

Numerical Ship and Offshore Hydrodynamics
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Lecture - 33
Time Domain Solution Using IRF (CONTD)

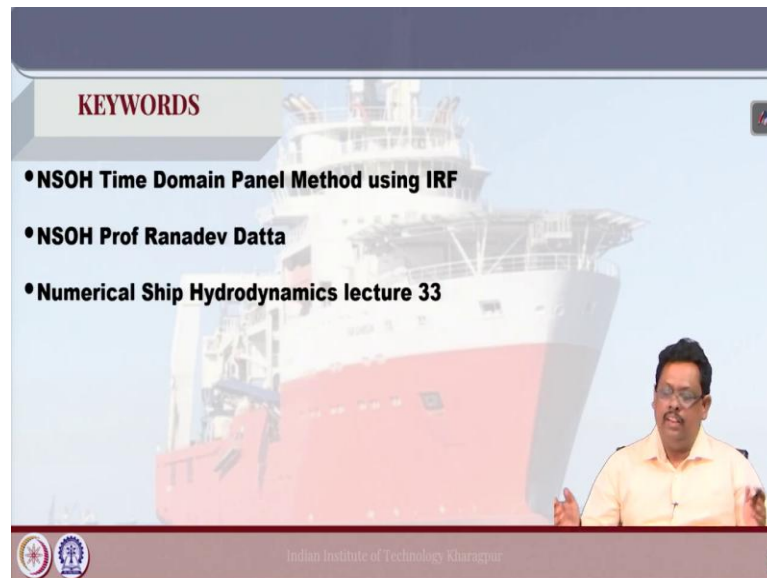
Hello, welcome to Numerical Ship and Offshore Hydrodynamics. Today is the lecture 33.

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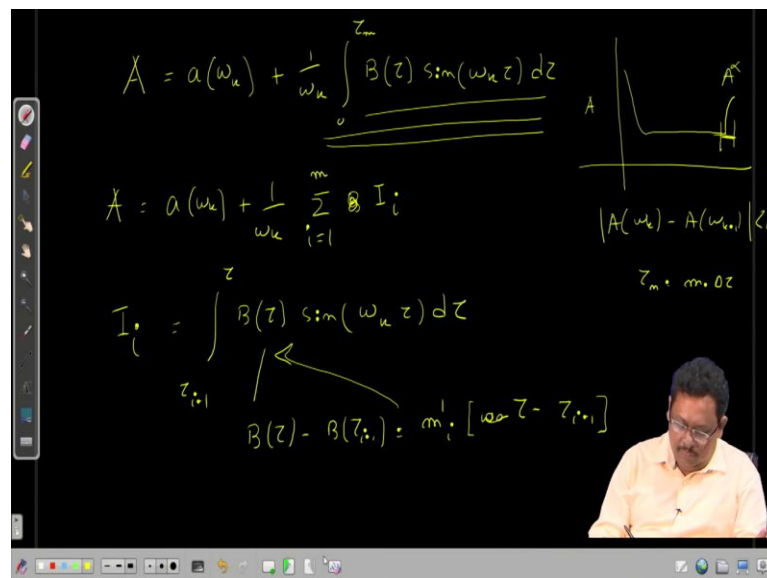
Today, we are going to discuss that how we are going to Solve the Equation of Motion that is the IRF Solution right, ok.

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So, and this is the keywords that you have to you know use to get this lecture ok. So, without further delay let us start.

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So, this is where actually we are in the last class. So, now, I know the solution for A and the solution for beta.

