Numerical Ship and Offshore Hydrodynamics
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Lecture - 32
Time Domain Solution Using IRF

Hello, welcome to Numerical Ship and Offshore Hydrodynamic course. Today, is a lecture 32.

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And, we are going to discuss the just continue from our last discussion how do I develop numerically to write this simple response function code ok.

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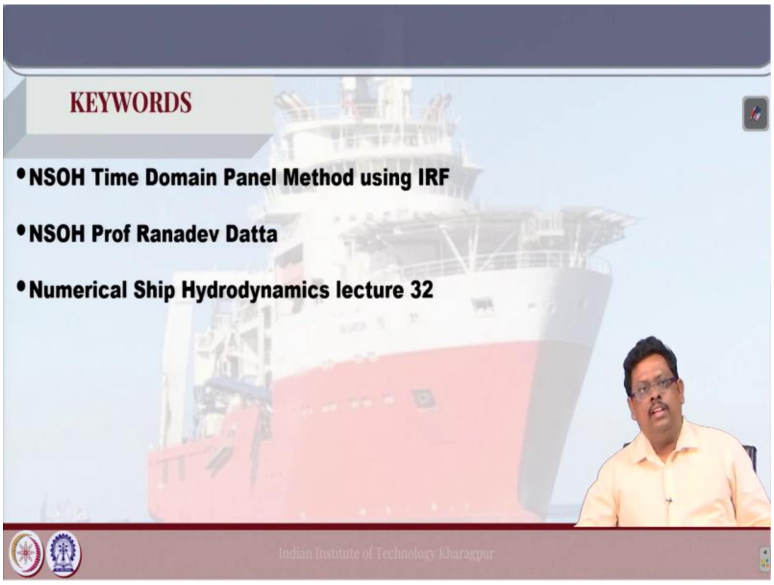

So, these are the keyword that you have to press to get this lecture ok.

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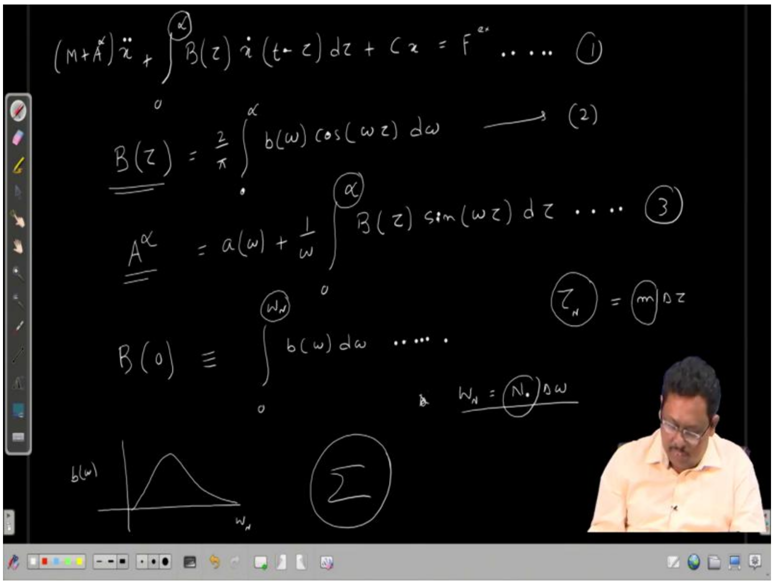

So, now let us discuss that how we can develop this code. Now, here if you remember in my last class what we did is let me write this equation again it is $(M + A^{\infty})\ddot{x} + \int_{0}^{\infty} B(\tau)\dot{x}(t+\tau)d\tau + Cx = F^{ext}$ So, this is how we can get.

Now, here also we understand that from this equation I get $B(\tau)=\frac{2}{\pi}\int_0^\infty b(\omega)\cos(\omega\tau)d\omega$.

So, let me write this equation number 1, let us say this equation number 2 and also this $A^\infty = a(\omega)+\frac{1}{\omega}\int_0^\infty B(\tau)\sin(\omega\tau)d\tau$. So, you can call this equation 3.

