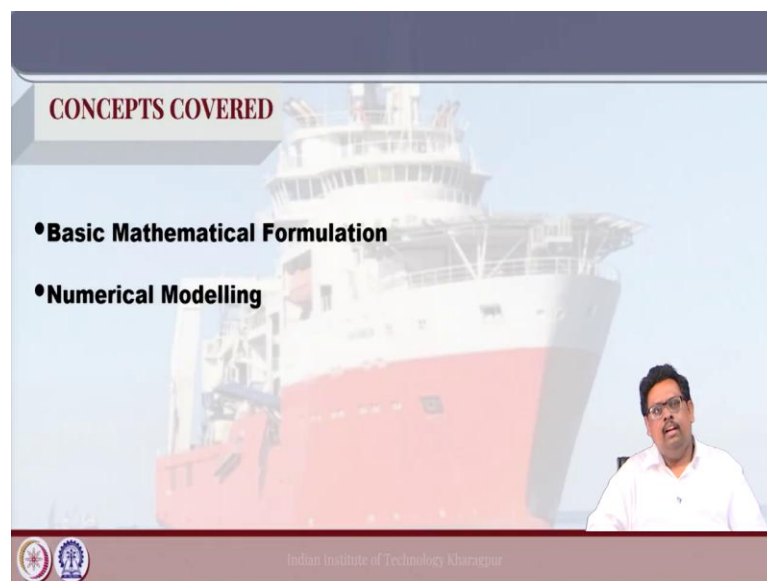


Numerical Ship and Offshore Hydrodynamics
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Lecture - 41
Strip Theory Part - 5

Hello. Welcome to Numerical Ship and Offshore Hydrodynamics. Today we have lecture 41.

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So, this is the concept we are going to cover today, it is continuation of the basic mathematical formulation that we have done for last few classes. Today, we are also going to discuss about the numerical modeling ok.

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KEYWORDS

- NSOH Strip Theory - 5
- NSOH Prof Ranadev Datta
- Numerical Ship Hydrodynamics lecture 41

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And, these are the keywords that we are going to use to get this lecture ok.

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The incident potential can also be represented by:

$$\Phi^i(x, y, z, t) = \zeta_a \phi^i(y, z) e^{ik_0 x \cos \beta} e^{-i\omega t}$$

$$\phi^i(y, z) = \frac{ig}{\omega_0} e^{k_0 y} e^{ik_0 z \sin \beta}$$

where $\phi^i(y, z)$ is the complex amplitude of the potential of a unit amplitude wave acting on the hull cross sections.

Using the former expressions, together with the strip theory geometric simplification, $ds \equiv d\zeta dx$, and the 2D unit normal vectors

Ship Froude-Krylov Forces

$$F_1^i = 0$$

$$F_k^i = \zeta_a \int_L \left(e^{ik_0 z \cos \beta} f_k^i \right) dx, \quad k=2,3,4$$

$$F_5^i = -\zeta_a \int_L \left(e^{ik_0 z \cos \beta} x f_3^i \right) dx$$

$$F_6^i = \zeta_a \int_L \left(e^{ik_0 z \cos \beta} x f_2^i \right) dx$$

Handwritten notes and diagrams: $f_2 \rightarrow$, $f_3 \rightarrow$, $f_4 \rightarrow$, $f_5 = \alpha f_3$, $f_6 = \alpha f_2$. A diagram shows a ship hull cross-section with a grid of strips.

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So, now in last class we discuss that to get the total Froude Krylov force over the body what we are going to do is as follows. We take a ship we take a ship and then we have done the cross section and each cross section if I take the sectional Froude Krylov force which is written by the f ok. So, let us take f_i is the sectional force and then if you integrate this sectional force over the line; that means, over the length we can get the total Froude Krylov force right.

Now, again here we have now you know that we have six mode right, and in last class we have discuss this f_1 is always 0. So, therefore, we need to take the sectional force only in three mode. So, what are the three mode the first mode is f_2 which is sway, then f_3 which is heave, and f_4 which is roll.

Now, if I know $f_2, f_3,$ and f_4 . So, definitely I can get this f_5 by taking the moment multiply the heave mode. And, if you get this for the sixth mode it is nothing but f_6 is nothing but $x f_2$. So, this is how actually we are going to get the total Froude Krylov force over the body right.

