Numerical Ship and Offshore Hydrodynamics
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Lecture - 36
Forward Speed Effects

Hello, welcome to Numerical Ship and Offshore Hydrodynamics. Today we are going to discuss on the Forward Speed Effects.

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So, today is a lecture 36, today we covered this following concepts that we have discussed something called encounter frequency, and then we have to discuss something on kinematic and dynamic. How what is the kinematic and the free surface boundary condition for a moving frame of reference ok.

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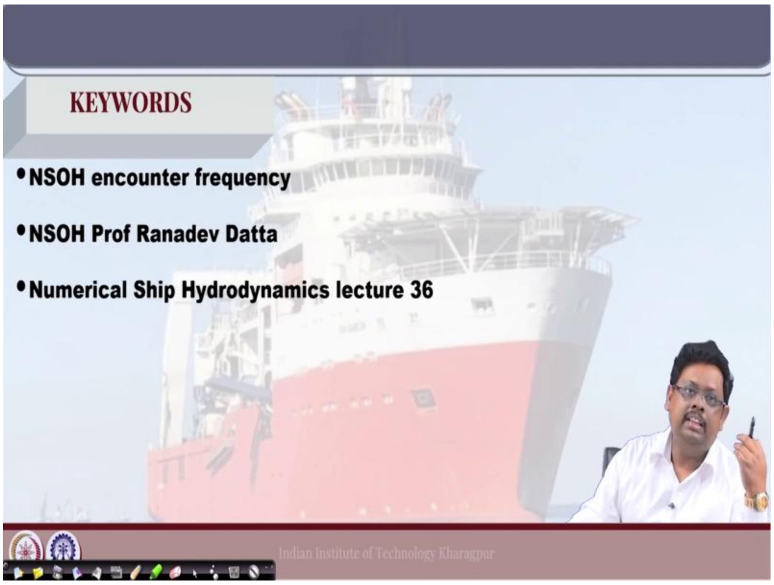

And these are the keywords that you have to use to get this lecture ok.

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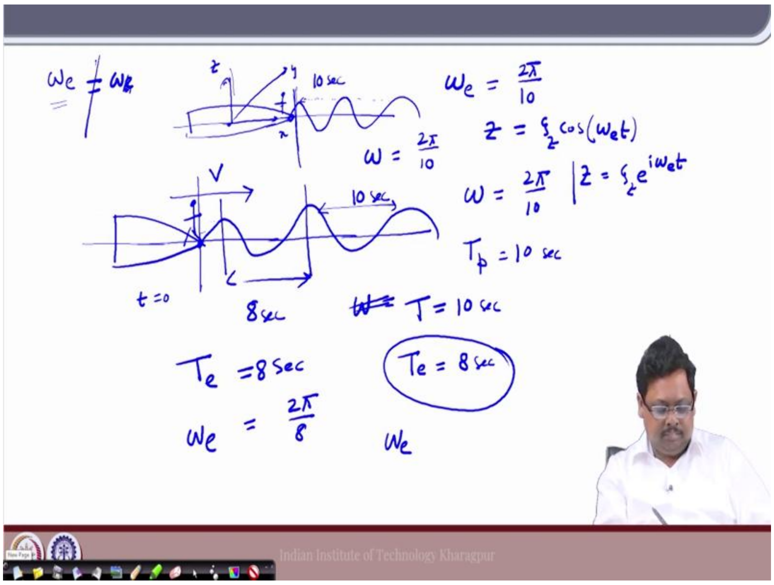

So, let us jump into the problem like that something that we are going to discuss something called the encounter frequency, and we call this as ω_e or we call the omega encounter. Now, what is encounter frequency or what is ω_e. Now, suppose let me draw one just a sketch of a ship from the top view, I am looking from the top. Now, if suppose this is not moving this ship is not moving at all ok with respect to position it is always fixed.