So, to solve equation 1 a priori we need to understand how I get this $B(\tau)$ and how I get this A^∞. So, we did not talk about how I get this A^∞, today we are going to discuss, but here in the last class what we discussed as follows that this $B(0)$ that I can get simply trapezoidal Simpson rule and it is nothing but 0 to some ω_N, right into it is $b(\omega)d\omega$. So, that is how we can go to get this $b(0)$.

And, then we have lot of discussions on how we can set that τ_N because we understand that for the large τ this whole expression goes to 0. So, therefore, even if this integration limit is infinity here, I can understand. Here also in this equation that integration limit is ∞, but however, it should be some finite value τ_N and you can call this would be is equal to some multiple integers multiplied by the $\Delta\tau$. So, that we understood.

Similarly, also we understand that from this graph of $b(\omega)$ this graph also cannot be infinitely long because at higher ω. So, at some ω_n and this curve also approaches to 0. So, therefore, even if this limit cannot be ∞ it should be some finite limit ω_n and similarly, I can call this may be this ω_n definitely some integer n multiplied by the you know $\Delta\omega$ kind of this.

Now, we need to because see at the end we need to convert this into some kind of numerical technique. So, therefore, we have to add certain things, right. So, we have to use this summation term. So, while doing this summation term that time I know I know that at which N the I should apply this summation form or at which m I should apply this summation, ok. So, that we discussed the last class.

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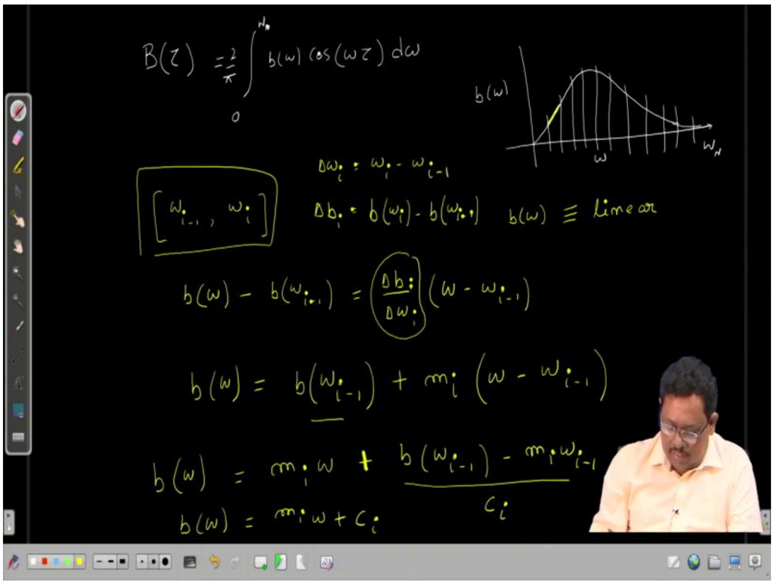

So, today let us try to find out numerically how could I get this solution for $B(\tau) = \frac{2}{\pi}\int_0^{\omega_N} b(\omega)\cos(\omega\tau)d\omega$ I just write it and then what we discussed in the last class that here how I approach this $b(\omega)$ curve.

So, now, there are several ways of you know approximation, right. So, this is ω and let us say this is ω_n and if this is the curve of my $b(\omega)$. Now, what we discussed last class we split into several sections here and within this section within this section we assume this $b(\omega)$ graph is linear. Now, once we approximate this $b(\omega)$ curve should be linear.

So, now, let us define something let us take some arbitrary interval. So, let us take some arbitrary interval you know this $[\omega_{i-1}, \omega_i]$. Let us take this arbitrary interval ok. Now, in this arbitrary interval then how I write an equation for a straight line? So, as I said that initially we thought of having it a constant to prove that this $B(\tau)$ goes to 0 when τ goes to ∞, but now here in reality see here we can apply many things.

