Numerical Ship and Offshore Hydrodynamics
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Lecture - 26
Frequency Domain Panel Method (Contd.)

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Hello welcome to Numerical Ship and Offshore Hydrodynamics. So, today we are having the Lecture 26 now in this lecture we are going to again discuss about the how I integrate the Rankine Part of the Green's Function. Now in last class we have discussed that how we can integrate analytically when source point and then field point are very close to each other.

Now, today we need to discuss about the how do, I integrate over the quadrilateral panel. Now if you remember I said in the last class that since it is like a rectangular hyperbola. So therefore, larger period of the space we are going to do the you know quadrilateral I mean that you know 2 Point Gauss Quadrature Rule. So, today actually let us try let us discuss about the how integrate over the quadrilateral panel using 2 Point Gauss Quadrature Rule.

Then you can actually take a homework to extend that theory for the 3 by 3 Gauss Quadrature Rule ok.

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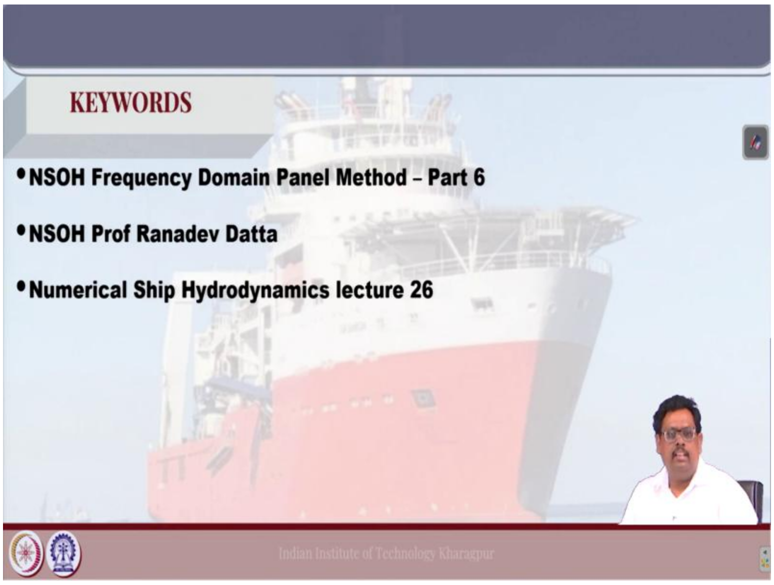

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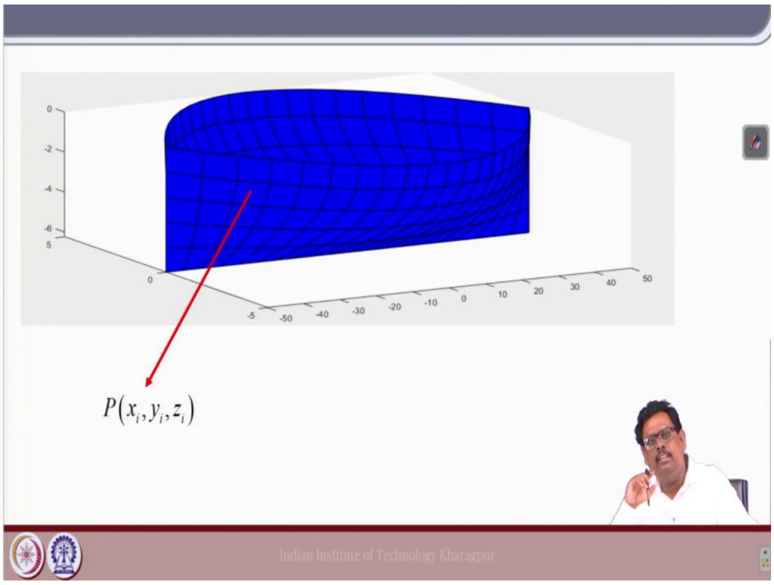

And this is the keyword that we are going to type to get this lecture ok. Now let us start now here if you look at this the ship model and it is a Wiggly Hull right. Then you can see that the point that panel the that the corner point at P x_i y_i z_i it is a point on a surface.

So, it is really not like a orthogonal plane it is not along that you know x_i y_i and z_i is constant then or y_i z_i x_i is constant or maybe y_i z_i x_i constant I mean not like that it is all varying right. So, essentially it is a 3D point right. Now if you know that gauss quadrature rule this we have to have a you know 2D point right because the gauss quadrature rule says it is integration from -1 to +1 for the x direction and - 1 to +1 for the y direction.