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$$(M + A^\infty) \ddot{x}(t) + \int_0^\infty B(\tau) \dot{x}(t-\tau) d\tau + Cx = F^{exc} \dots (1)$$

$$(M + A^\infty) \ddot{x}(t) + \int_0^{\tau_m} B(\tau) \dot{x}(t-\tau) d\tau + Cx = F^{exc} (2)$$

$$C_{33} = \rho g A \omega p \quad C_{55} = \rho g \nabla G_{ML}$$

$$C_{44} = \rho g \nabla G_{MT}$$

So, now again let us go back to the equation of motion again. $(M + A^\infty) \ddot{x}(t) + \int_0^{\tau_m} B(\tau) \dot{x}(t-\tau) d\tau + Cx = F^{exc}$. Now, you see in this equation there is there is lot of things are there, right and here only we only know how to calculate the $B(\tau)$.

Now, we need to so, now, it is a time domain equation. So, finally, you have to solve everything in time domains right, but here only we know something it is we required it before even we start the solving the equation of motion, right. So, I so, by now we know how to get this $B(\tau)$, by now I know how to get this A^∞ , right.

Now, let us see that what is more required to solve this equation. Let us say give equation number 1 what more is required to solve this equation 1. Of course, I know $B(\tau)$ of course; I know infinity let me again write it little bit because this I do not integrate till infinity as such. So, frankly speaking it should be this into $\dot{x}(t)$ plus 0 to some tau N or if we use this the nomenclature that I do in the last class.

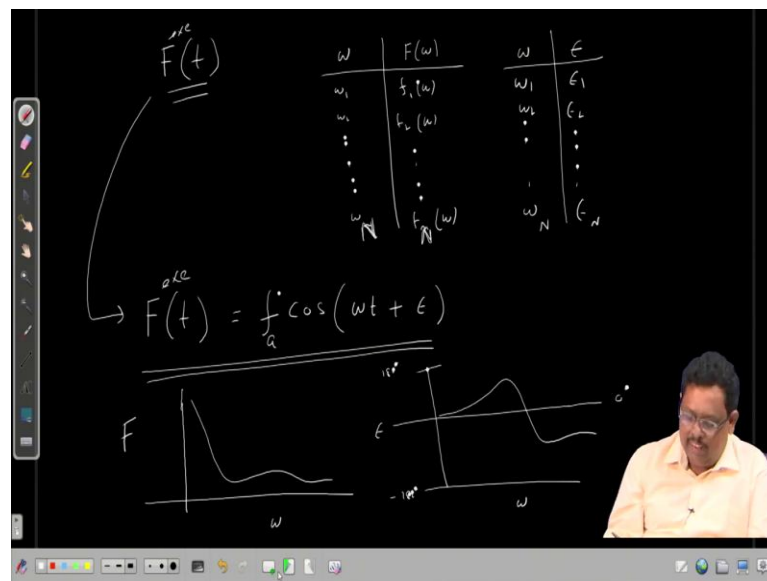
So, let me use instead of τ_n let us use the τ_m , ok and then it is $B(\tau) \dot{x}(t-\tau) d\tau + Cx = F^{exc}$
 Now, here we know how to get this, we know how to get this, but really still we need to know many more thing before actually starts solving this equation what are the does this

parameter. So, one parameter is C and one parameter very important parameter which is F^{exc} .

Now, today I mean in fact, this all thing is required before we solving this equation of motion, I have to have all this data before we start this solving this equation of motion let me call this equation 2 ok. So, now today let us start with how we can calculate the C , the restoring coefficient first I do not spend much time on this because we spend lot of time initially to get the C .

Now, you know that for C he or called the C_{33} this is nothing but it is $\rho g A_{wp}$, right. So, let us solve let us think that I solve for C and then you know that what is the solution for C_{55} it is nothing but $\rho g \nabla GM_L$ and in case of C_{44} it is nothing but $\rho g \nabla GM_T$. So, we know all these things before, right. So, we really do not discuss on this.

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So, we do not discuss on how I calculate C_{33} , but we need to discuss how I can calculate my $F^{exc}(t)$. This is very important. Now, how I get this $F^{exc}(t)$ here because I have no clue about you know how to get this exciting force in the right hand side. Now, this also we can get from my frequency domain solution, how? Now, if you solve like a frequency domain solution or let us say vomit type solution, then you have a data.