We can apply quadratic curve also right, but that time you need if you want to do the cubic line then you need to have two points and also the two slopes at two end points side and in quadratic also you need at least three intervals. So, some intermediate point is required. So many things we can do. We do the simplest way we assume it is it should be a straight line between two intervals.

So, then how I write? It is everybody knows it is a straight-line equation see $b_{\omega} - b_{\omega-1}$, right ok before that for convenience let us define my $D\omega_i = \omega_i - \omega_{i-1}$ and also let us define you know $\Delta b = b\omega_i - b\omega_{i-1}$. So, let us define this. So, then it should be equal to $\frac{\Delta b_i}{\Delta \omega_i}(\omega - \omega_{i-1})$, is it not?

So, therefore, I can assume my solution of $b(\omega)$ it $b(\omega_{i-1}) + m_i(\omega - \omega_{i-1})$. Or to be very specific so, now, I can call this $b(\omega)$ equal to now this and this term is constants ok. So, I just again split into two terms. So, I can call $m_i\omega + \frac{b(\omega_{i-1}) + m_i\omega_{i-1}}{C_i}$. I do this and this actually we can call some constant. So, I can call it C_i.

So, finally, I have $b(\omega) = m_i\omega + C_i$. So, that is how actually I can write the equation in the interval between ω_{i-1} to ω_i, right.

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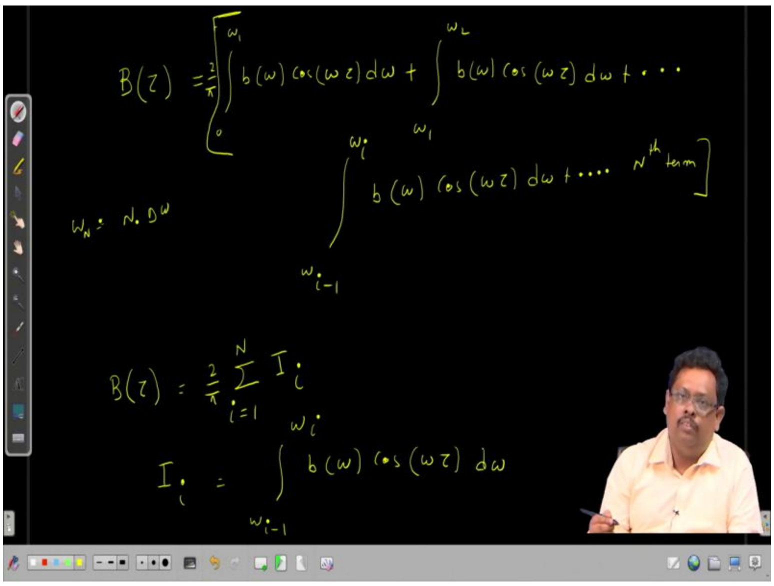

So, having said that now what I do is this $B(\tau)$ I split into several interval. So, I started with 0 to ω_1 and each interval I am performing this operation integration $b(\omega)\cos(\omega\tau)d\omega$, ok. Of course, $\frac{2}{\pi}$ is in the front, right. So, then so, I just fitted this way. So, then it is $(\omega_1 - \omega_2) \times b(\omega)$ into ok. So, just I just make one thing I just make this whole thing $\frac{2}{\pi}$ outside. So, it is $\cos(\omega\tau)d\omega$ and so on.

So, in the i-th interval it should be ω_{i-1} to ω_i, right and then it is again $b(\omega)\cos(\omega\tau)d\omega + \ldots\ldots\ldots\ldots\ldots N^{th}$ term. So, then things actually I can write in more simple way it should be then $B(\tau) = \frac{2}{\pi}\sum_{i=1}^{N} I_i$ that is what I talked about like when this that is why write that you know your $\omega_N = N \times \Delta\omega$, right.

So, then is $\frac{2}{\pi}\sum_{i=1}^{N} I_i$. So, this is how actually I can express this $B(\tau)$ very nicely and then the expression for $I_i = \int_{\omega_{i-1}}^{\omega_i} b(\omega)\cos(\omega\tau)d\omega$. So, this is how actually I can think of doing it, ok.