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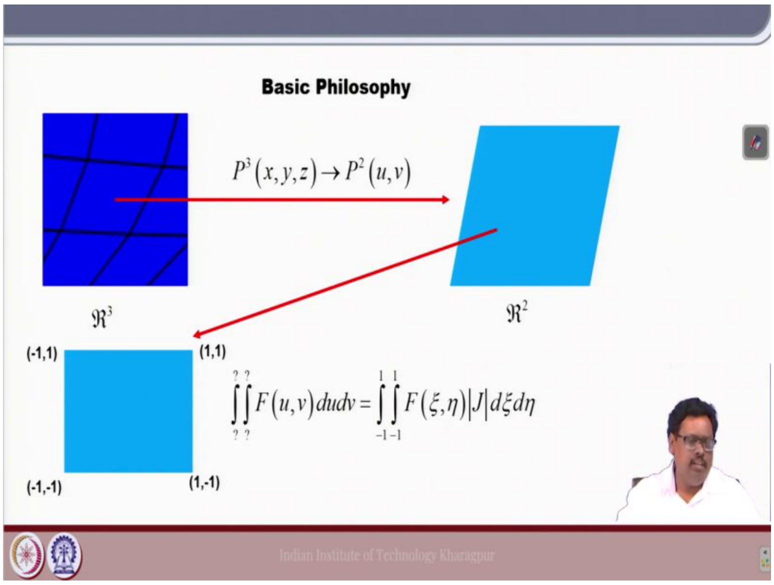

So, now so that what is the basic philosophy is as follows, I have now this R 3 domain this coordinate is 3 dimensional coordinate of course right. I mean 3 dimensional when I say the 3 dimensional is a 3 dimensional point right. So, I need to first transfer this 3 dimensional point to the 2 dimensional quadrilateral surface.

So, both are quadrilateral but it is inclined plane and then it is a 2 D plane. Now there are many ways to do that you can do some angle operation from you know this side and that side. So, this is one way you can change that to that, but there are several other approaches also available. So, and most of the time not most of the time in very I would say that very few time like not necessarily that all these 4 point lie in the same plane. Now everybody knows that with respect to 3 point I can define a plane not necessarily that fourth point should lie on that plane.

Now, here in this scheme what we are going to do that even if it is. So, actually we can take a projection so that all the 4 equivalent all the 4 point lie in a same plane that we make sure and we will see how we are going to do this. Now once it is possible to get this R 3 point into the map to the R 2, then again we are using another mapping from R 2 to this -1 + 1 this square map. Because this gauss quadrature rule should be you know this that limit should be -1 to +1 for x and then -1 to the + 1 for the y and we can call this a natural coordinate system now.

So, what we are doing here from this P 3 dimensional point we are going to have. Let us say 2 dimensional global coordinate system u and v now from this 2 dimensional global coordinate system again we map back to the natural coordinate system that you can call is a -1 to +1 for the this horizontal direction and -1 to +1 for the vertical direction right.

And then after doing this we are going to do this integration right and then of course that you know that limit as I said that this is the more tricky part like what would be the integration limit in the global coordinate system that we really do not know. So therefore, but however in natural coordinate system of course it is integration from -1 to +1 into the functional value of course, $F(\xi,\eta)|J|\,d\xi d\eta$.

So, this you know this it is known to everybody right. Now let us try to figure out how we can do all these things and then get the value for integration of $\frac{1}{R}$ and integration of $\frac{R.n}{R^3}$.

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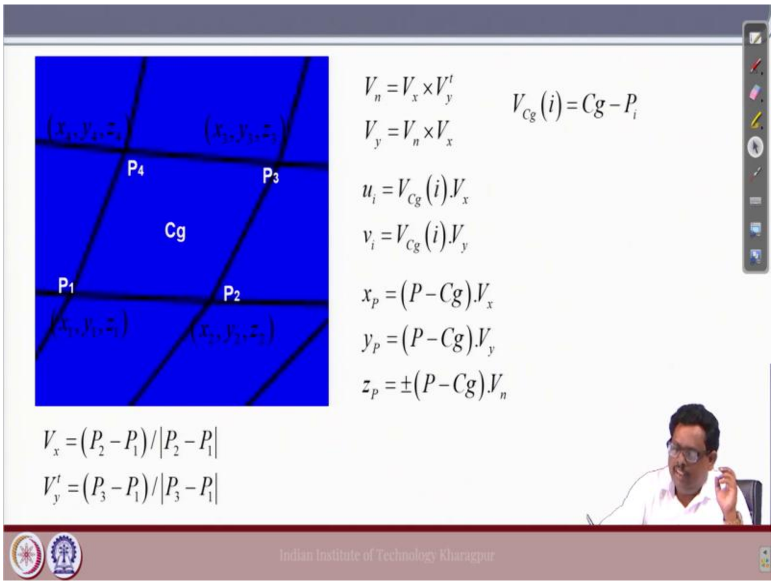

Now, here so first let us try to see that how from this the 3 dimensional point P_1 P_2 P_3 and P_4 we can get the equivalent 2 dimensional point ok.

Now, here we define one vector along this horizontal axis you can see it is $(p_2 - p_1)/|p_2 - p_1|$ it is a unit vector along x and then we can find out that unit vector along it is a temporary like. So, what we are going to define we are going to define one is along P_1 P_2 another vector is along P_1 P_3. Now if I take a cross product definitely I am getting the normal. So, V n is nothing but the normal to the vector which is V_x and V_y^t. So, it is a normal.