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represents the 2D Froude-Krylov force due to unit amplitude waves:

$$f_2' = -i\omega \rho \int_{C_0} \{\varphi'(y, z) N_2\} d\zeta$$

$$f_3' = -i\omega \rho \int_{C_0} \{\varphi'(y, z) N_3\} d\zeta$$

$$f_4' = -i\omega \rho \int_{C_0} \{\varphi'(y, z) N_4\} d\zeta$$

$$f_k = -i\omega \sum_{i=1}^N \psi(y_i^k, z_m^k) \cdot N_i^k$$

Handwritten notes on the slide include: $\phi = -\rho \frac{\partial \phi^F}{\partial t}$, $\phi^F(y, z, t) = \psi(y, z) e^{i\omega t}$, and $f_i = -[\rho] i\omega \psi(y, z) \cdot N_i d\zeta$. A diagram shows a ship's hull cross-section with axes y and z , and a curve C_0 . A lecturer is visible in the bottom right corner.

Now, here we are going to check that how we can get the sectional force. Now, in sectional force you know that it is force is nothing but the integration of the pressure. Now, where I going to do the integration now here this f since it is f is in the $y z$ plane because, I am drawing the cross section.

So, it is with respect to the y direction, this is the z direction and this is your curve. So, we can call this is C or C_0 why 0, 0 is basically I am integrating upon the mean weighted surface. If I want to do for exact weighted surface in that case it should be C_b .

So, let us not go into that let us restrict our attention into the linear case and then also we know that if I look this previous thing that it is harmonic ζ is harmonic multiply with

$e^{i\omega t}$. Now, if I do that here this pressure then it is nothing but $p = -\rho \frac{\partial \phi^I}{\partial t}$. So, here so, $\frac{\partial \phi^I}{\partial t}$ now again this ϕ^I let us take that sectional ϕ^I which is let us take $\phi^I(x, y, t) = \psi(y, z)e^{i\omega t}$ right? So, if I do that apply over here then pressure is coming $-i\omega\psi(y, z)$.

Now, if I want to integrate it. So, get this normal. So, if I try to find out that. So, it is let us say this pressure integrated in the sway mode or surge mode whatever, now if you integrate then we get this let us take the sectional force in the i^{th} mode it is nothing but this integration. So, I just make $-\rho$ over here $-\rho$ this and it should be multiplied by that type normal and over this dC or dC_0 or here I am get a dx_i .

So, that is how we are going to get the force. Now, here you can see I just leave this part, I did not write minus over here why? Because sometimes instead of $e^{i\omega t}$ many people take $e^{-i\omega t}$. If you take $e^{-i\omega t}$ then this become plus because this minus will this minus will cancelled out. So, therefore, based on the situation it should be minus or it should be plus.

If you take $e^{i\omega t}$ definitely you have a minus over there, and if you take $e^{-i\omega t}$ definitely you have the plus over there. And, now this is very simple for you for numerical calculation because this ζ is a analytic function, if I go back I can see this ζ is a analytic function. Now, so what we need to do is I need to; I need to take the section over here and then I need to divide this number of panel and then you need to take the center point of this.

Now, in this case panel is nothing but a straight line because a two dimensional curve and I take the center of this point and in this center of this point, I calculate this numerical value right, and then I multiply by the normal at this particular point. So, which is here and then I sum up everything. So, these things essentially this f_i equal to now I just use minus $i\omega$ because I am taking $e^{i\omega t}$.

It should be sum of is $i = 1$ to the number of straight line. So, I take I have some N number of straight line. So, then I just put this functional value at the middle point of this each section. So, I can just called y_m, z_m and of course, it is i . So, it is basically the middle

point between i and $i + 1$, in this section taking the middle point. I just find out the value of ζ over there right.

And, then I multiply this by that now in which mode let us say it is in now, I should not use i here because I already use i . So, let us say in the k^{th} mode. So, it is basically for the k^{th} mode for the i^{th} segment. So, this is how we are numerically we are getting the exciting force. It is similar right we have done this for before also we have done for tunnel method then omitted solution also.

So, we aware of how to do this earlier we have done it for the surface; now, here it is even easier because now surface requires panel, but curves requires collection of the straight line. We are coming we are maybe later on if we get time we will show that you know numerically how could we do this, but right now let us leave this point because we know how to calculate the Froud Krylov force. Most interest or more interesting and more complex thing is how I get the diffraction force.