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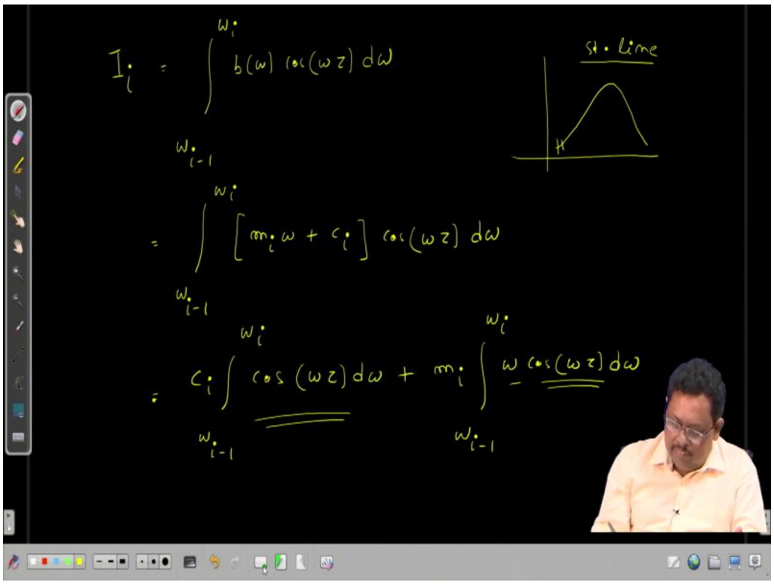

So, now find out this integral $I_i = \int_{\omega_{i-1}}^{\omega_i} [m_i\omega + C_i]\cos(\omega\tau)d\omega$. Now, you see that this is now I think it is elementary. Because now I just write the expression, I assume this is the linear interpolation. I will tell you that final expressions mathematical expressions are easy, right, but this final expression required some kind of approximation technique some kind of you know simplifications, right. So, it is up to the who is writing the code, right.

Now, if you take is a quadratic function this $b(\omega)$ between two intervals, then you can take a bigger patch so that you can you if you can you afford to take care about the curvature of the $b(\omega)$. Now, then a solution become little bit more complex the expression comes little bit more complex. Now, if you consider it is a linear then you cannot afford to take a longer $\Delta\omega$ because of this the curvature of the curve.

So, here if you take a as a straight line, then definitely this interval become very small. So, you can say that within this short interval this variation of this $b(\omega)$ can be considered as linear. So, this depend on the coder they who is writing the code, their understanding, their expertise, the experience everything actually comes to that is why the is a very small thing like.

But, then how you can define this small interval ω_{i-1} to ω_i that is also not that you know easy I would say because you need to check lot of things that whether this solution is convergent enough or not with respect to that interval, right. So many things are there, but anyway once you are you find out that interval and all the remaining part is trivial, right. So, now, here what we need to do is this ω_{i-1} to ω_i.

So, I just check that what this is the expression for my $b(\omega)$. So, let me use this expression. So, here it is some $\left[m_i\omega + C_i\right]$ and then it is $\cos(\omega\tau)d\omega$. Now, you see the things are very trivial. So, we have two interval one is ω_{i-1} to ω_i. So, I just take out the C_i outside, it should be $\cos(\omega\tau)d\omega$ and second one is plus I can take out this m_i outside integration sign so, it is ω_{i-1} to ω_i. Now, you have $b(\omega)\cos(\omega\tau)d\omega$.

So, this is the thing that you have now this is integration is trivial it is sin omega tau and now here. So, you have to do little bit more mathematical work to this integration by parts you need to do to find out, right. I know that this you know very well these things, but still for the completeness.

