Numerical Ship and Offshore Hydrodynamics
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Lecture - 39
Strip Theory Part - 3

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Hello, welcome to Numerical Ship and Offshore Hydrodynamics. Today is the lecture 39. Today we are going to discuss some basic mathematical formulation of Strip Theory. To be very specific, we are going to discuss mainly the boundary value problem for the radiation problem. Now, by this time you aware very well that mainly we are going to deal with the three sorts of hydrodynamic forces. One is Froude-Krylov, one is diffraction, one is radiation. So, today we are going to focus on the radiation force ok.

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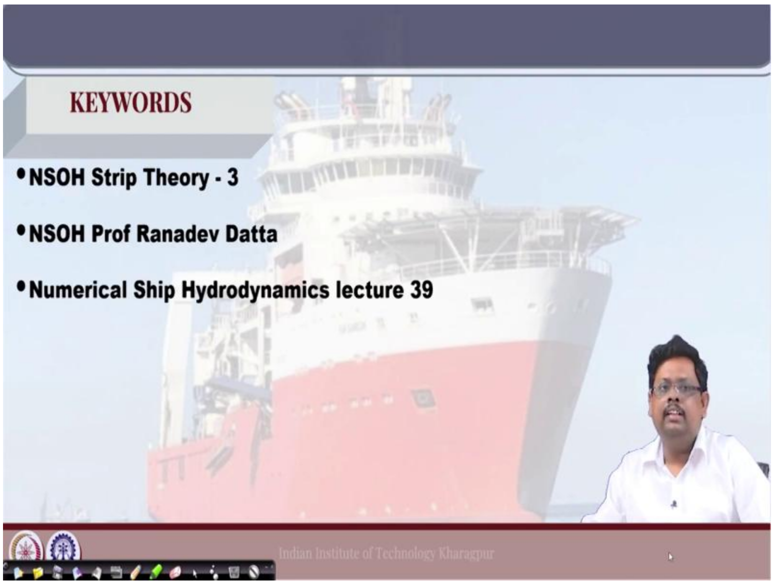

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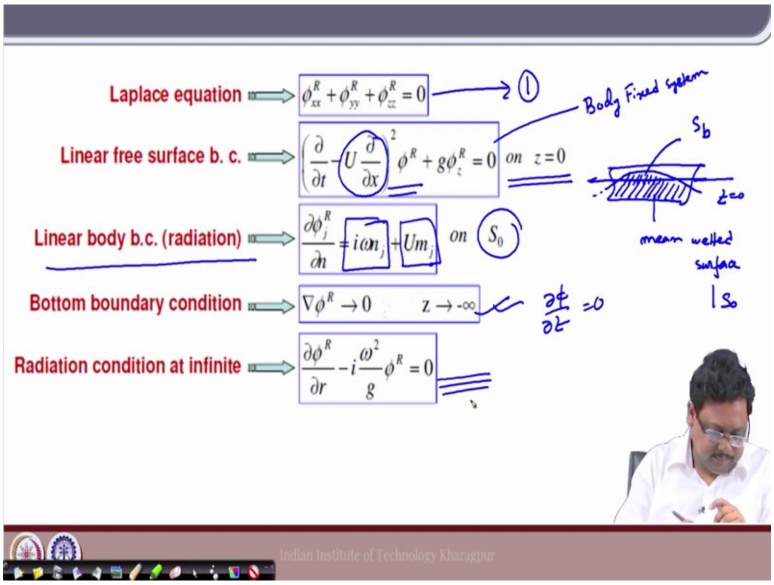

And, these are the keywords that you are going to use to get this lecture ok, let us start. Now, if I write a differential equation for the radiation problem so, definitely ϕ has to satisfy the Laplace equation which is the equation 1, this one. So, ϕ must satisfy this Laplace equation, you can call that as equation 1 right.

And, then it must satisfy the linear free surface boundary condition. Now, looking at this expression, we understand this section is written in the body fixed system right. Because, in case of a earth fixed system, that this second term this one will not be present ok.

Now, frankly speaking that we have not yet discussed very elaborately about this body reference frame and the fixed reference frame we are going to do this when we when you are going to do for the three-dimensional panel method, that time definitely discuss again all these things ok.

But, right now for this moment it is sufficient to know that in case of a body fix system, we are having one extra term which is coming over here. And, already in our previous lecture we discussed how this expression comes ok. Now, also this is the body boundary condition and it is applied on the I of course, I forget this must be applied z = 0 right.

Now, in case of a linear body boundary condition, it is definitely on S_0. Now, if you remember that we draw actually there are just again I am drawing over here, like we have a shape. Now, this is your z = 0 and we can call is the wetted surface at z = 0.

And, it is the mean wetted surface; we can call this as mean wetted surface right. However, if I superpose the wave onto this so, some part will going out of the I mean of this mean wetted some part will be out from that water part and some part will include. Now, if I consider this and this we can define this is S_0. Now, if we consider this exact wetted surface, we can call that is S_b.

So, these are the nomenclature that if you are very much familiar with then it is useful. So, when whenever I said it is S_0, you have to understand it is on the mean wetted surface. Now, again this part already known to you right because you know that we are defining, we have in case in Wamit type solution also we have used this one.