Now, here I understand that V_x and V_n both are perpendicular to each other. However, V_x and V_y^t is not perpendicular to each other right. So, but what I need is I need a local coordinate system which is you know it I mean all these axis should be perpendicular to each other. So therefore, I redefine this $V_y^t \times V_y$ by having this $V_n \times V_x$ by doing this what actually you know we did that now I am having a you know local coordinate system which is V_x V_y and V_n right.

So, this is the first thing that we are going to do when I have a panel with respect to that I am having the mutually 3 perpendicular vector one is along the horizontal direction, one is along the transverse direction and one is the normal to this plane right. So, this is the first job that we are going to do. So, when we are having it now let us then it is very easy to get the corresponding 2 dimensional point let us see how we are going to do that. Now then again I define the 4 vector which is the distance from the point.

So, the therefore I can define one called the $V_{C g}$ (I) which is the distance from the C g is a 3 dimensional point. So, it is (Cg - P_1)then you can get V_g 1 then I can get(C g - P_2) you can get V g 2 I can get C g minus P_3 I will get V g 3 and then I can get (C g - P 4) so I get V g 4 right. So, this I do and then my ui is nothing but the dot product of V_g (i)with the V_x right. Now this is because I am taking this component of this along this horizontal axis.

So, then this ui is basically the point on the x coordinate system so, now in my local coordinate system V_x V_y and V_n.

Now, u_i is falling on this local coordinate along the line V_x because I am taking this component along the V_x. Similarly the similar way I can take the if I take a dot product with the v_i and now remember this v_i also the mutually perpendicular axis. So, therefore,

v_i is again along this axis u_i right. So, this is how I am transferring my and also the local coordinate because this is a local coordinate system the another component definitely I am going to get 0 take 0.

So, this is the point that u_i and v_i which is nothing, but the on that particular point this which is P_1 P_2 P_3 P_4 would in the surface now I transfer that to the u_1 v_1 u_2 V_2 u_3 v_3 and u_4 v_4 right and where it is on the local coordinate system which is u and v right fine.

Now, here now what actually let us see that what actually we did and what is more required to be done as follows.

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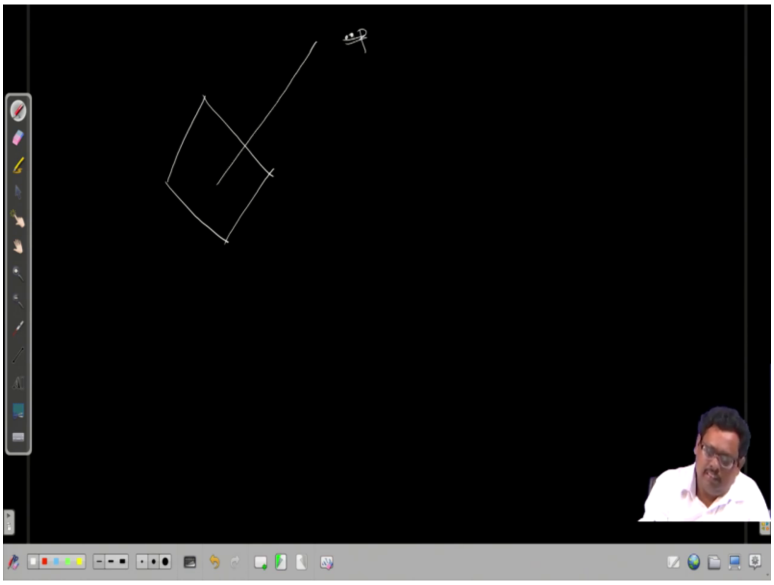

Now, what we did as follows now I have let us take a inclined plane like this. Now with respect to this is my point P right. Now let us not this let us do it again.

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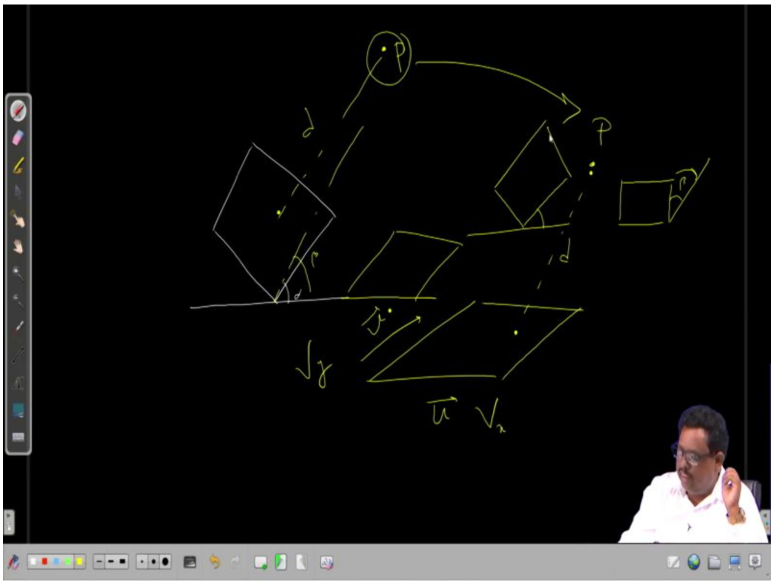

Suppose this is my that inclined panel right and it is inclined on both side let us say this is this is the angle alpha and also if I make a transverse direction somewhere here. So, this angle also it has let us say beta something like this. So, it is a 3 dimensional point.