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Diffraction force

Diffraction Force:

$$F_j^D(x) = \int e^{ik_0 x \cos \beta} h_j(x) dL, \quad j = 2, 3, 4$$

$$F_5^D(x) = - \int_L e^{ik_0 x \cos \beta} \left(x + i \frac{V}{\omega} \right) h_3(x) dL$$

$$F_6^D(x) = \int_L e^{ik_0 x \cos \beta} \left(x + i \frac{V}{\omega} \right) h_2(x) dL$$

$h_j(x)$ are the sectional diffraction force amplitudes,

$$h_j^D(x) = \frac{\omega_0}{c_0} e^{ik_0 x \cos \beta} \int \left\{ (iN_3 - N_2 \sin \beta) e^{ik_0 y \sin \beta} e^{ik_0 z} \phi_k^R \right\} dS$$

2D and 3D normals

$n_1 \approx N_1 ; n_2 \approx N_2 ; n_3 \approx N_3$

$n_4 = y n_3 - z n_2 \approx N_4$ ✓

$n_5 = z n_1 - x n_3 \approx -x N_3$ ✓

$n_6 = x n_2 - y n_1 \approx +x N_2$ ✓

$\phi^F = \frac{a_3}{\omega} e^{k^F x}$

$\sum_{i=1}^6 (k_n \cos \beta)$

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Now, in case of diffraction force there as I said there are lots of theories are available, but we are really not going into the theories. Now, may be in the next lecture I am some reference I will add, if you go through this reference then definitely you get to know about the theory behind this formulation ok.

But, at present what we do is how this theory I do not say discuss on how this theory works, but what is the final results and where we are going to proceed that definitely we are going to discuss. Now, in case of a diffraction force it is similar that the three dimensional diffraction force here, I can get if I know the two dimensional diffraction force ok. Now, here this $h_j(x)$ represent the two dimensional diffraction force and now this is basically particularly for a section.

Now, here if you remember we said that wave might come, but different angle right. Now, diffraction for the logic is this body is now here the thing is that body is moving forward it is not in a it is not fixed it is not for a zero speed, remember that in case of a like added mass we are considering the zero speed and then we do the forward speed correction.

Now, here suppose it is having diffraction force in the forward speed then of course, we have this three dimensional thing. So, we have the encounter wave angle β . So, therefore, at some particular section it is not simple x it should be this one right because we have already know that just go back here like if we do this. So, that is how actually we have defined this sectional Froude Krylov force right.

Because it is, if we remember our previous classes we have discuss that it is now normally this ϕ^l the way we are writing is or ϕ^l or ϕ^D it is nothing but you know $\frac{ag}{\omega} e^{kz}$.

Now, instead of $\sin kx - \omega t$ we used to write as follows we are writing this into $\sin kx \cos \beta + kx \sin \beta$.

Now, we have here this $kx \cos \beta$ and that is why you are having these things here. Anyways, and these terms again as I said this is again a forward speed correction. So, because of the forward speed it is coming and really we are not going in detail into this theory how these terms comes actually here. So, if you follow this Faltinsen I mean STF paper the classical STF paper then all these discussions is there.

But, here our intention to how to get this sectional diffraction force and after all applying all this theory and we define, now here this capital N is nothing but the two dimensional normal and small n is actually three dimensional normal. So, $n_1 = N_1$, $n_2 = N_2$, $n_3 = N_3$ and then n_4 is nothing but the moment. So, it is and then n_5 is again the in this r.n the

second term and this is r cross a the third term. Now, how it can change to the two dimensional strip this is written over here.

So, now the sectional diffraction force of any mode j , it can be defined as this expression. Now, how this comes as I said we do not discuss, but physically we can see some interesting fact is that you can see that my sectional diffraction force, I represent as a sectional radiation force.

Now, this is something very interesting right. Now, you see there is a two different phenomena is it not one is that there is no waves the still water and then body start oscillating and then the in case of a diffraction we can say the body stands still and wave is hitting. Then, how actually I can represent the radiation force a radiation potential to get the diffraction force.

Now, the idea is like it is a matter of perception. Now, suppose if you think this way now there is a wave is stand still and then the body actually pushing back then it is a radiation force right, is it not? Now, radiation force is the is water stands still and we just oscillate the body. Now, in diffraction this body is still and then wave is hitting.

Now, instead of wave hitting if you consider some picture that wave is still and this body is pushing the wave then actually we can say that ok that could be you know some kind of a radiation component right. So, this is some very vague I do not say vague, but very simplified physical understanding of this equation and lot of mathematics involved here to do this, but we do not discuss that, but only thing is that why this is important that this sectional added mass we can get through many ways.