Now, this additional term which is related to the m terms, this is only coming because of the forward speed right; as you know that if you write the expression in terms of moving reference term, again one point you should be very careful to know that, this all this formulation it is based on moving reference frame. In case of a fixed reference frame, this we do not have the same terms ok, fine. Now, here also you know so, this term is coming because of that and then this is the you know this is your bottom boundary condition.

So, at bottom that $\nabla\phi = 0$ or to be specific normally we go with $\frac{\partial \phi}{\partial z} = 0$. However, this in all direction it is 0 and then this is the so-called radiation condition. So, this is the general formulation for I will I would say that forward speed seakeeping problem in frequency domain ok. I repeat this is the formulation for the forward speed seakeeping problem in frequency domain ok.

So, however, in this is the general am and this is for the three-dimensional body right. However, we need to do some kind of simplification as we mentioned in my previous class that we need to do some simplification by assuming something right. So, what we are going to do is let us find out that what are the assumptions that, we are making right.

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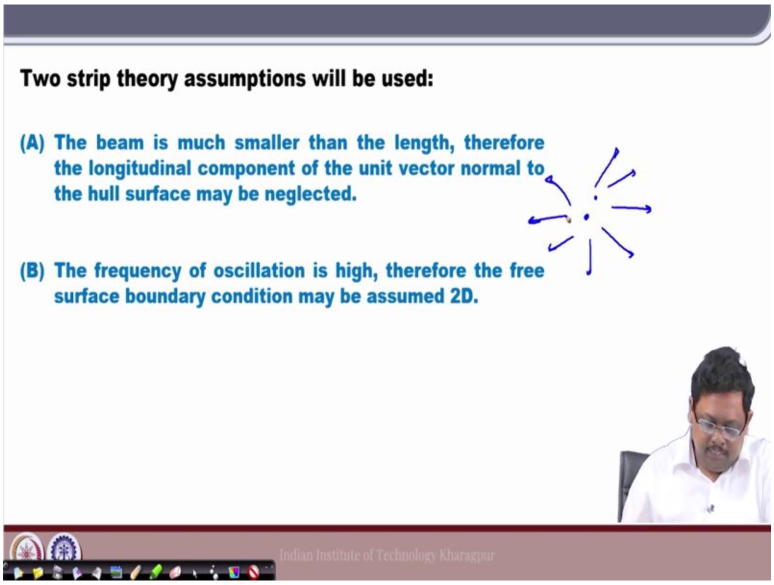

So, now here also is the same we you discuss in the last class also. So, here also why I am telling so many times just is kind of hammering, like these things is so, important you should have should not have any kind of confusion about the assumptions. So, here the assumptions are the beam is much smaller than the length. Therefore, the longitudinal component of the unit vector to the hull surface may be neglected.

So, therefore, in case of now if you see that the Laplace equation ok, in Laplace equation here this term, the variation here I mean we could neglect. Actually, you can say that the variation along the x axis is not that important ok. So, this is one and second is the frequency oscillation is high therefore, the free surface boundary condition may be assumed 2D.

Now what is that? We discussed that in last class also, just again I am just repeating the thing; now, as you said that the free surface when you create a disturbance, the wave is propagating in all directions right normally. So, if you create a disturbance over here, then this it propagate in all direction, in circular fashion right.

Now, actually just to see that I can you can see that our radiation, that our radiation condition if you look at here, it is about the $\frac{\partial}{\partial r}$ right; that very much telling you that is always it is in the radial direction right fine. Now, here in case of a high frequency phenomena, if I consider this as a high frequency phenomena.

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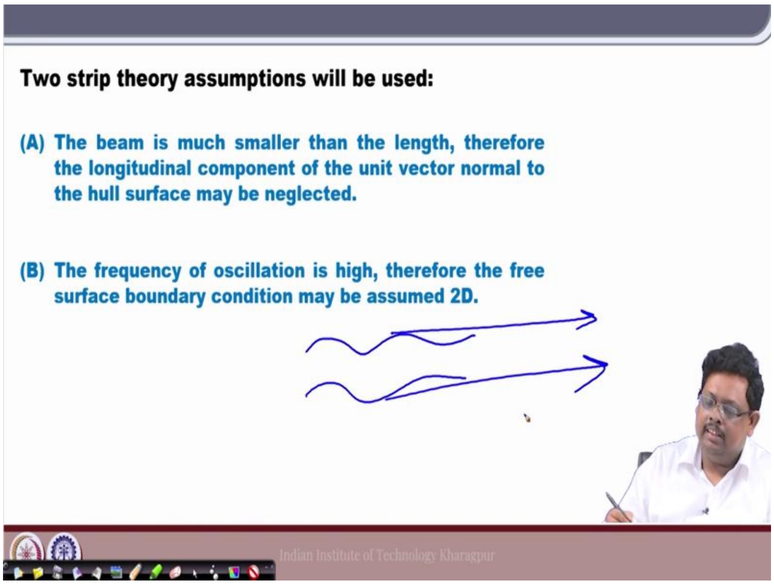

We can assume that this all this propagation are parallel to each other. So, it is on only on this horizontal direction. So, it does not propagate in all direction. So, this is the underline assumption when you say that we are taking the high frequency oscillations. So, therefore, the free surface boundary condition may be assumed at 2D ok.