So, let us speak a coordinate system also, now this is the coordinate system of the vessel ok. So, let us fix x y and z. Now, with respect to this coordinate system the ship is not moving at all. Now, you can and then the observer here suppose there is some observer is standing here and he is observing that when this wave crest actually approaching to this point ok. So, now, he observe the first wave crest here, then this wave crest slowly slowly coming and hitting here, and then this time he find out that this time taken to coming this crest here and then slowly, so this will come.

So, consecutive with these two crests this time let us say ten second. So, that means, the observer can see over here that each 10 second that wave is hitting to the ship ok and then subsequently we can understand then that frequency or in that case the encounter frequency omega and which is $\omega_e = \frac{2\pi}{10}$.

And, if you look at this what is the wave frequency here. Now, wave frequency also remains same because $\omega = \frac{2\pi}{10}$. So, therefore, in this case this encounter frequency is equal to the wave frequency omega e right. So, we have no doubt about this like if there is the ship is standing still and it is not moving with some some velocity there that moment of time the observer, if I see the time period that wave is approaching and hitting to the ship is a 10 second that is exactly what the wave time period. So, there is no doubt about this.

Now, let us take a second situation. Now, in this time what is happening? Now, again we have the ship now at $t = 0$ it is here and then again some observer here is standing and just looking at this the whole process and then wave is slowly slowly coming and approaching right, it may not be very good waves anyways.

Now, what is happening this when these waves approaching this point by that time that ship moves little bit forward because it is moving with some velocity V. Now, what is happening now if it is a period of 10 second now in this case if the wave frequency or let us say this $\omega = \frac{2\pi}{10}$ or may this time period of the wave $T_p = \frac{2\pi}{10}$ second. Now, for this observer he might I mean he can observe that this time taken between these two consecutive waves are not 10 second.

Because this ship is moving forward it could be some 8 second let us say. So, for the observer what we he can observe as follows: he observed that for him the time period is nothing but the 8 second, and consequently that encounter frequency it is nothing but $\frac{2\pi}{8}$.

Now, in this case you can see that ω_e not equal to the wave frequency ω. Now, then the question is then at which frequency the shift should oscillate right. Now, here you have two questions right? Now, you see that is ω it is let us say or let us say let us try to find out T, we are much more comfortable with the time period. So, let us say T = 10 second and then you can see that encounter and time period is 8 second. Now, the question is about which time period or which frequency the ship should oscillate.

Now, the answer is very straight forward the answer is it should oscillate about this encounter time period or encounter frequency ω_e right. So, this is how I mean this is the first difference between the fixed case and then a ship advancing with some velocity V right. Now, if the ship advancing with some velocity V, then actually this wave frequency ω is not the same because of this forward velocity V. So, ship cannot oscillate with respect to omega it should be ω_e.

Now, if I try to find out the motion of this ship then let us say vertical motion z, then z should be some amplitude ξ_z, let us say and then it should be $Z = \xi_z \cos(\omega_e t)$ that should be the motion of the body; in other words or if you write in complex domain $Z = \xi_z e^{i\omega_e t}$. So, this is the underlying concept of the encounter frequency right. So, now, this encounter frequency let us now quickly go to some kind of definition, which is some one is angle of attack ok.