So, it is inclined this way and also it is tilted this way. So now, here now with respect to that I have the point P over here right and then now if I change now essentially when I do the dot product what I am doing is I am actually incorporating the angles right. So, I change the orientation. So, I make this as a flat panel like this and something like this is the idea right.

So, I have a inclined in a 3 dimensional surface point let us take this way this one and now if I apply this rotation then I can get this panel with like this. Now may be in this horizontal axis again I have the beta, now if I apply this beta angle also. So, finally, it is lying on my you know u v plane right. Lying on my this you know this plane let us call this u v and in fact we can call this a V_x and this were in the along the length V mutually perpendicular.

Now, what happened to the point P now, because P must gone through all this coordinate transformation. So, that this distance the normal distance let us say d that must be you know that must be invariant under all such transformation law. So therefore, it is not only shifting the panel from the 3 dimensional to the 2 dimensional, at the same time I need to shift the point P from here to here also right.

So, let us see how we do that? Now here again it is again very simple that x P now what is P is the source point right. So, then actually I can define that x P is nothing but the now here it is the distance the x component is nothing but that the distance the horizontal. So, it is P of x minus C g of x.

So, that is the x component of that vector right and then that vector actually i multiply by the V_x (I) mean that dot product of V_x then y P again. Similar way that (P - C g) is a vector and then it is dot product by y and then the z p is again this (P - C g) , but now I am having the dot product with the normal.

Now, why I said plus minus depending on the situation when taking the positive upward like inward to the fluid outward to the body or maybe the inward to the body outward fluid, I mean that is based on that all the formulation can change. So therefore, it is your choice how you define your normal how you write that integral equation right, you have to adjust this sign plus minus everywhere. So, here that is why I keep as plus minus plus minus P minus C g dot V n.

Now, you see after applying all this thing what I now what I do is this point P_1 P_2 P_3 now I have a equivalent you know point on u_1 v_1 u_2 v_2 u_3 v_3 and u_4 v_4 and also remember when you have a coordinate transformation definitely there is a possibility that you know that area might change right.

But if you do this vector application you can check that area remain invariant right. So, therefore, you really do not need to introduce any kind of Jacobean here ok. Now what is the definition of Jacobean why the Jacobean is required, definitely we are going to discuss today only later on. But at this point what I am trying to say is as follows if you apply this you know coordinate transformation then that area so like ok.

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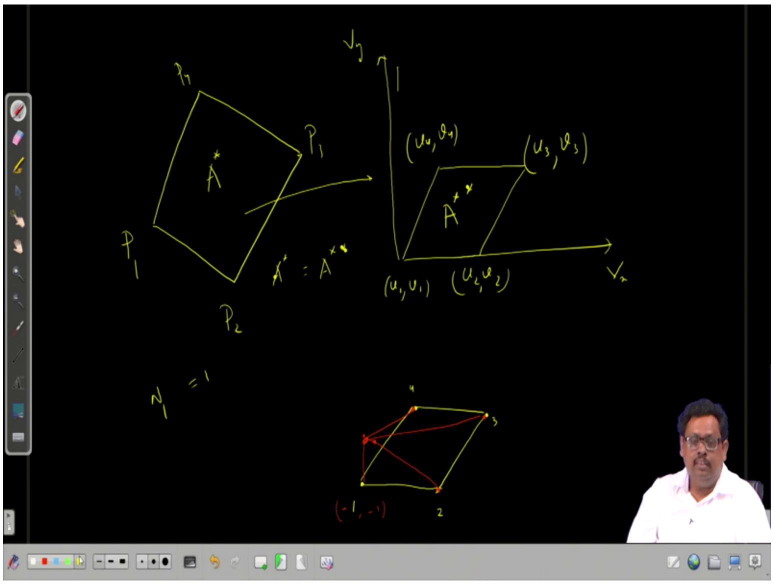

Let us go back here again what I say is as follows in case of this the global 3 D point that may be your the quadrilateral that P_1 P_2 and P_3 and P_4 and let us say area is A star or may be A star ok. And then you do this coordinate transformation and you can now having it is on the flat surface along this is here V_x and may be along this in V_y and then you have let us say u_1 v_1 u_2 v_2 u_3 v_3 and then u_4 v_4 and if the area is A star star.

So, my point is this a star is equal to A^{**}. So, this transformation actually that area is under invariant under this transformation. So, that is the whole point I want to make ok. Now having said that it is this area is invariant under this transformation.