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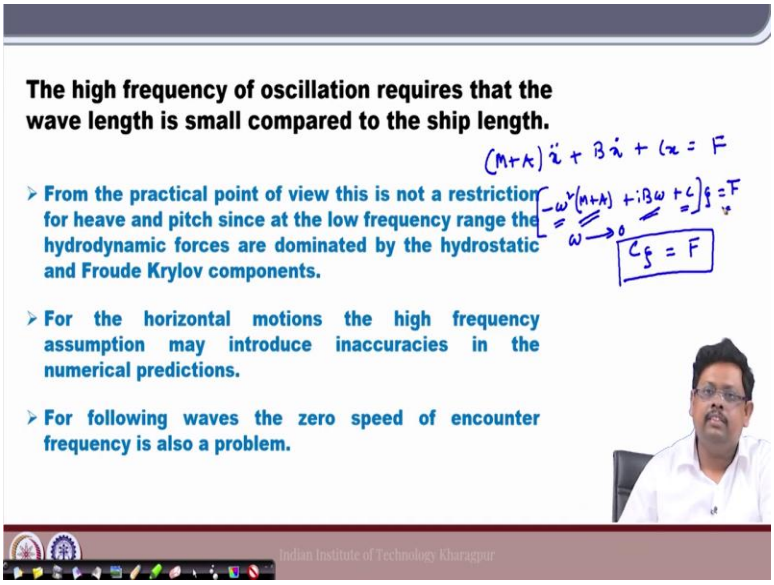

So, now very specifically this from the practical point of view, this restriction that it is a two-dimensional for heave and pitch it is not that important ok because, in case of a low frequency region mainly the hydrostatic and Froude-Krylov force are dominating right. So, that also we have discussed very thoroughly or extensively in our previous lectures, that you know that I many times I give an example.

Like when you go to the ocean and go for a bath and wave is coming, you are simply going out and going down. So, when you are doing this that moment that there is not much waves are radiated by you and there is not much wave got diffracted by you as well. So, you are simply moving up and down with the waves that time that Froude-Krylov force and the hydrostatic force, these two forces are mainly played a significant role.

Now, mathematically also it is true because, I just take a few minutes because you know if you write the equation of motion, sorry if you write the equation of motion mostly we are writing here like M plus ok, it is $(M + A)\ddot{x} + B\dot{x} + Cx = F$.

So, this is something that we are writing. Now, if you take this x is harmonic so, what is happening this, this term comes with $\left[-\omega^2 (M + A) + \mathrm{i}B\omega + C\right]\zeta = F$ right?

Now, if you take this for the low frequency region; that means, in omega tending to 0. So, in low frequency region $\omega \to 0$, we can simply say $C\zeta = F$ right. So, therefore, in low frequency region I can understand, this added mass quantities and this the damping part, that does not have much significance; mostly the restoring and the exciting force that plays a significant role ok.

Now, but however, for the horizontal motion and the high frequency assumption may be introduce some kind of numerical you know accuracy it is possible ok. And, even in case of a following speed also it could be a problem because, that leads to you know that as I mentioned when you do the encounter frequency and all that following sea wave is always a problem.

However, we are lucky that most of the time that major hydrodynamic force actually most important in head wave conditions right. So, I mean that is why it is better not to use this strip theory for the following sea condition, because sometimes that you can have some numerical inaccuracy or the code might not work properly anyway. Let us go ahead.

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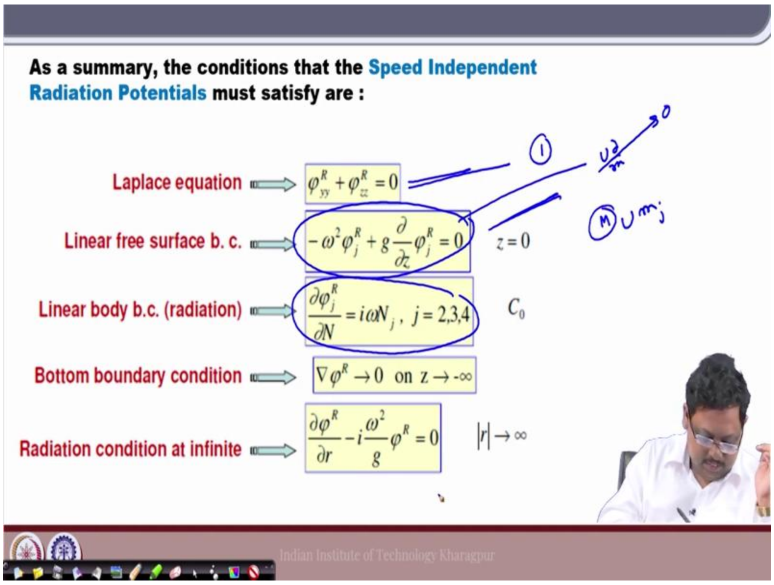

Now, if you remember I said that in case of a finding out the radiation force, it is we are doing for the zero speed problem. And, then we introduce some kind of forward speed effect when you are doing for the finding out the added mass for the three-dimensional body. Now, here also from this equation also one can understand that the two-dimensional problem we are solving in case of a zero speed.