The data tells you two things – one is that about that each ω what is the exciting force F of ω one thing and also at each you know ω what is the value for the phase or you can say ε these data always there in any kind of frequency domain solver.

So, which means that we have $\omega_1, \omega_2, \dots, \omega_N$ and then you can have now I just make it a $f_1(\omega), f_2(\omega), \dots, f_N(\omega)$. In fact, you know just make it an equal nomenclature, call it capital N. Similarly, here also $\omega_1, \omega_2, \dots, \omega_N$, we have we can say $\varepsilon_1, \varepsilon_2, \dots, \varepsilon_N$, right.

So, then $F^{exc}(t)$ I know these all are harmonic in nature so, $F^{exc}(t)$ also a sinusoidal function; $F^{exc}(t)$ also harmonic. So, I know my $F^{exc}(t)$ should be in the form of some amplitude $f_a \cos(\omega t + \varepsilon)$. So, I know my nature of the solution for the right-hand side this if this is exciting force should be in form of some amplitude multiplied by the $\cos(\omega t + \varepsilon)$.

Now, here we have this data. Now, here we have some specified data; that means, we have again a graph. Now, if you have some kind of idea like how this graph should look like. Now similarly if I try to plot let us say for the heap my F^{exc} versus ω it looks like something like this, let us say. And, then let us say the phase, now here again this ω and here is you see you say the epsilon and if you like a take here it is 0 degree and then it is positive 180 degree, a negative minus 180 degree.

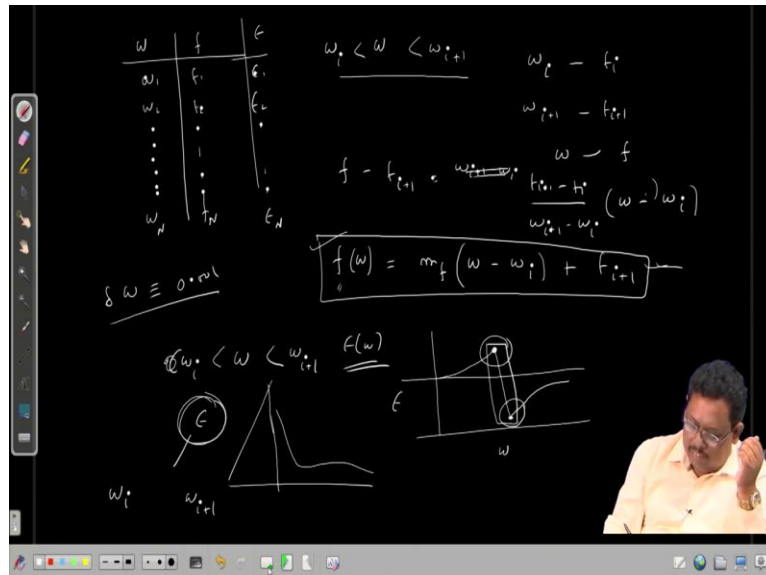
So, it should be started with 0 then go up go down something like this let us say. So, this should be the picture for F , this should be the picture for ε . Now, you see that many people everybody has their own way to write this exciting force. Now, what I can what I am teaching you that is how actually you know I solve this problem right ok. So, it this is so philosophical like and there is so many ways to do these things. So, one really cannot say that whatever I am doing is the 100 %, correct. So, one has to understand this, ok.

You know if you it is a purely kind of research problem I would say and there are many theories developed with respect to this and of course, all these papers related paper, related theories, related journal paper everything will be distributed to you, but as a classroom teaching, we are always going with the very stick with the very basic thing

very simple way of you know approaching to the solution ok. And, then let us try to find out at the end the solution I am getting whether it is really a realistic solution or not.