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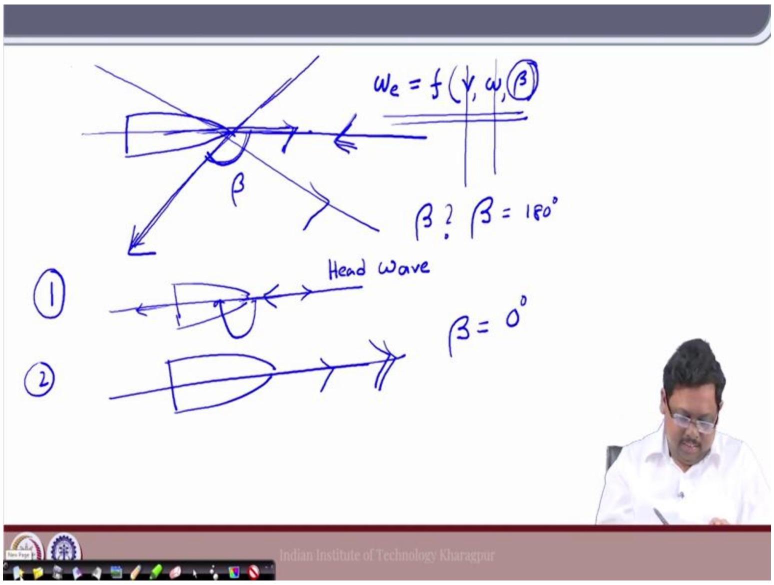

So, now what is angle of attack because you see like now you see the problem is the ship normally, all the time I am not encountering the waves which is coming only on this direction. Sometimes, wave might come with some angle like or sometimes it can come from this other side also this side also.

So, then this angle I mean some angle when they define because it always it is not coming from the front right not possible because it is a three dimensional phenomena wave can approach from any direction.

So, therefore, I can understand this ω_e may be, I mean not may be definitely the function of first that velocity that ship is travelling and also that omega that means, the wave frequency not only that there must be some angle between them which was called the angle of attack ok. So, this definitely depending on this three parameters and there is always one parameter which is the depth like we will see later.

So, now first we need to define the all 3 parameters. So, we have we know what is V. we defined V, we defined ω. Now, we need to define what is called the β. Now, β is suppose if this is the direction of the ship and if this is the direction of the wave normally we can define this angle is my angle β, which is started from here to here ok. So, with this definition let us quickly find out that what is the you know angle of attack for some situation. Let us take that let us take the case 1.

Now, in case 1, now let us say now I am not drawing the ship or I can draw also. So, this is one it is going this way and then the wave is coming this side then what is the β in this case? Let us say now as you know as I said that it should be from this direction to that direction. So, it definitely this is the angle so, in this case my $\beta = 180^{\circ}$.

So, I understand for head wave situation. The heading angle we can also call the heading angle or angle of attack. So, that is also is a 180 degree. Now, for let us say the follow we can call this is a head waves, and second is called the following waves. Now, following waves now this is my the ship also going this direction and also the wave also going in the same direction.

Now, then this angle is beta now in this case this $\beta = 0^{\circ}$ right? So, therefore, in following sea condition it is 0° right? So, in that way, this is the definition of my β, okay?

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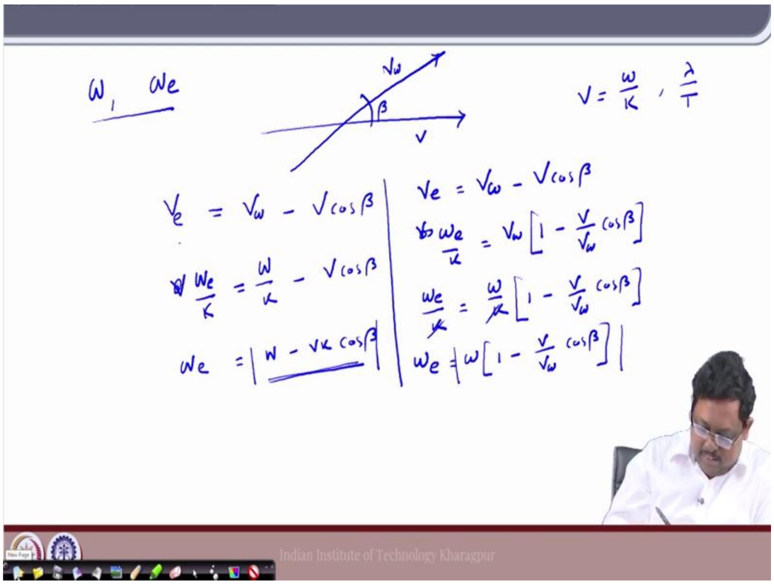

Now, with respect to this definition, let us try to find out the relationship between the wave frequency ω and the encounter frequency ω_e okay?. Now, let us take. So, let us say this is your the ship is moving with the velocity v and let us take this wave is moving with the velocity v omega. So, definitely this is the angle β.