Now, you see that here we are dropping the x term so, the Laplace equation is simplified right. So, this the equation 1 is the Laplace equation which is simplified. And, then also we are doing this the second equation also, that free surface boundary condition also, we are replacing the ϕ_{tt}, because this u term we are we did not drop the u term because of the zero speed condition.

So, it is as same as the floating body problem right. So, this is the equation in case of a floating body. Here, see when I say the j, it is the jth mode right ok. And, then if you look at this the third boundary condition this one and also you know from here how I get it? I just drop that u into you know del this term, if I drop this term $U\frac{\partial}{\partial x}$, then you get this one.

And, here also I am dropping that m terms right, that m terms actually Um_j that terms actually, I am dropping because again I am saying that the that is the main flavor of the strip theory. I am solving this boundary value problem in case of a zero speed and then we develop some theory to incorporate something. So, that I mean incorporate the forward speed effect in other words ok. But, when you solve the two-dimensional problem, it is always in the zero speed.

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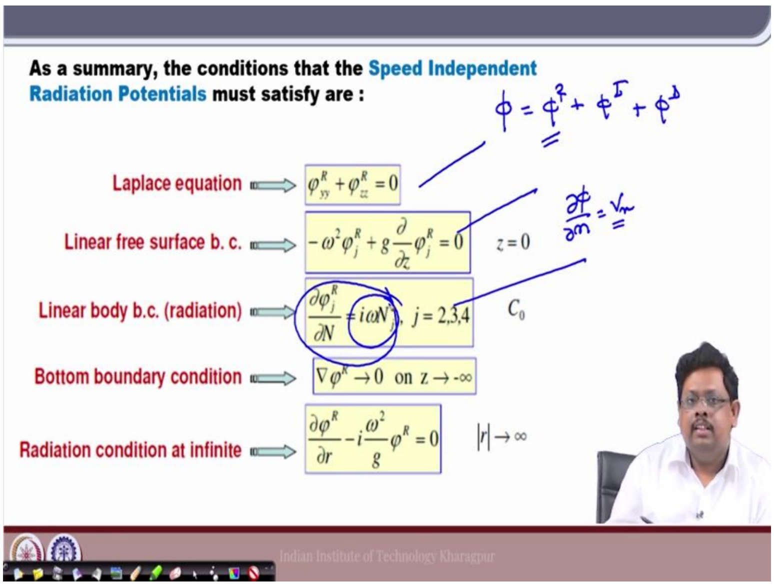

So, these are the major three equations. So, these are governing equation 1 and then we are going to solve this equation based on this linear free surface boundary condition and as well as the linear body boundary condition. And, these two are radiation condition and bottom boundary condition is always there. Now, here also you know to be very specific, here I did not mention about the splitting of my total potential ϕ.

Because, we understand already this total potential ϕ is combination of ϕ^R plus ϕ^I and then ϕ^D right. So, we are only discuss about this ϕ^R right. So, at this point today we are only discussing about the radiation problem. We are not discussing about the excitation problem or diffraction problem that is why I am writing the modifying boundary condition which includes the exciting force component also right.

So, here it is only we are applying $\frac{\partial \phi_j^R}{\partial n} = V.n$ right and then now you understand then this is $V.n = i\omega n_j$. And, you know very well from our potential flow theory that wamit solution, how we can obtain this $i\omega n_j$ right. So, here we understand two things, one is I am dropping the forward speed term number 1.

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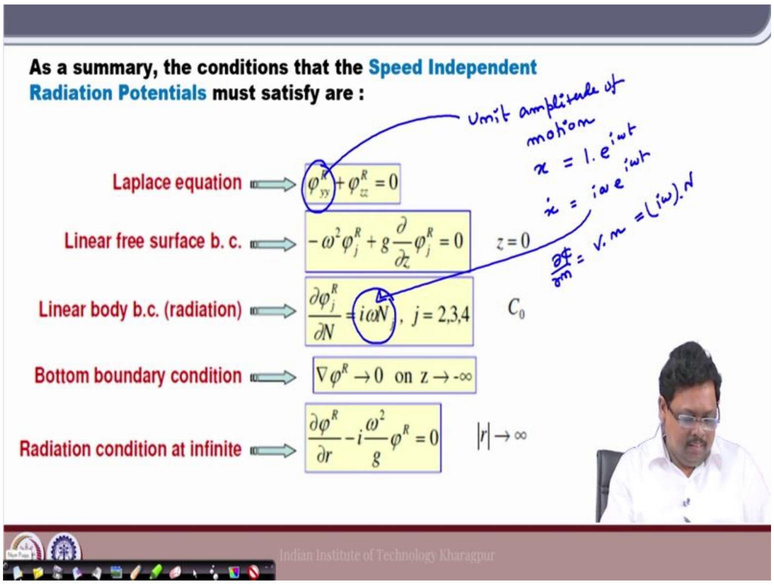

And, also this ζ, the radiation potential this is only for the unit amplitude of motion right. So, that is why? Because otherwise you do not get this $i\omega N_j$ right, because I assume my motion expression let us say $x = e^{i\omega t}$.