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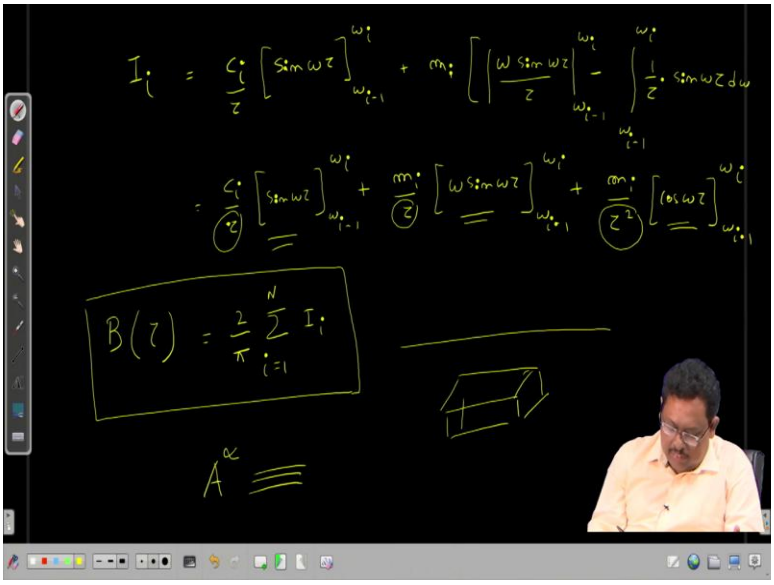

So, the first case this equal to sin of, so C_i is there. So, then it is $\left[\frac{\sin\omega\tau}{\tau}\right]_{\omega_{i-1}}^{\omega_i} + m_i$. So, you take this as a first function, second functions and then you can carry on. So, it should be $\left[\omega\frac{\sin\omega\tau}{\tau}\right]_{\omega_{i-1}}^{\omega_i} - \int_{\omega_{i-1}}^{\omega_i}\frac{1}{\tau}\sin\omega\tau d\omega$. So, now, here so, I just let me leave it like this way it is only these expressions become big because computer is doing it. So, anyways so, it is $\frac{m_i}{\tau}[\omega\sin\omega\tau]_{\omega_{i-1}}^{\omega_i} + \frac{m_i}{\tau^2}[\cos\omega\tau]_{\omega_{i-1}}^{\omega_i}$. Now, why I am actually these things I am not interested like this is ok fine it is computer will do that the important part is the nature actually I am able to capture.

Now, here you can see this it is divided by tau and this is divided by tau square. So, this confirms me that when τ goes to ∞ because this under the square bracket this term are constant I mean lying between -1 to +1 or something for a higher ω may be little bit more. However, denominator is rapidly going to infinity, right. So, therefore, the overall contribution become 0 at τ goes to ∞.

So, this nature is very much visible here also. So, finally, once I know this term I_i so, then $\frac{2}{\pi}\sum_{i=1}^{N} I_i$. So, now, this is how I get the solution for $B(\tau)$. Now, definitely in fact, maybe in future classes maybe next or next to next class I am going to show all kind of nature of this curve $B(\tau)$ and then a infinity how to calculate this, ok.

Then it will actually you can see that the how this curve varies because you know later on when you develop this code then actually you can see for a comparison ok this would be the nature of the curve whether it is following it or not ok anyway. So, now, I know how to get the $B(\tau)$, but secondly, you need to find out how to get an infinity. Now, it is an infinitely infinite frequency domain infinite frequency added mass. So, it is actually a geometric property, right.

Now, if you remember our first the introduction to this numerical method started with a infinite domain problem where when we take a object floating in inside the ocean right. This I mean this is inside an ocean and then we try to find out the added mass of this object that we did, right.