So, then if I call the relative velocity or we can call the V relative or velocity encounter velocity definitely, $V_e = V_\omega - V\cos\beta$, right? Fine. Now, remember one thing that the velocity, $V = \frac{\omega}{k}$. So, that is or $\frac{\lambda}{T}$ both is known to you right.

So, therefore this V definitely we can now from V, we can write this $\frac{\omega_e}{k}$ equal to now V_ω is now it is $\frac{\omega}{k} - V\cos\beta$ and then if you multiply throughout the k. So, then we can get $\omega_e = \omega - Vk\cos\beta$. This is one way you can find out there is I think I mean there is many way you can get it.

So, this is this is 1. Now, we can make alternative expression also for example, I can take this $V_e = V_\omega - V\cos\beta$ and then I can write $V_e = \frac{\omega_e}{k}$ and here I can take common V_ω. So, I can get $1 - \frac{V_e}{V_\omega}\cos\beta$ and then I can write $\frac{\omega_e}{k} = \frac{\omega}{k}\left[1 - \frac{V}{V_\omega}\cos\beta\right]$. So, I can cancel k. So, I can get another expression something like $\omega_e = \omega\left[1 - \frac{V}{V_\omega}\cos\beta\right]$ right. I mean you can take anything right at the end everything you know should be same.

Now, since this ω_e cannot be negative, but there is some situation you can see from here that ω can be negative right. So, normally when you do the mathematics or when you solve it normally we try to take the not normally we take the modulus of it. So, that to make sure that encounter frequency always become positive ok.

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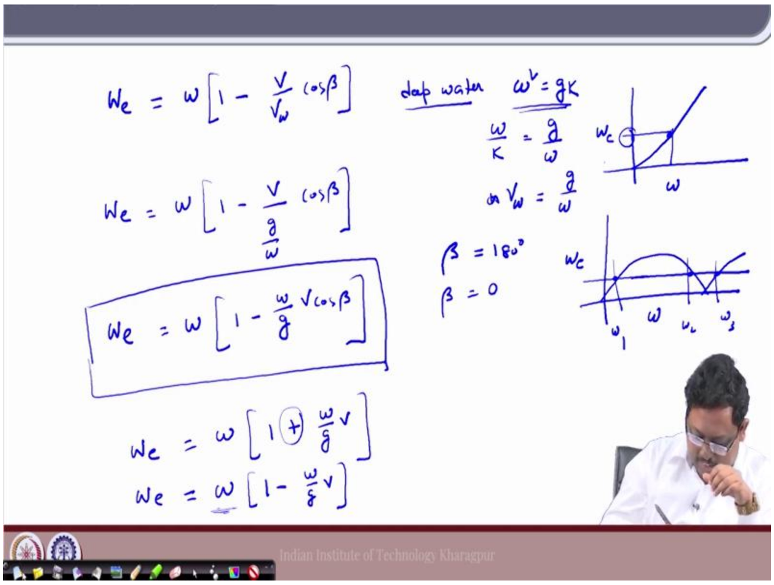

So, so let me write again the expression that $\omega_e = \omega$, let me go with the second one $\frac{V}{V_\omega}\cos\beta$. Now, here if you take that deep water situation or deep water case where the dispersion relation becomes $\omega^2 = gk$ and then we know that $\frac{\omega}{k} = \frac{g}{\omega}$. Now, this $\frac{\omega}{k} = V_\omega$ over here. So, $\frac{g}{\omega} = V_\omega$.