So, my velocity $\dot{x} = i\omega e^{i\omega t}$ and that actually I substitute over here, because my $\frac{\partial \phi_j^R}{\partial N} = V.n$. Now, $V.n = i\omega N_j$, the normal. I hope we discussed a lot before also, today also I am discussing. So, I hope that this things is clear to you right now.

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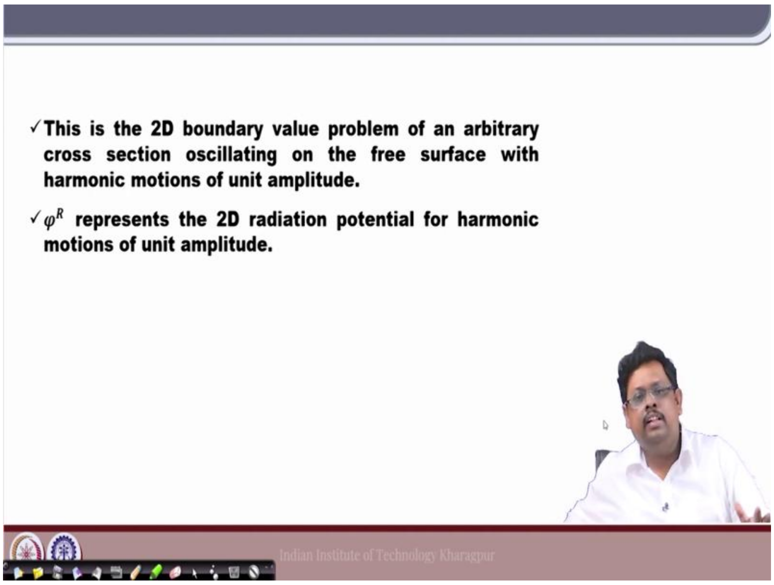

So, this is what is written over here. This is the 2D boundary value problem of an arbitrary cross section oscillating on the free surface with a harmonic motion of unit amplitude right. And, ζ^R represent the two-dimensional radiation potential for the harmonic motion of unit amplitude right. And, why we are choosing this unit amplitude and what is the benefit of using this unit amplitude that we discussed a lot before. So, today I am not going to discuss this anymore ok.

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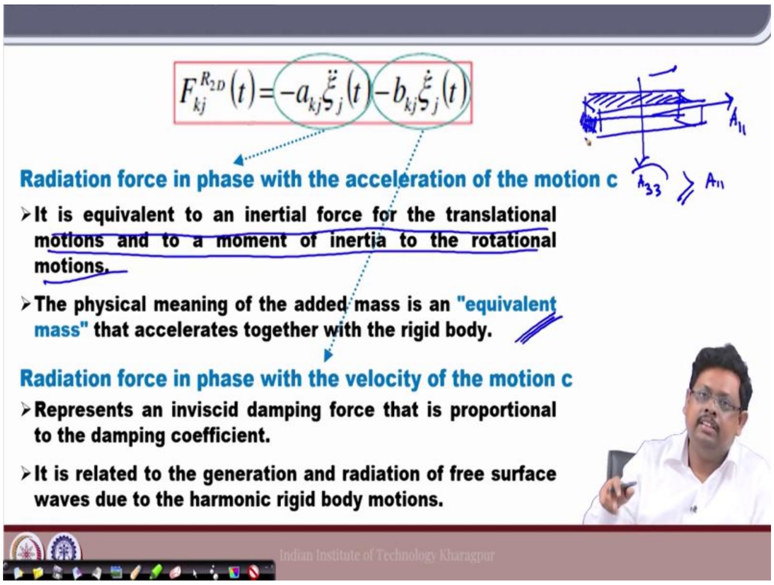

So, now here assume that I am getting the I mean today mainly we are focusing on the overall description of the radiation waves ah. So, we are not really going to detail analysis of how we are obtained this the two-dimensional added mass coefficient a and two-dimensional damping coefficient b, in our future lecture definitely we are going to discuss the same. But, here today our idea is having the you know this two-dimension added mass and damping suppose I have this; then how could I write the radiation force?

So, now, this radiation force has definitely has two component. So, one is go with the acceleration and we call that is an added mass. So, that is what I written is this that. So, it is equivalent inertial force for the transcendental motion and moment of inertia of the rotational motion. Now, here people might have problem with this physical explanation which is equivalent mass that accelerates together with the rigid body.

However, I would say that the correct mathematical way to describe the added mass as we create and disturbance the oscillate body in unit motion amplitude. And, then because of this oscillation, we have a pressure field around the body. And, now if we integrate the pressure field, then we can then we can see that some component is going with the acceleration and some component is going with the velocity.