So, in that respect I can guarantee you would like whatever we are doing here at the end even it is much more simplistic than the proper way the researchers are doing engineers can accept ok in the research term you can say it is a within the engineering accuracy ok. So, let us see that how actually we approach over here because you know many there is a many alternative ways, many methods are also available. It is purely a very sophisticated process and I am doing it with absolutely simplicity ok.

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Let us see now, what again I am doing over here the problem is I have a table with ω versus the f and also this ω versus ϵ , but it is only for specific data $\omega_1, \omega_2, \dots, \omega_N$ it is f_1, f_2, \dots, f_N and it is $\epsilon_1, \epsilon_2, \dots, \epsilon_N$.

So, this is the this is the data that I am having, but suppose I need to find out in some arbitrary ω . Now, this ω not necessarily all this list of ω that I am having with me because when you solve this frequency domain method you really do not have any such of idea that I am going to do for the IRF analysis also we are going to use the same omega I that I am using for my frequency domain solution, it is not possible it absolutely not possible.

So, then here this ω not necessarily fall within this list, but this ω definitely within some interval ω_i to ω_{i+1} . Now, you understand what I am. So, what we do over here that we do the simple linear interpolation, ok. So, that means, I know at some ω_i then this is my f_i and then at ω_{i+1} I know this is my f_{i+1} .

So, then at ω what is my f ? This is my that is what I need to do and I do exactly is a linear interpolation. So, therefore, it is definitely $f - f_{i+1} = \frac{f_{i+1} - f_i}{\omega_{i+1} - \omega_i} (\omega - \omega_i)$. So, this is a simple linear interpolation, right.

So, we can get my functional value at that particular ω which is $f(\omega)$ is nothing but now you can call this again a slope. So $f(\omega) = m_f (\omega - \omega_i) + f_{i+1}$. So, this is the formula with this formula I can get the solution of $f(\omega)$. So, now, you see like this is again I say that this is absolutely I cannot say that it is simple, but this is not the only way to do this ok.

In fact, I would say it is a very simple way of doing it, but you know again as I said that if I have the sufficiently small ω_i like $\omega_1, \omega_2, \dots, \omega_N$. If all are very small interval. So, I have lot of data like if I take this ω is really the $\delta\omega$ is really small which is some order of 0.001 or so, and then if you plot all this ω then definitely this will give you quite reasonable answer.

And, similarly, the phase also with this simple equation I can find out the phase of this phase of $\epsilon\omega$ also that is a similar way. Again, I can do this linear interpolation I can get it. But sometimes you know there is a problem. The problem is as follows like if you look at the phase the nature of the phase it might not be very good idea all the time like it goes somewhere up and then suddenly is go down and then coming up like this.

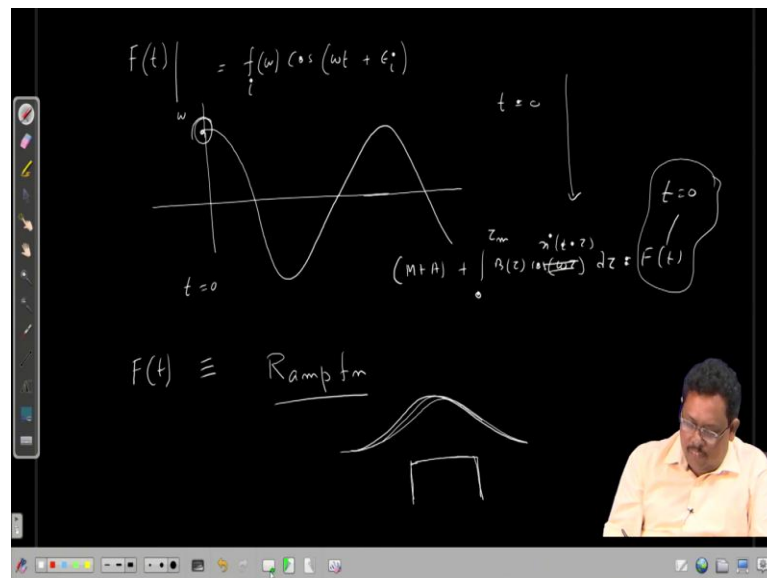
So, the problem is if your ω is fall in this between so, then really you know there are situation like you can have problem. Now, when there is a sudden change of this phase and which is very realistic, sometimes it is going up and suddenly it has shift change abrupt change that time this linear interpolation might not work.