Now, if I substitute over here. So, we can get $\omega_e = \omega\left[1 - \frac{V}{\frac{g}{\omega}}\cos\beta\right]$ and then finally, you can get $\omega_e = \omega\left[1 - \frac{\omega}{g}\mathrm{V}\cos\beta\right]$. So, for in case of a deep water we are going to use this relationship and actually in this course we are really going to use this because we are always dealing with the deep water situation where this disperse relationship holds ok.

Now, there are so many discussion is possible over here ok, I just tell you only two possibilities let us take the two extreme situation when $\beta = 180^\circ$, what is happening then we can see my $\omega_e = \omega\left[1 + \frac{\omega}{g}\mathrm{V}\right]$ because $\cos\beta = -1$.

And if you take this $\beta = 0$ so, that time we can take $\omega_e = \omega\left[1 - \frac{\omega}{g}\mathrm{V}\right]$ because at that time $\cos 0^\circ = 1$. Now, if you look at these two equation you can see that ω the nature of ω is a quadratic and here it is always positive because this is plus. Now, if I try to draw a graph between let us take ω_e to ω you can see this is quadratic graph. So, it should go like this way.

Now, here we have one advantage this advantage as follows that if we talk about this and this. So, for each ω I have one ω_e for sure, but now if you look at this one, now if you try to plot a graph here it is ω_e versus ω then you can see this graph should be go like this way.

And, then since it is not negative I should go like this way. So, both are quadratic, but the nature is different now here the problem is in this following sea condition you know we can have three same ω encounter frequency for different ω_1, ω_2, and ω_3.

So, therefore, in case of a following waves it is really difficult to find out at this if this is your encounter frequency what is the corresponding wave frequency right. You know this is I mean this all discussion it is actually should do in separate sea keeping course. So, we are not going much detail into this. So, we drop this the concept of encounter frequency here and let us try to discuss the other two concepts which is the, what is the forward speed effect on the kinematic free surface boundary condition.

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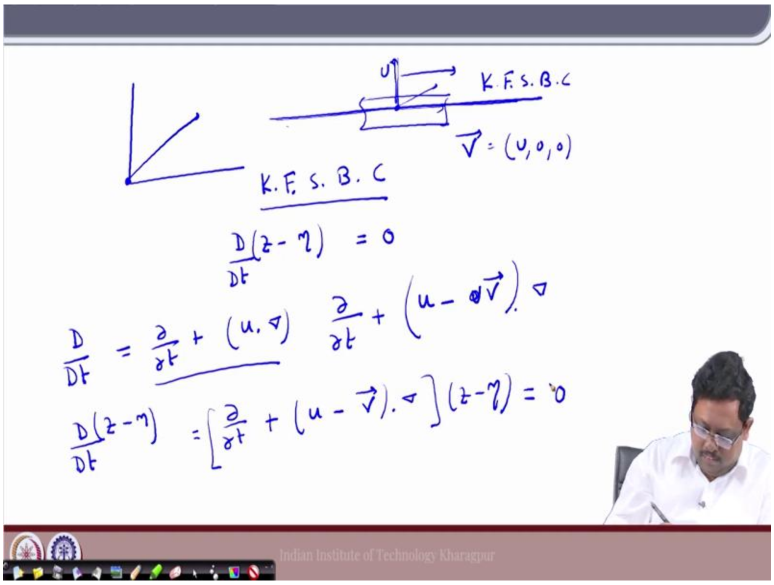

Now, in case of a now here we need to define two reference frame. Now, one reference frame is in earth fixed system and one and then you I we can draw another reference frame let us say which is let us say on the ship and with which is let us say moving with a velocity U with the ship.

So, here I have another reference frame. Now, if I try to write the kinematic free surface boundary condition with this moving reference frame, how do I do that? So, if it is with respect to fixed space does not matter, but now I need to write the same thing with respect to the moving reference frame ok.