Now, that part is going with the acceleration, we call the added mass. Now, this is mathematically fine, but then in realistically suppose you are in ship yard and suddenly you try to find out the natural frequency of some vessel. So, then if your boss tell that see you have some 5 to 10 minutes time to predict the what would be the natural time period, then that time you need to estimate that added mass. Then, how do you do that? So, that time normally you are taking some for heave added mass and all you taking some 80 percent, 90 percent of the mass right.

So, that time when you approximation, approximating this added mass that time mostly we are actually following this concept only ok. It might not be mathematically very correct, but in practical application, people used to do this sort of thinking to estimate the added mass. So, if you remember that is why I said now suppose if you have a body, a body a long body like this. And, then if I ask you a question that can you tell me that if this added mass in this direction is more or added mass on this direction is more?

So that means, I ask you if the added mass A_{33} is magnitude is of A_{33} is more or magnitude of A_{11} is more? Now, you people definitely going to answer that a magnitude of $A_{33}>A_{11}$, because if you oscillate in this vertical direction then you are actually oscillate this larger part of the mass right. You are moving the larger part of the mass and if you oscillate in this direction, you are oscillating the smaller area.

So, in that way we understand that surge added mass is lesser than the heave added mass, may be roll added in much lesser than the pitch added moment of inertia right so, anyway. So, those are the, you know engineering way of looking at the problem. Now the second part is of course, which is going with the velocity we called as a radiation damping ok.

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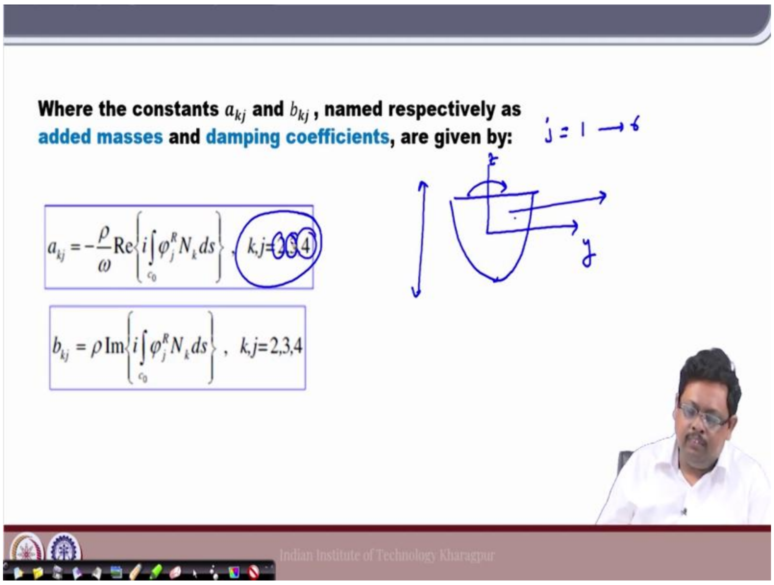

Now, suppose I have these two things, how I get this? Now, again I am using the same that is the result for from WAMIT, this is same result. I mean the for frequency domain search, this is the expression for the added mass and damping. We have already discussed the expression of the added mass and damping before. And, believe me this the way getting it, it is exactly the same the way we are getting for the three-dimensional sequence domain problem.

There is nothing, there is absolutely no change. In that case we are taking a panel. Now, what we are going to take for, in case of two-dimensional definitely we are going to discuss later on. But, the process, the differential equation, the boundary condition, the splitting of the surface, getting the boundary condition at each panel; now in that case each segment all are same and then the final expression also same. Only thing you can see here, that in case of a three-dimensional body; I used to go for everything.

I used to go from this j = 1 to 6. However, in case of this, the strip theory the j is only oscillating 3 modes; it is 2, 3 and 4 which is sway, heave and then the roll right. Why? Because you see this is a two-dimensional body. So, therefore, it can only oscillate, now it is if you take this is y and z direction.

So, therefore, it can oscillate about the - I mean on the z direction. These are translation motion which is which is nothing but my 3. It can go in the y direction right, which is the 2 and also we can have a moment which is the 4.

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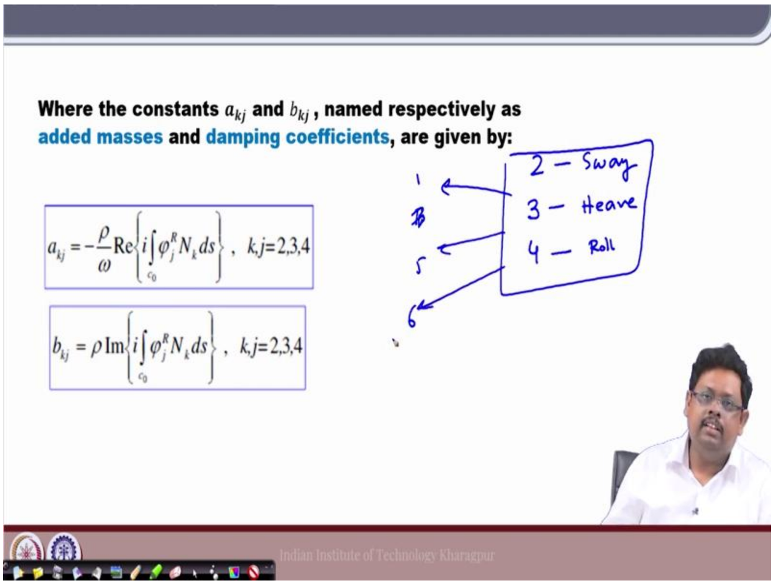

Now so, therefore, when we solve the two-dimensional problem, we only solve for the mode 2 which is sway, we solve for mode 3 which is heave and we solve for mode 4 and which is roll ok. And, from there when you do the integration, we can get 1, 3 and 5 from these three results, yeah sorry 1, 5 and 6 from these three results. We can always getting all remaining 3 modes. If we have this 3 mode value, we can from here we can get all this remaining 3 modes, that we are going to see later on ok.