So, in that is why it is sometimes it is safe that you know you take epsilon that let us say if this ω in between this some ω_i and some ω_{i+1} , it is always better to do this verification that how this your ϵ is changing. So, if this epsilon changing is abrupt then you have to again you have to decide like whether to go with this phase or whether to go with this phase, right. So, you know it is it comes with lot of you know what I said like experience something like that, right.

It is not a you know is a uniform way of expressing all these things right, but anyway in general what you could do is this $f(\omega)$ definitely it is a very continuously changing curve for this you can have this linear approximation. But while calculating the phase sometimes normally what we what trying to do is you can take the phase at either ω_i or you can take the phase as you know ω_{i+1} .

You can take any phase if the phase is not that rapidly changing and when you do the rapidly changing then we have to see that what the other possibilities that we can explore ok. But, let us forget this part, but in other part that linear interpolation was absolutely fine, ok.

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So, now I understand how I get this amplitude $f(\omega)$ so, therefore, this $F(t)$ of some frequency ω definitely it is this amplitude of this $f(\omega)$ there we which we can get

using the linear approximation multiplied by the $(\cos\omega t + \varepsilon)$ or we can call this εI whatever we can call this $f(i)$ something like this.

Now, here there is a problem not a problem, but you know thing is that what is happening over here normally if you look take a plot, then it is a cos term so, its cos goes like this way. So, now, at you know at $t = 0$, you can experience a very large force over here and that might cause you some kind of numerical instability. Why because at $t = 0$ you know I really do not want a very large force acting on the vessel.

So, then how actually, but again it is here like this. So, then my then the question should be that how I can write this $F(t)$ the lot of problem writing this, right. Initially I do not know about the phase there is a there is an always some kind of more rigorous analysis required to which phase I should attach with this. Second one is that how I do the linear interpolation to get this f this f omega naught.

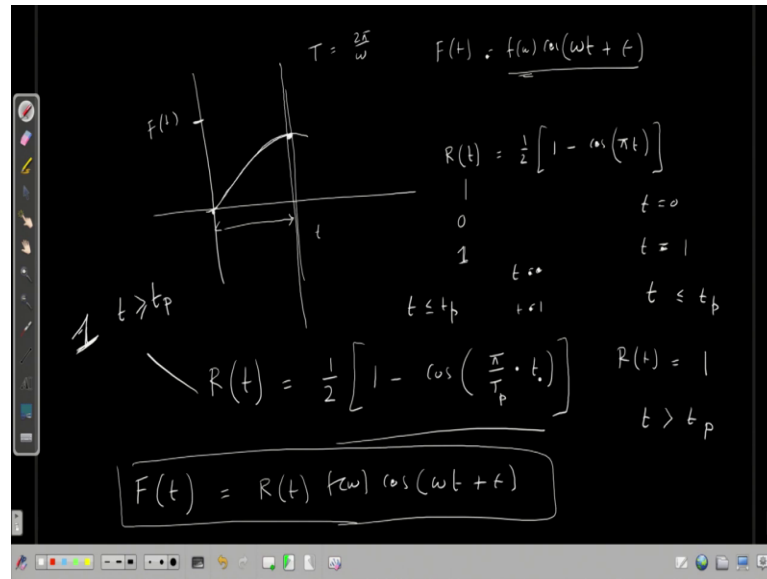
And, third now which is more important and very crucial that at $t = 0$, when you do this

equation like when you do this $(M + A) + \int_0^{\tau_m} B(\tau)\dot{x}(t - \tau)d\tau = F(t)$, now at $t = 0$ this goes

to a very large force. Now, the question is that if this in a very large at start starting point it might cause some kind of a I mean that we are not ready for it I would say that way.