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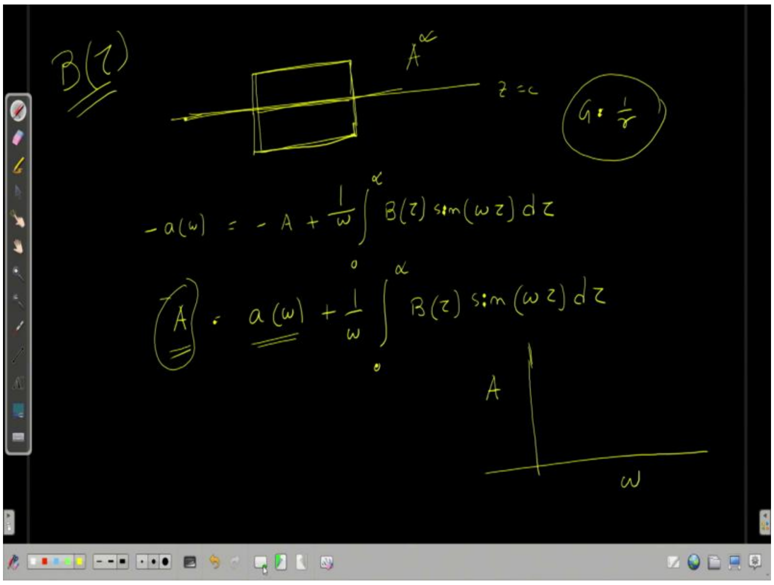

Actually, this is this the same problem actually you can use like if you have this you know Z = 0, and then if you take this object and then its image and then you are solving the same problem for infinite frequency that infinite depth problem that we did right we take that G = $\frac{1}{r}$ and we have solved that problem, right you remember it, right, initially?

So, I think the I mean that is the key I mean if you have that code with you so, then this is your infinite added mass only thing that geometry you need to put over here and then it is the weighted surface and you take the image part. And, then you solve this with this Green's function and then simply solve this boundary value problem then you get this infinite frequency added mass, ok.

Since we have already discussed that there really, we do not discuss over here, but there is some popular software available here like OrcaFlex, I think they show that infinitely frequency added was through this coming's equation again. Now, if you remember in the last class, I live this way it is I know $-a(\omega) = -A + \frac{1}{\omega}\int_0^\infty B(\tau)\sin(\omega\tau)d\tau$, right.

And, then I can write this $A = a(\omega) + \frac{1}{\omega}\int_0^\infty B(\tau)\sin(\omega\tau)d\tau$. Now, before we try to find out the A, we must have the value for $B(\tau)$ which I just discussed how to get the value for $B(\tau)$, ok. And, then we need to find out this added mass for each frequency now what is happening what I am going to do is as follows I am try to plot it this function A with respect to all ω, ok. So, that is actually my objective, right.

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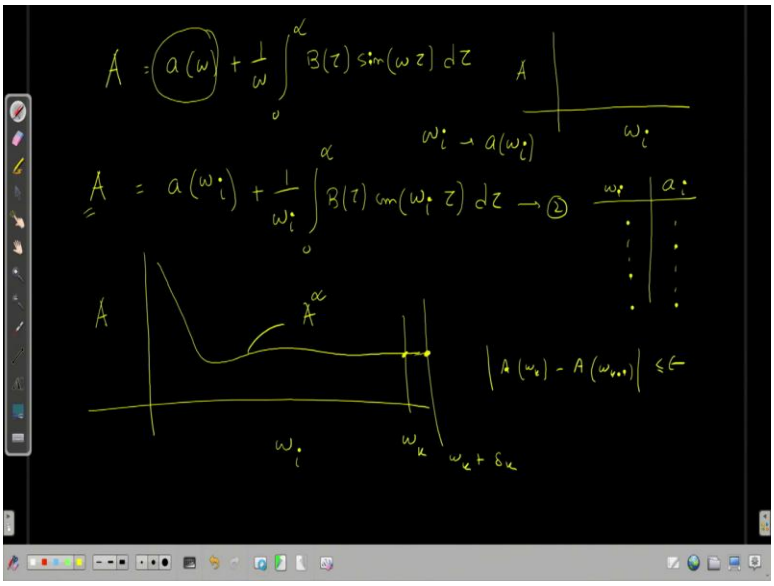

So, how to proceed? Now, here let me write again this $A = a(\omega) + \frac{1}{\omega}\int_0^\infty B(\tau)\sin(\omega\tau)d\tau$.

So, this is my equation, right. Now, what I said I need to apply for I try to find out a for each this ω. Now, I have this solution for an ω I have this solution for A ω for each i so, that means, what I have with me for each ω_i i have this solution for ω_i. So, this list I have.