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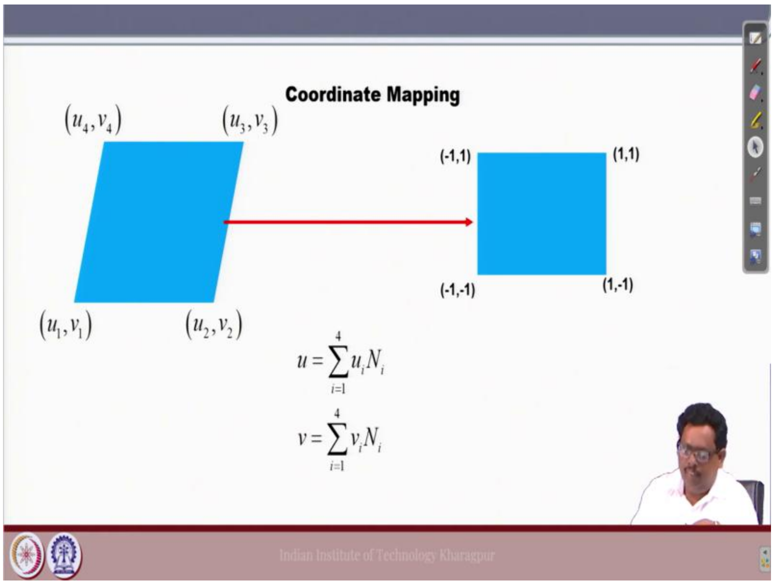

So, now I am having the 2 dimensional points that u_1 v_1 u_2 v_2 u_3 V_3 and u_4 v_4, then with this respect to this now I need a serious coordinate mapping from here to the our natural coordinate system which is $-1-1-1-1{,}1$ here right.

So now, how I do that? Now in if you do this isoparametric things in finite element I mean if you have those courses then definitely you know that how we are going to do this. But in case you do not have so I am just trying to just give you some idea like how we can implement that and how we can do the integration ok.

Now, here in general any point u can be defined as the sum of e 1 into N_i, which is this Ni is nothing but a shape function now what is shape function. And you know what is this importance everything that is definitely is a part of the study we are not interested here at this point. However, like you know in I if you say that this isoparametric things material and everything there. So, is a very standard way to define a point on this transfer from global coordinate which is u_1 v_1 u_2 v_2 I mean u v system to the natural system through this shape function.

Now, what are those shape function? Then what is the property of this shape function?

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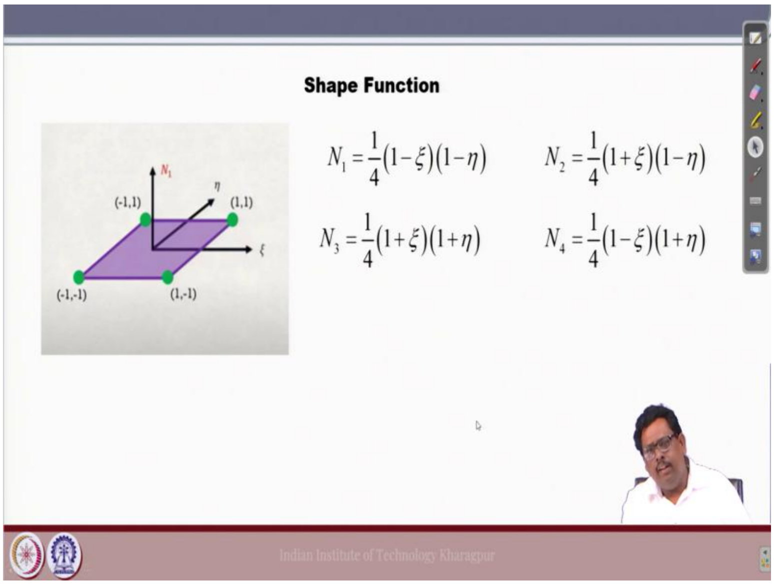

Now, here the Shape Functions this property is that at the node actually I am interested that shape function it takes the value one there and then in other 3 point the value becomes 0.

So, just you know like let us say for a one now if I do this if I do this point, now it is let us take it is this node 1 it is a node 2, let us take this is node 3 and let us say node 4 then in N_1 at node 1 it takes the value 1 in other situation it takes the value 0. So, if I try to draw the plot of N_1 you know this curve you know this should be something like this value you know something like this. It is here it is 1 and in all points it is 0 and then some way it is going from 0 to 1.

Now, this what is the expression for the N_1 in that case you can you say that like in this node, so that means at $(-1,-1)$ it takes the value 1 and in all other point it takes the value 0.