And, may be from the you know the next class onwards, we are going to see that how we can get the sectional added mass maybe today also in later part we are going to discuss that. So, thing is that we really do not have to find out the ϕ diffraction and ϕ radiation. If I get the ϕ radiation using that ϕ radiation we can get the diffraction ok well.

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Now, let us take a special case. So, I take this $\beta = \pi$. So, then $\sin \beta = 0$ and then $\cos \beta = -1$. So, therefore, now this equation now I can see in this way like that $h_j^D = \omega_0 e^{-ik_0 x} \int_{C_0} \{ (iN_3 - N_2 \sin \beta) e^{ik_0 y \sin \beta} e^{k_0 z} \phi_k^R \} dC$ because N_2 is having $\sin \beta$. So, it goes to 0 and then this $y \cos \beta \sin \beta = 0$. So, this expression so, now, again from here we do not have to do anything.

So, now in case of a heave in case of a heave I just see that we have to solve for the ϕ_3 , right and then we have the multiplication of the N_3 . So, now, if I rearrange this further. So, we can see that $h_3^D(x) = \omega_0 e^{-ik_0 x} \int_{C_0} \{ (iN_3) e^{k_0 z} \phi_3^R \} dS$. Now, you see this is something actually very known to us.

Now, if we look at this part. So, this is the part actually we know for the radiation it is very it is very very known for us right. Now, suppose if you know ok we know this result of course, now you see from this result I know that $N_i \phi_3$ it is nothing but you can represent in terms of added mass and the damping. Now, you see this expression and this expression is very similar only thing is that I have in integration e^{kz} .

So, now if I try to if I do something for example, if I try to approximate e^{kz} and if I this z , if I replace by some constant value then these terms also can go outside and that only

leads to $\omega^2 a_{ij} - i\omega b_{ij}$ equal to you know I can use this into this value. Now, you see the idea is here idea is that in this integration I need to do some kind of approximation to get this outside.

Now, if I follow the STF paper classical STF paper they have written this expression as $e^{-k.d.s}$. Now, here this d is nothing but the sectional draft and then s is nothing but the sectional area coefficient. So, d they are using at sectional draft and then this s they using the sectional area coefficient ok.

Now, so, if I apply this then this expression then I can rewrite as follows, it is nothing but $i\omega_0 e^{-ik_0 x}$. Now, I have C_0 sorry and also this now I can take out $e^{-k.d.s}$, and now I have over this $C_0, N_3 \phi_3 dS$ or dC whatever like and then if I do that then I can use this as a added mass and the damping right.

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$$h_j^D(x) = \omega_0 e^{ik_0 \cos \beta} \int_{C_0} \left\{ (iN_3 - N_2 \sin \beta) e^{ik_0 y \sin \beta} e^{k_0 z} \phi_k^R \right\} dC$$

For $\beta = \pi$, $\sin \beta = 0$ $\cos \beta = -1$ $(\omega^2 a_{i,j} - i\omega b_{i,j}) e^{i\omega t} = \left(-i\omega \rho \int_{C_0} N_i \phi_j dS_0 \right) e^{i\omega t}$

$$h_j^D(x) = \omega_0 e^{-ik_0 x} \int_{C_0} \left\{ (iN_3) e^{k_0 z} \phi_k^R \right\} dS$$

$$h_3^D(x) = \omega_0 e^{-ik_0 x} \int_{C_0} \left\{ (iN_3) e^{k_0 z} \phi_3^R \right\} dS$$

$$h_3^D(x) = i\omega_0 e^{-ik_0 x} \int_{C_0} e^{kz} \left\{ (N_3) \phi_3^R \right\} dS$$

$$h_3^D(x) = -\frac{\omega_0}{\rho \omega} e^{-k_0 d.s} (\omega^2 a_{33} - i\omega b_{33})$$

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So, if we apply everything here and then finally, you know we get as follows, we get the now this is the very special case for head waves, if I try to figure out how to get the diffraction force then this is the expression, and you know that this in also it is very well known that $\beta = \pi$ is the condition where we are interested to get all these wave loads because it is assumed that when for the head waves it is much more dangerous compared to the other wave angle.