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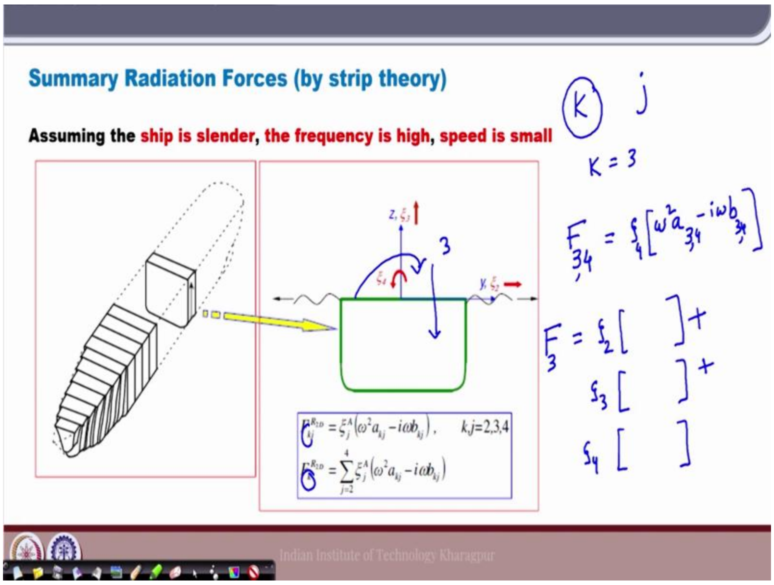

So, now if you have if you have the slender body and if you have take this section out of it right. And, now from the previous slide, we saw that it is actually component of added mass and damping. So, that is actually I am writing over here right. In two-dimensional case it should be, now here the indexing you should not confuse with the indexing, that k is the index.

The first index that k, it is that which mode I am trying to figure out my radiation force. And, then the second index which is j, it is that which mode I try to oscillate my body. It means that suppose I have this body and now I want to find out what is the added mass in the direction of 3. So, in that case my k becomes 3, K=3. However, I can oscillate the body, let us take in roll also.

So, in that case I am trying to figure out that force at the mode 3, while I am oscillating body in mode 4. So, in that case it is F_{34} right. So, therefore, it is equal to it says that ζ that amplitude of - so, oscillation of that mode which is 4 and multiply by the ω^2 into added mass, $\omega^2 a_{34}$ at which I try to find figure out the force; in that case it is 3. And, then the second index is at direction I am trying to oscillate which is 4, $-i\omega b_{34}$.

So, damping coefficient also the first component that side which I try to figure out that I am interested to get the force on that direction, in that case it is heave and then I am a similar body in the direction of 4. So, this is how actually we write here over here. So, you can see the expression. So, $F_{kj}^{R_{2D}} = \zeta_j^A(\omega^2 a_{kj} - i\omega b_{kj})$, where this k stands for the mode which I am interested to get the force and j stands for the mode I am oscillating the body.

So, therefore, if I try to find out, then what is the force? My 3rd mode so, definitely it is a summation right, summation is suppose if I oscillate the body in the 2nd mode and then I try to figure out what is the force in the 3rd mode, plus if I oscillate the body in the 3rd mode, then what is the force I am getting in the mode 3? Plus if I oscillate the body in the 4th mode and then what is the oscillation of the body right.

So, that is how you can see over here this summation j =4; so, this j stands for that at which mode I am oscillating the body. So, therefore, j = 2, 3 and 4. However, you can see my k is here; that means, at which mode I am interested to get the force. Now, if I if you understand this very correctly, then we are done actually.

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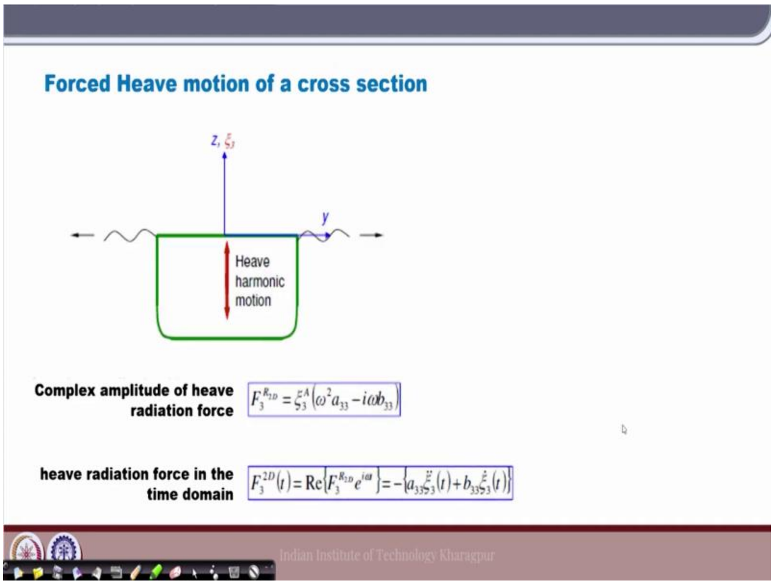