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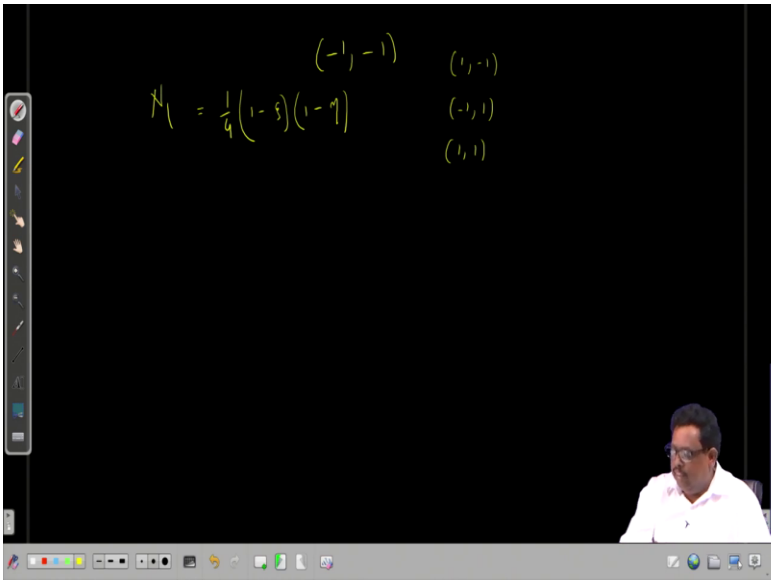

So, what would be the node function N_1 remember I said it is at $(-1-1)$ it takes the value 1 or in all other point it takes the value 0.

So, it is very easy you can say there is only unique way you can do that it should be $(1-\xi)(1-\eta)$. Now if you do that you can check that in case of if you put the value (1,-1) or (-1,1) or take (1,1) then you can see that all the points this value becomes 0 and if you take this (-1,-1) then you can say that value of N_1 become 1 right.

Now, using this understanding this is enough for doing our job ok, we really do not need more theory just to do this integration of 1 upon R for using this quadrilateral panel this is enough. Now, if I do this then definitely I understand this $N_1 = \frac{1}{4}(1-\xi)(1-\eta)$, then if I use the same logic so definitely for N_2 the value should be that $\frac{1}{4}(1+\xi)(1-\eta)$ I am talking about the this point right.

Now, in N_3 it should be $(1+\xi)(1+\zeta)$ because I am talking about this point and now in N_4 it should be $\frac{1}{\xi}(1+\eta)$. Now once we have this shape function. So, definitely I am ready to define that any point on the natural coordinate system using you know this coordinate this law using this result. Now I am ready to define the point on u and v on the natural coordinate system ok fine.

Now, in order to do the integration that I need to find out what is the Jacobean? Now what is Jacobean? You see here if you look at this integration scheme in before here we can we can see that it is the integration we have the some term which is mod of J and this is nothing but the Jacobean. Now, what is the Jacobean and why it is used?

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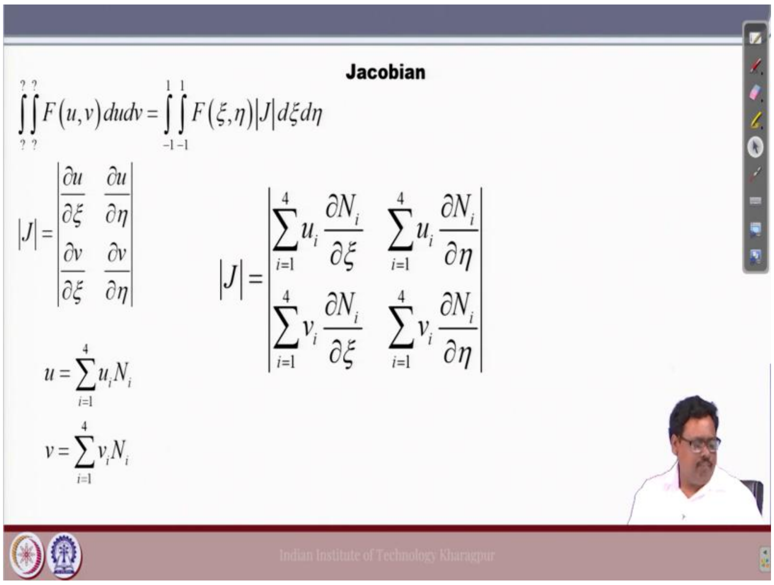

So, let us try to discuss it is important to introduce the Jacobean here. If you remember I just discussed that when you do this previous transformation from this 3 dimensional coordinate into this 2 dimensional u v I mentioned that it preserves the area right. So, area does not change. So therefore, the ratio of the 2 area equal to 1. So, then actually really we really do not need any Jacobean, because Jacobean exactly doing the same thing like now when you do the coordinate transformation it is possible that area is not invariant.

So, therefore you need to multiply the Jacobean to adjust the area itself. Now you know unknowingly during your school days you know you do any coordinate transformation for you know one dimensional thing.