Now, I know that the kinematic free surface boundary condition it is $\frac{D}{Dt}(z-\eta)=0$. Now, how I write this $\frac{D}{Dt}=\frac{\partial}{\partial t}+(u.\nabla)$. Now, here in case of $(u.\nabla)$ because the in the reference is itself moving with a velocity. Now, if you do this $\vec{V}=(u,0,0)$.

So, definitely this equation going to modified as $\frac{D}{Dt}=\frac{\partial}{\partial t}+(U-V).\nabla$. Now, once I do that then my kinematic free surface boundary condition becomes $\frac{D}{Dt}(z-\eta)=\left[\frac{\partial}{\partial t}+(U-V).\nabla\right](z-\eta)=0$.

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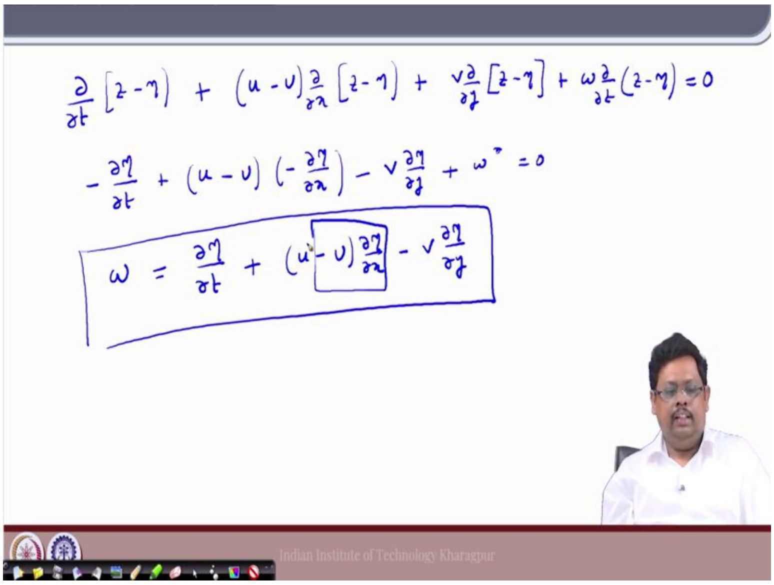

Now, if I do this then what I get as follows where, we are having this $\frac{\partial}{\partial t}(z-\eta)+(U-V)\frac{\partial}{\partial x}(z-\eta)+V\frac{\partial}{\partial y}(z-\eta)+\omega\frac{\partial}{\partial z}(z-\eta)=0$.

So, now remaining part is very simple it is $-\frac{\partial\eta}{\partial t}+(U-V)(-\frac{\partial\eta}{\partial x})-V\frac{\partial\eta}{\partial y}+\omega=0$. So, then I got $\omega=\frac{\partial\eta}{\partial t}+(U-V)\frac{\partial\eta}{\partial x}-V\frac{\partial\eta}{\partial y}$. So, this is my kinematic free surface boundary condition for the moving frame of reference ok. Now, you see the only difference is we are having one extra term ok because of this forward speed fine. Now, let me write the dynamic free surface boundary condition and after that today we are going to end this lecture.

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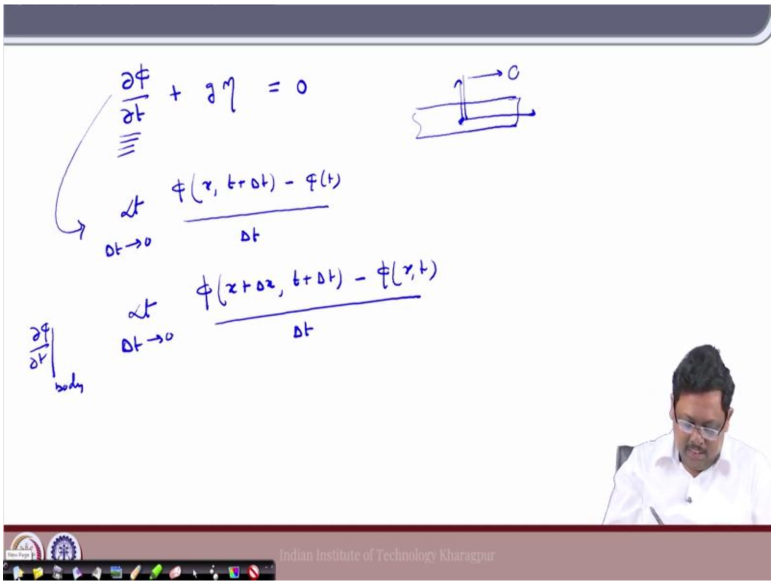