So, therefore, for some particular let us say ω_i, my added mass is the frequency domain added mass is a ω_i. Now, remember this equation is satisfied for all ω, right. So, what I did is so, first then actually for each ω_i I can solve this $\omega_i + \frac{1}{\omega_i}\int_0^\infty B(\tau)\sin(\omega\tau)d\tau$ Now, mostly why I am looking it I try to find out when at which ω this value of A become you know constant like so, I expect this graph will some kind of this. Now, when here it is let us say some ω_k if this is the value for A and it is let us say $\omega_k + \delta_k$, if this is the value of k now, if the value of $|A(\omega_k) - A(\omega_{k-1})| < \varepsilon$ ok . So, I that point of time I can say this is my infinite frequency added mass A^∞, right.

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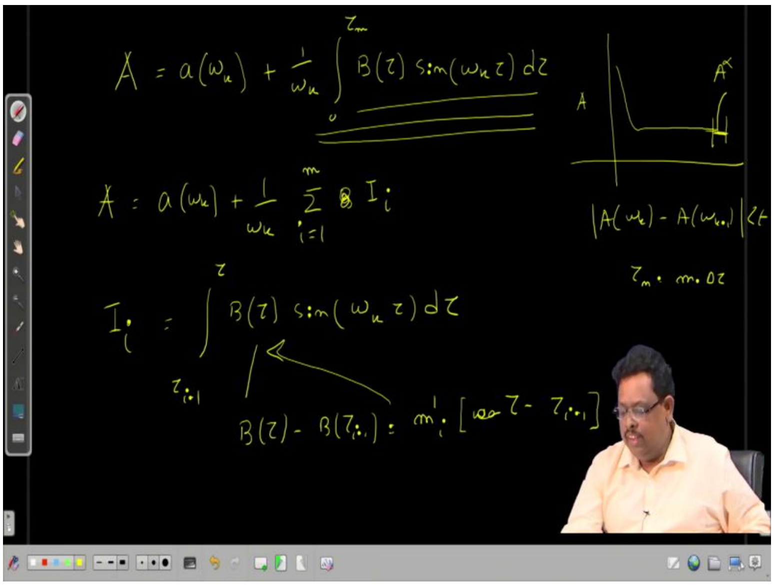

So, the idea is I am applying this ω_k I mean for a particular ω I am doing this integration and definitely it is not infinity now. I understand it should be some τ_m finite τ_m and

$\int_0^\infty B(\tau)\sin(\omega\tau)d\tau$. Now, how to perform this? So, it is very simple, I do that. I perform each omega k and I see the graph when I can see this graph is you know is a constant so, that time I call. So, this is it is my A^∞ because $A(\omega_k) - A(\omega_{k-1})$ if you take a mod this less than sum of ε which is my choice, fine.

Now, this whole thing the whole exercise even how I can model this integration. So, now, I also know how to do this because just right now the similar way I did that for $B(\tau)$, right. Here the thing is that I define this integration again I can write as A into $a(\omega_k) + \frac{1}{\omega_k}\sum_{i=1}^{m} I_i$, right because I said that my $\tau_m = m \times \Delta\tau$, right. So, then it is then it is integration of some I_i, where $I_i = \int_{\tau_{i+1}}^{\tau} B(\tau)\sin(\omega_k\tau)d\tau$.

We already we know that now how to do this right because again I assume this $B(\tau)$ is a straight line. So, I can say $B(\tau) - B(\tau_{i-1}) = m_i^1\left[\tau - \tau_{i-1}\right]$. So, I write this equation I substitute this equation here and again I know that how to do this analytically, right.

So, now, today's class what we understand that how to get the solution for $B(\tau)$ and then solution for A^∞ using you know I am not doing this for infinite frequency panel method that you can do from that also. But this is the other way of doing it you plot the graph get that way it is convergence and that is your infinity, right. So, and then, so now, today I know that how to get these two parameters, but still, I need to solve this the whole equation of motion and that involves the convolution integration also. So, in the next class we are going to discuss that.

Thank you.