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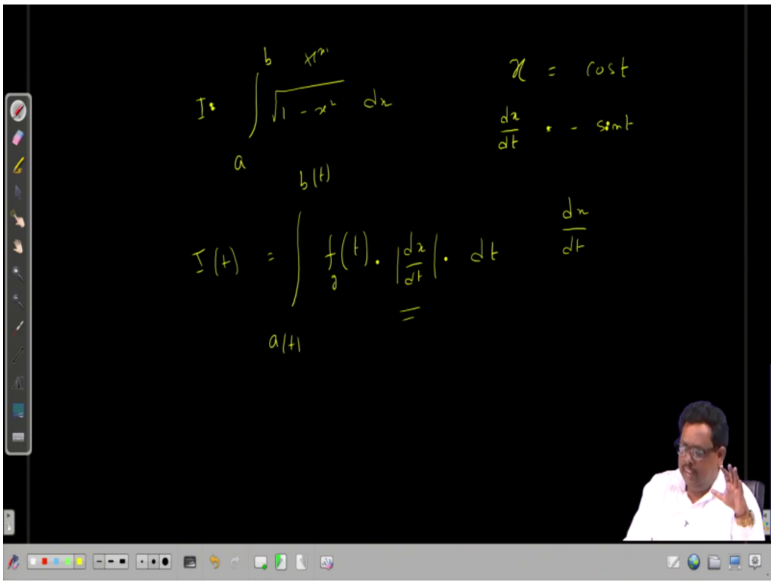

For example, let me give you this thing let us say if you try to do integration some a to b some integration I and then you try to do it is $\sqrt{(1-x^2)}dx$, let us see you try to perform this integration.

So, what you do is here if you remember you put that you take x equal to you know some cos of t and then what you do is you do $\frac{dx}{dt}$ which is nothing but minus sin of t. Now here then you change this I from ax to t and you change the limit now earlier the limit the function. So now, it is changes to the t. So now, this a is now function of t and then the b also then the function of t right and then what you do is you change is this the function of f x.

Now, this function of f x also change to some you know function of t and then you multiply the Jacobean which is $\frac{dx}{dt}$ that is called the Jacobean, which is minus of sin t and then you integrate in the domain t.

So, that is what we are doing. Now if you look at this what is dx? dx nothing but the length and what is d t? dt also nothing but the length. So, what I am doing here I am multiplying this coordinate system with raise to keeping the area in case of one dimensional which is the length. So, the ratio of this 2 length we call that is the Jacobean. So, this is the physical explanation of the what is the Jacobean.

Now, this is very important we need to do that. So now, how we are going to do over here. So now, here as I said this integration limit is unknown to me for this you know this u v domain; however, in natural coordinate system is definitely - 1 to + 1. But then you know there should be some area should be I mean should be multiplied with the area because area may be same may not be same with I really do not know. Now here this is how we do the Jacobean.

And this is the standard formula I mean everybody know about this is the standard formula of the Jacobean. Now how to apply over here now here? I know this is by definition for u it is u_i N_i and v is v_i N_i. So, I know this definition right then this then this you know this Jacobean is nothing but you just replace by u over there. So, it turns out to be $u_i \sum_{i=1}^{4} \frac{\partial N_i}{\partial \xi}$ and similarly the other 4 right. This is how I can find out my Jacobean.

Now, one this Jacobean I know how to his is a determinant of this of course, this is the Jacobean I mean mod of Jacobean is a determinant. So now once I know this so now you can see this everything is ready for me minus 1 to plus on the limit I know minus 1 to plus on limit for eta that also I know Jacobean now I know because I know the shape function. So, therefore the differentiation with respect to ξ and differentiation with respect to η that also I know.

I can add everything and I know how to get the value for the 2 by 2 the determinant is if equal is A_1 it is A_2 it is A_3 it is A_4 then A_1 A_4 minus A_2 A_3 that everybody understand right ok.

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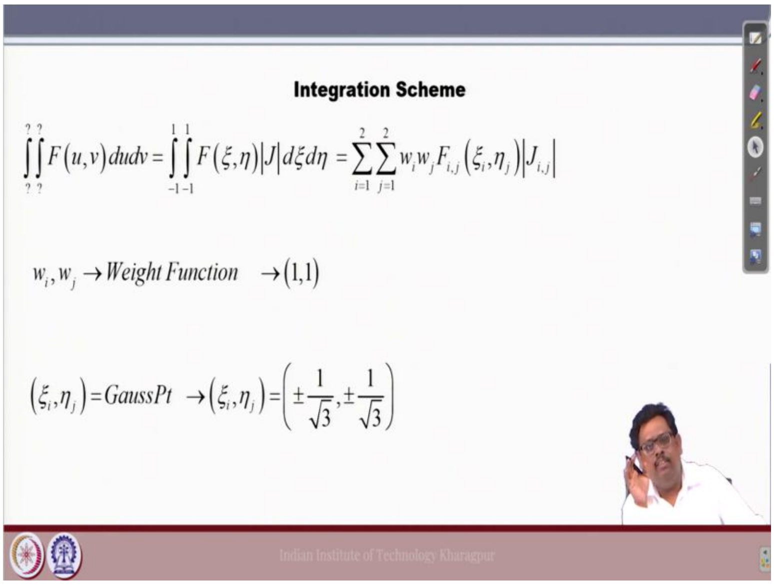

So, now the integration scheme the 2 point gauss quadrature scheme, actually it is this turns out to be i = 1 to 2, j = 1 to 2 , w_i w_j F_{ij} multiplied by the Jacobean. So therefore, it is a standard scheme for every any rule it is a quadrature rule it is a you know other integrant technique trapezium everywhere you have a functional value, you have a weight function you multiply this right.