Now, to write the dynamic free surface boundary condition you know that it is nothing but $\frac{\partial \phi}{\partial t} + g\eta = 0$. I mean I am taking the linear only ok let us take the linear case only. Now, here now we need to understand this is for the fixed reference frame. Now, what should be the moving reference frame this thing. Now, if we know that this the here the definition is nothing but it is $\underset{\{\partial t \to 0\}}{Lt} \frac{\phi(\mathrm{x}, \mathrm{t}+\Delta \mathrm{t}) - \phi(\mathrm{t})}{\Delta t}$ something like this right.

So, I am fixing this position and I am taking the displacement of the body some (Refer Time: 24:36) qualitatively we can see that. But, now what is happening in case of a fixed reference frame when you try to measure this, the body itself moves some velocity and then the position will shift from here to some other position.

So, what is happening with respect to the body fixed coordinate system, thing is what is happening is at we can think of it is $\{\partial t \to 0\}$. Now, ϕ actually is the position moves from some Δx and $\underset{\{\partial t \to 0\}}{Lt} \frac{\phi(\mathrm{x}+\Delta x, \mathrm{t}+\Delta t) - \phi(x, \mathrm{t})}{\Delta t}$ something like this is happening. So, this is we can say that $\frac{\partial \phi}{\partial t}$ with respect to the body something like this is something is happening ok. Now, let us see that how we can relate this to ok.

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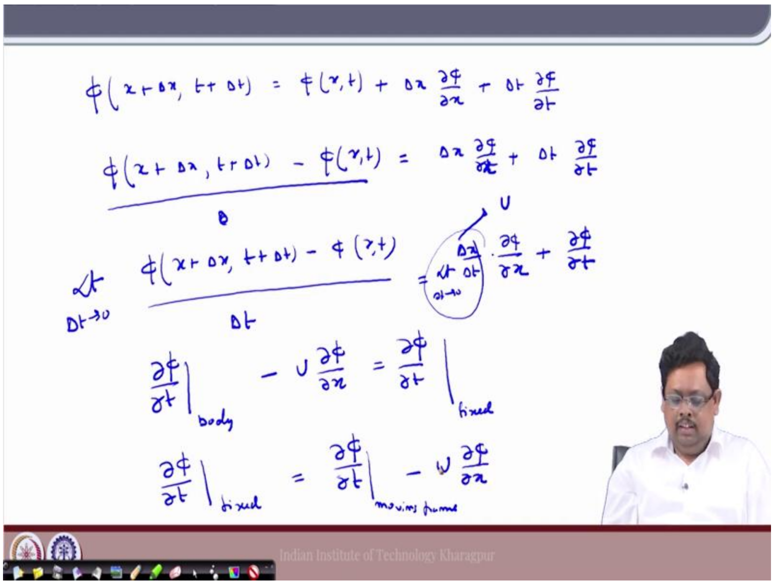

So, now let us do the Taylors expansion of $\phi(\mathrm{x}+\Delta x, \mathrm{t}+\Delta t)=\phi(x, \mathrm{t})+\Delta x \frac{\partial \phi}{\partial x}+\Delta t \frac{\partial \phi}{\partial t}$.