Then how to write it? So, normally we write with the help of some kind of ramp function. Now, what is ramp function? Ramp function is something like you know that it this it is going little bit like this is your ramp function right; that means, it is of course, is in a speed breaker. Now, but this speed breaker we always take a continuously this. So, slowly slid go up and then go down, we really do not want we really do not want a speed breaker like this, right. We really want this ramp function.

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So, what graphically that we really do not start with a very large value. So, what I need this the F if I call this force F of t with respect to t , I really do not want to start from this particular point. What I want is it should be here go up and then after some time t it reaches the peak peak may be after the first cycle because you know this $F(t)$ is nothing but it is some kind of $f(\omega)\cos(\omega t + \epsilon)$.

So, this is a harmonic function, right. So, maybe it is periodic I mean not maybe it is periodic. So, maybe we can the first period we do not really do not want it. So, I want it after using some ramp function to leads you from here to here. So, how I write this ramp function? So, now, remember that ramp functions cannot be for the all the time. It should be acted here and if this ramp function should end over here, right.

Now, so, how to write it, right? So, let us see that how we can write it there is again there is a different way of write you can write a ramp function. So, let us try one ramp function. So, we just call this $R(t)$ some ramp function you can say it is

$$R(t) = \frac{1}{2} \left[1 - \cos\left(\frac{\pi}{T_p} \times t\right) \right].$$

Now, let us see what is the behavior of this function $R(t)$. So,

when we use the $t = 0$, we can say that is this is this $= 1$. So, then $R(t)$ goes to 0, right.

Now, then when this then when the $t = 1$ so, at that time it is actually going to $\cos\pi$ which is -1 . So, at that time this $R(t)$ goes to 1. So, therefore, if I use this the ramp

function then I understand this at $t = 0$ it is 0 and then at $t = 1$ second it actually goes to the one value. So, if I attach this ramp function with this. So, maybe after 1 second I can reach that, but then I really do not want to make it at 1 second. So, I want to make it I need to devote at least 1 time period.

Now, you know that time period $t = \frac{2\pi}{\omega}$, right. So, we know that. So, this ramp function

I might write in that way. So, $R(t)$ if I write this ramp function

$\frac{1}{2} \left[1 - \cos \left(\frac{\pi}{T_p} \times t \right) \right] t = \frac{2\pi}{\omega}$. Now, if I write this ramp function this ramp function goes to

-1 when $t =$ time period, right.

So, therefore, at least I can the at least I can use this ramp function to make my signal beautifully arranged for the first time period I am devoting it to set up the force and then actually we can do this force continuously. So, you can get a very the nice force. So, this $F(t)$ I must use with this $R(t)f(\omega)\cos(\omega t + \varepsilon)$.

This should be the way to write this ramp function and I must say this ramp function is this when $t = t_p$ and then the value of ramp function $R(t) = 1$, when $t > t_p$, ok, fine. So, today let us stop over here now today we discuss how we can write the ramp function, right.

So, now, this is the ramp function when this when $t < t_p$ and this ramp function goes to 1 right this ramp function goes to 1, when that $t \geq t_p$, ok. So, today we are going we discuss these things right, but then still we need to write the equation of motion ok.

So, now, still we are there like we first discuss how we can get the B tau, I can how I can get the A^∞ , here we can discuss how we get the exciting force the generate the exciting force. So, now, we are ready and also, how we get the C, the restoring coefficient. Now, we are ready to write the equation of motions. In the next class, we are going to write the equation motion and then if time permits, we can show you the MATLAB code, the graphs ok. So, till then so, we stop today at this point, ok.

Thank you.