So, now here what is w_i and w_j for our case which is called the Weight function. Now in case of minus one to you know in case of this Gauss quadrature rule 2 point Gauss Quadrature Rule this weight function is 1. So, fine so there is nothing wrong with this. So, it is it was simple and also ξ_i and η_j those are the Gauss point, but this Gauss point in the in this - 1 , + 1 into. So, what is that this is also very well-known you can get it from the G of the Legendre polynomial we are really not going to discuss all this mathematical thing ok.

So, here we are just use those results and try to do this integration. So, here it is again $\pm\frac{1}{\sqrt{3}}$. So, this is now I know everything over here I know in the integration or the weight function the value 1 and 1. I know all these gauss points now it is minus 1 by root 3 minus 1 by root 3 then plus 1 by root 3 minus 1 by root 3 then plus 1 by root 3 plus 1 by root 3 and then plus 1 by root minus 1 by root 3 into plus 1 by root 3. So, these are the

4 points I am getting ok. And then simply I use this and I can get the value for this integration.

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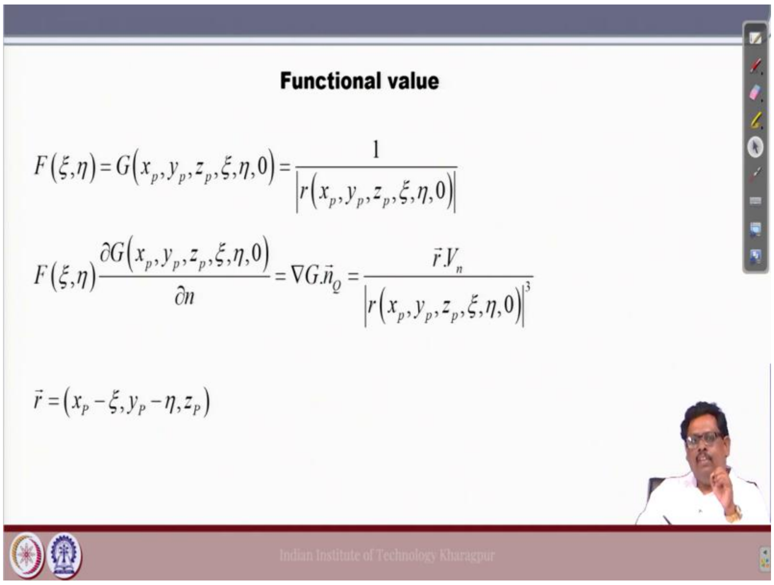

Now, only thing we have not yet discussed which is the functional value what are the functional value over here. Now you know this we have 2 integration 1 is G which is $\frac{1}{R}$, now this $\frac{1}{R}$ is nothing but the distance from this source point which is x_p, y_p, z_p to the field point which is (ξ, η, 0).

Now, here in gauss quadrature rule the 4 gauss point is nothing but my the source point right fine. And then that another function value F (ζ, ξ, η)which is $\nabla G.\vec{n}_Q$ and that is nothing but $\frac{\vec{r}.V_n}{|r|^3}$, that is also you know where the(x_p, y_p, z_p) are the source point. And (ξ, η, 0) is my sorry (x_p, y_p, z_p) at the field point and (ξ, η, 0) is nothing but my source point. And what is the r? r is a vector which is $(x_p-\xi, y_p-\eta, z_p)$.

So, now I know my functional value. So, what I need to do is very simple I have the 4 Gauss point and each Gauss point I am trying to find out what is the value for G and what is the value for you know $\nabla G.\vec{n}$ right. And then I apply and in this previously summation in previous summation form I put this values and I and also those point also I need to find out the Jacobean right.

Now, here you can say you have a 4 Jacobean point right. So, then what is the 4 Jacobean point now, if you do this differentiation then it is a function of again ξ and η and in this ξ . And η you have to if you put the value for ξ η $_i$ then definitely you are getting a number right. So, from this Jacobean also we can get the number.

Because it is $\frac{\partial n}{\partial \xi}$ which is nothing but a function of eta right and this η if I put the value of η_i and then again $\frac{\partial n}{\partial \eta}$ we get the function of ξ. In that function I take the value of ξ and similar similarly $\frac{\partial n}{\partial x}$ and $\frac{\partial n}{\partial \eta}$ for v_i also I am getting.

So, at the end in each point I am getting the value for the Jacobean because at each point I know what exactly my η_i and ξ_i need to substitute in this and I get the value for Jacobean. So, that Jacobean value we are going to apply over here, for each Gauss point each Gauss point I am trying to find out what is my functional values and weight function is of course 1. So therefore, I can get the integration right.

So, now this is how actually we are performing the integration for 2 Point Gauss Quadrature Rule for the Quadra panel and now it is you know I just give I like to give a homework try to extend this theory for the 3 Point Gauss Quadrature Rule ok. So, let us stop today.

Thank you very much.