on this. So, if I differentiate with respect to z definitely I am getting $-z\iint \left(\frac{1}{r^3}\right)\partial\xi\partial\eta$. So, this actually my minus of F. So, I got $\frac{\partial \psi}{\partial z} = -F$.

So, then I can get psi is nothing but the integral of those that thing. So, I am getting $\psi = zdF - zF$. So, this is how I am getting the so, this is how I am getting the integral value, right. So, now, I understand that $\psi = \int zdF - zF$. So, equation 3 we need to find out to get the solution for ψ, ok.

Now, this is all about the thing. So, first I need to find out the integral of F, the moment I am getting the integral of F. I am getting the value for the ψ if I integrate the $\int z\, dF$ ok.

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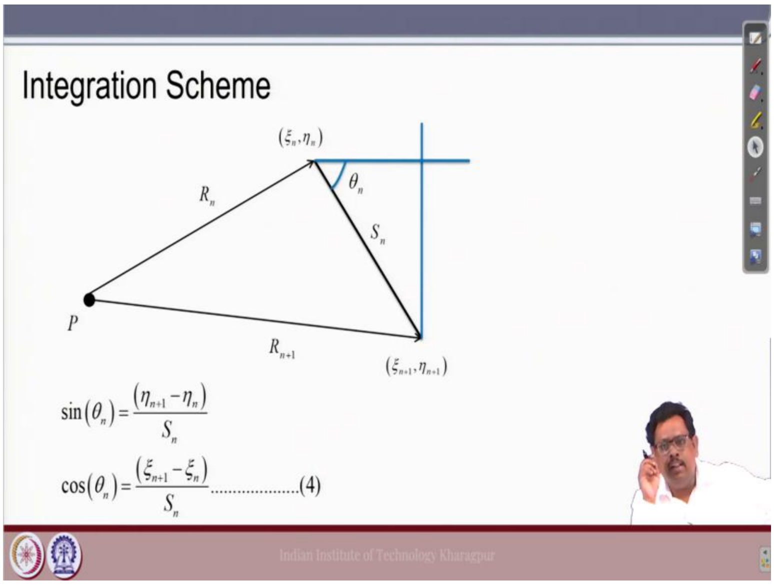

Now, here referring to that paper actually let us try to figure out actually how we can do that numerically. Now, let us take now as you know that it turns out to be the line integral. So, now, this is actually a ξ_{n+1}, η_{n+1} and ξ_n, η_n this is two arbitrary line in that quadrilateral, ok. Now, if you look at this line this thing you have to understand we are going is a clockwise direction, right.

Now, P be the source point. So, from P to this; this ξ_n, η_n, let us take the distance is R_n and then it is ξ_{n+1}, η_{n+1} if the distance is R_{n+1}. Then we can define sin theta n and then cos theta n, right. It is simple and S_n is nothing but the distance. So, what is that? It is nothing but that if I have a quadrilateral panel. So, then this the two point it should be xi ξ_1, η_1 and ξ_2, η_2 in that way. So, it should be, but you have to take care that it should be in the clockwise order, ok.

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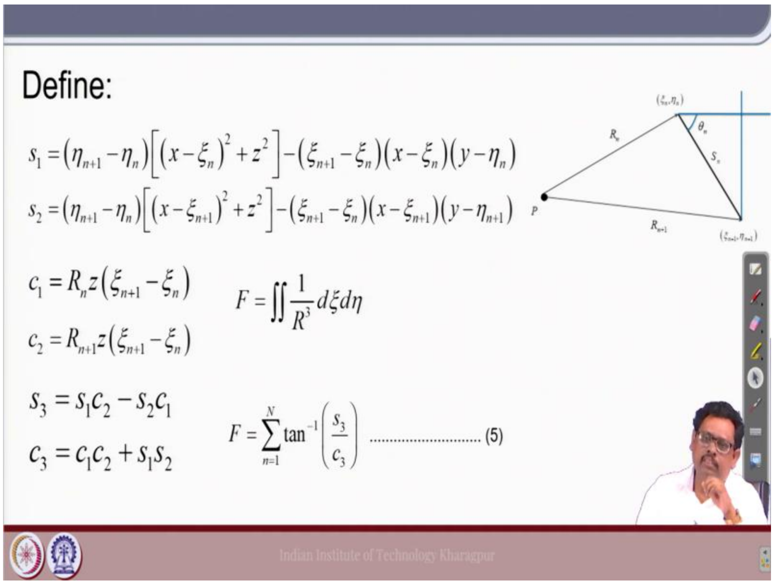

Now, if it is so, let me take this out here ok, yeah. If it is so, then we have to define certain parameters. So, we are defining this parameter. Now, this is purely mechanical ok s_1 in terms of ξ and η we are finding out s_1, in terms of ξ and η we are finding out the s_2 and then in terms of ξ R_n and z we are trying to find out the c_1 and c_2. So, these are something.

Now, you see like now is the radical scheme become very simple right it is only the mechanical thing there is nothing I mean nothing special about it. So, theory is complicated, but this application is really easy. So, then I can define another parameter s_3 and c_3 based on this c I mean this s_1 s_2 and c_1 c_2, and then we can find out this integral that F is $\frac{1}{R^3}$ which is the dipole that integral is it becomes the tan inverse s_3/c_3, right.