Then, very easily you can understand this is the expression that I am writing for the heave mode. Now, heave mode is not coupled with any other mode. In this equation, this heave mode is not coupled with any other mode. So, therefore, I have a single expression for the heave.

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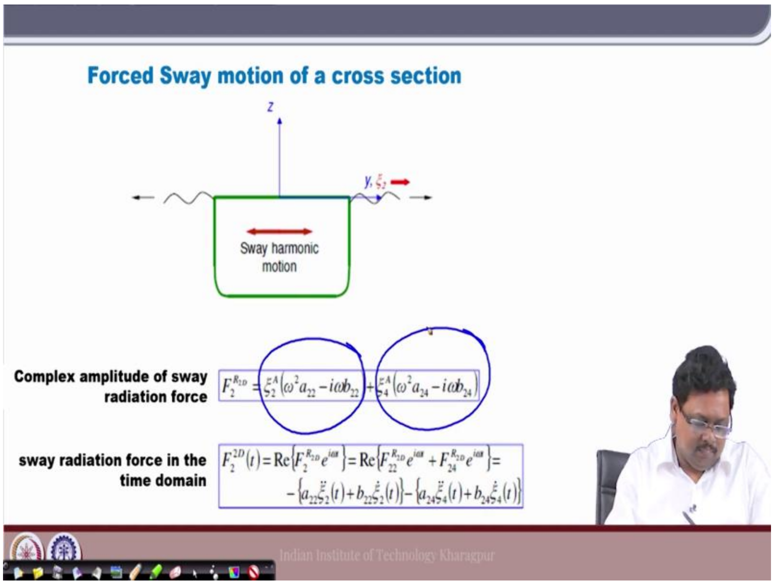

However, if I go for the sway, now sway is always coupled with roll ok. Now, you see that that now sway means you know so, let us take - I mean better is not the pen may be this mobile. Now, what is happening? sway means it is oscillating this way. Now, what is happening when this going in this direction, it can you know roll also. Now, you see it can roll in two way, like you see it is with phase going left side, I am rolling this way, going right side rolling this way. This is one orientation right.

And what is the other orientation? Other orientation is I am going and then I am having this way, also can I can use that. So, I can see that roll and sway are coupled each other. So, therefore, I have to use the coupled equation of motion for the sway. So, this is what is you can see from this expression right. We have the first term, the first term is for the you know we have this first term. This first term is for the sway mode and again we have this second term, this second term is for the roll mode ok.

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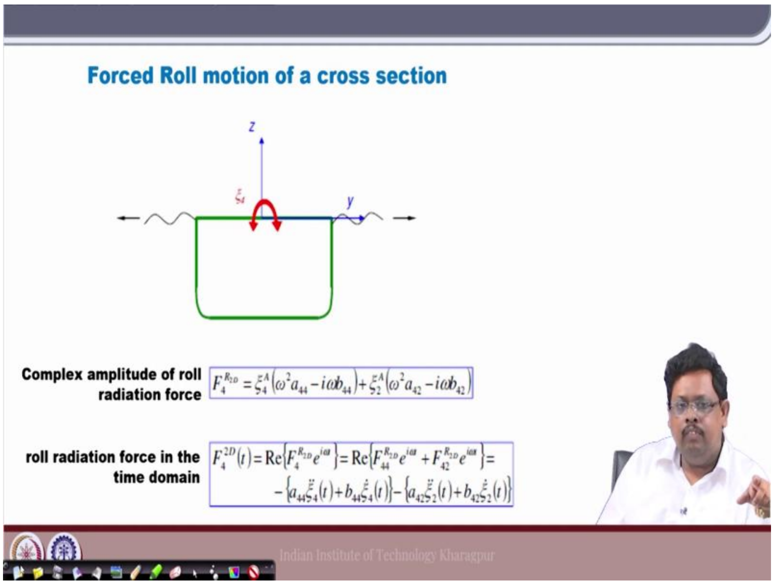

So, similarly we can understand that when we consider the roll motion also, that also coupled with the sway. So, here also I can see the first term is the you know that this is the for the role and then the second term, which is basically for the sway. So, now, I understand that how I can write the two-dimensional you know radiation force for a particular section now.

So, now maybe we stop today at this point and in the next class we start from this point. Now, writing it in a two-dimensional section so, how can we write for the three-dimensional body, how we can write it so, that thing we can discuss from next class. And, also if you have time then we can discuss for the exciting force also which is combination of the diffraction and the Froude-Krylov ok so.

Thank you very much.