Now, I divide the everything with respect to Δt and taking limit tending to 0. So, I have $\underset{\{\partial t \rightarrow 0\}}{Lt} \frac{\phi(\mathrm{x}+\Delta x, \mathrm{t}+\Delta t)-\phi(x, \mathrm{t})}{\Delta t}=\underset{\{\partial t \rightarrow 0\}}{Lt} \frac{\Delta x}{\Delta t} \frac{\partial \phi}{\partial x}+\frac{\partial \phi}{\partial t}$.

Now, if the limiting position this becomes my velocity u and this of course, I am getting this $\frac{\partial \phi}{\partial t}$ of body. Now, if I take this side so, $\left.\frac{\partial \phi}{\partial t}\right|_{body}-U \frac{\partial \phi}{\partial x}=\left.\frac{\partial \phi}{\partial t}\right|_{fixed}$. So, therefore, this is the relationship between the fixed frame and then moving frame. So, whenever I write $\left.\frac{\partial \phi}{\partial t}\right|_{fixed}=\left.\frac{\partial \phi}{\partial t}\right|_{movingframe}-U \frac{\partial \phi}{\partial x}$.

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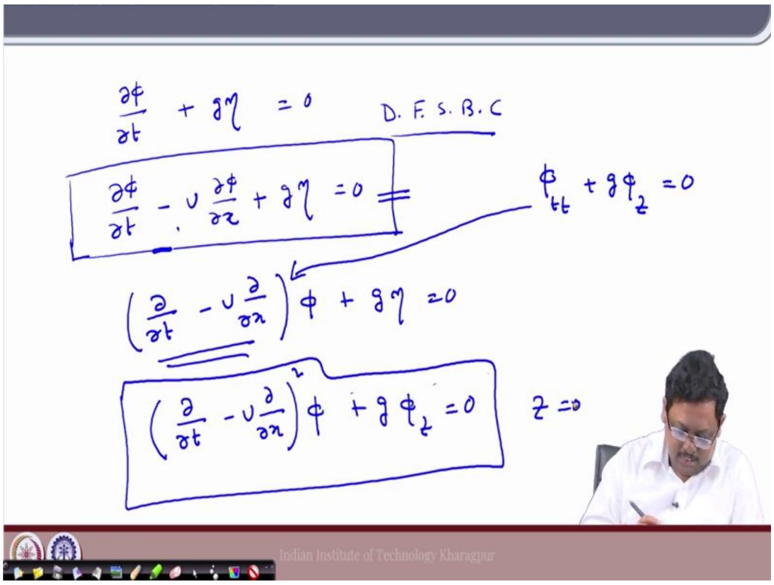

Now, with this knowledge so, now, I just replace this dynamic free surface boundary condition is $\frac{\partial \phi}{\partial t} + g\eta = 0$. Now, I am going to write as follows it is $\frac{\partial \phi}{\partial t} - U\frac{\partial \phi}{\partial x} + g\eta = 0$. So, this is going to be my dynamic free surface boundary condition.

Now, from here directly actually we can write the linearized free surface boundary condition which is nothing but. So, now, let us try to write the combined free surface boundary condition, now how do I write it. Now, from this equation I can modify this as $\left(\frac{\partial}{\partial t} - U\frac{\partial}{\partial x}\right)\phi + g\eta = 0$.

So, the important part is I can write this one like this way. So, now, if you know that the combined free surface boundary condition is $\phi_{tt} + g\phi_z = 0$. So, this actually I need to replace by this. So, combined free surface condition with respect to the moving coordinate system is as follows right.

So, this is the combined the free surface boundary condition that should be applied *Z=0*. So, now, here when from the next class onwards we are going to discuss on strip theory we are going to use this kinematic free surface boundary condition for the moving reference frame ok. So, that is all for today. So, we are going to meet next class we are going to start strip theory from the next class ok.

Thank you.