So, therefore, now in case of a quadrilateral that four time you need to do that, right. So, you have to do for you know ξ_1, η_1 to ξ_2 η_2, then ξ_2 to ξ_3, ξ_3 to ξ_4 and ξ_4 to ξ_1 in that way you need to perform this integration you can get the value of F.

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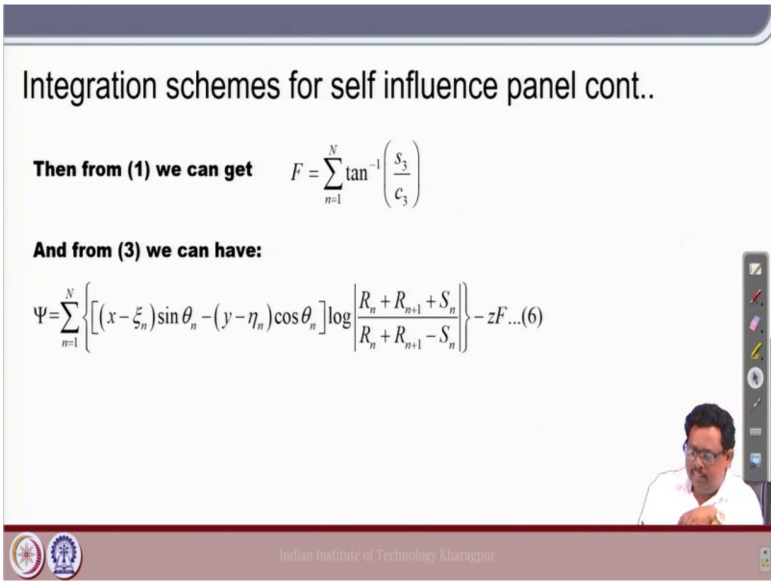

Now, once you know this value of F is known to you. So, from 1, we can get this is the value for my F. Now, if you remember that from the 3, that it equation is that integral z dF-zF. So, then you will see here that integral z into F also again referring to the same paper this is the formula is given, ok.

So, again you can see everything is already you have obtained the moment you defined it the moment you define the point p $_1$ ξ_1 η_1 and p $_2$ ξ_2 η_2 all these parameter R_n R_{n+1} S_n and then or ξ_n η_n x and y is the field point cos theta and everything is known to you.

So, this value for psi also you can get from this expression, ok. So, this is actually that is what we are doing for the self influence panel that time we really do not want to go any 2-point Gauss quadrature rule or 3-point Gauss quadrature rule and nothing we simply use this analytical expression.

Now, you know that as I said the theory part is really complicated and we are not discussing it. But, however, the results actually that we are getting from that theory it is really very easy to implement for the solution, right.

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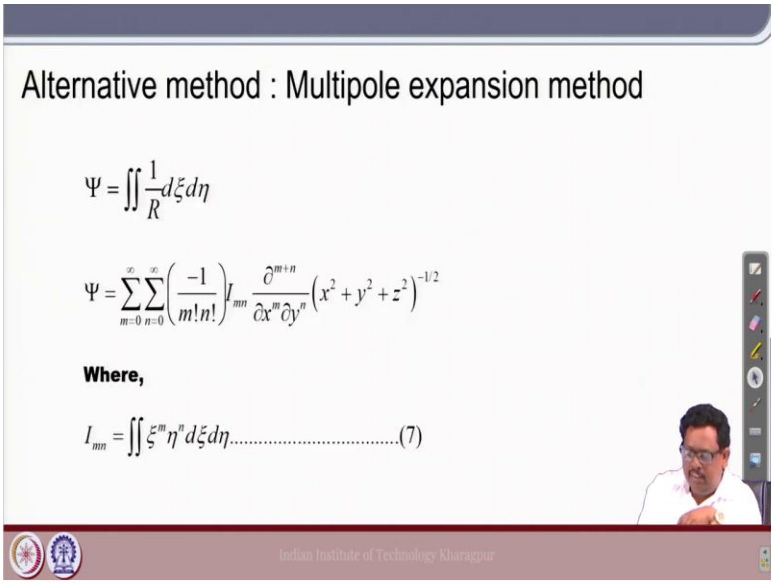

So, today we are going to stop over here and this is the another alternative method which is call the multipole expansion method. So, but this is again you can use this to get the value for the $\frac{1}{R}$, we really do not worry about the value for that $\frac{\partial G}{\partial n}$ because from the geometry understand it is the solid angle right, ok.

So, today let us stop here and in the next class we are going to discuss about the another numerical scheme like this 2-point Gauss quadrature rule for arbitrary shaped panels, right. It is easy like 2-point Gauss quadrature is easy for a rectangle, but how to, but sometimes when you do the paneling it is really not that always you are getting the rectangular panel or square panel, you can get an arbitrary shaped panel also. That time how you perform it, that is we are going to discuss from the in the next class ok; so till then.

Thank you.