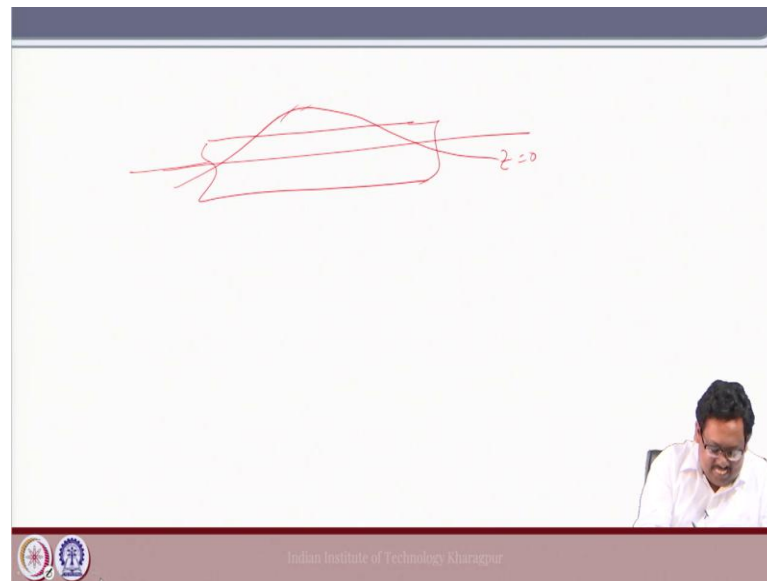
So, in case of a sea keeping if you try to figure out heave pitch coupling etcetera etcetera then you know beta equal to π is the most crucial you know as I said that more crucial heading angle, and then in that heading angle I can get this diffraction force is much more simpler way ok fine.

Now, let us jump into this now you see that as you know as you will see here that there is lot of theory available and very complex theory to calculate the diffraction force, but however, the exciting force is much easier. Now, I would like to suggest you not suggest you like propose this is not proposals, it is somehow like some simplistic way to calculate the exciting force. Now, suppose you are trying to get trying to write some code sea keeping code using the strip theory, but you are you really do not know how to handle the diffraction force right.

So, I mean then suppose, but somehow as you know that the added mass and damping I discussed in very first class, it is available the added mass and damping is available through some chart and what is the chart definitely I am going to discuss now. Now, suppose assume a situation you know that added mass and damping these values you have some chart of for this to get the added mass and damping of the ship.

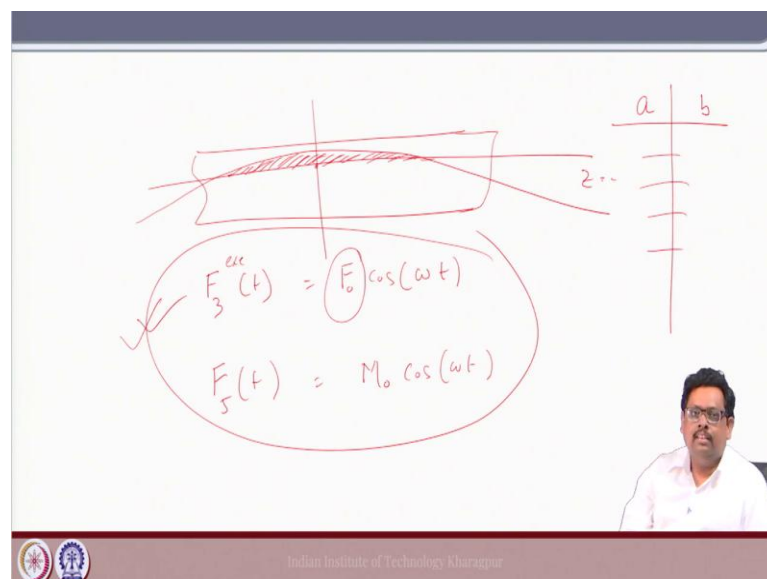
But you really have no clue how to get the exciting force, then how you write the strip theory code. So, I just discuss it and then you try to write a strip theory code based on that the chart for added mass and damping definitely can be provided or it is available in internet.

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Now, how we do that as follows suppose if you have a ship. And, then now if is let us say is the $z = 0$ line, and now if you superposed the wave here oops no it is not very bad way to superpose the waves. So, not like that ok let us do it again. So, yeah.

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Now, this is your $z = 0$ line let us say and then it is probably this is the superposition of the wave. Now, what one can do that now they can take this extra buoyancy is the magnitude of the exciting force. Now, you know that the exciting I mean the exciting

force is the harmonic function. So, I can write that $F_3^{exc}(t) = F_0 \cos(\omega t)$ this is how we write it right.

Now, we are we do not know that what is the magnitude F_0 . Now, what you can do that for in case of a heave, you can take this extra buoyancy force as a magnitude of the exciting force. Now, again if you know this magnitude then if you take a moment. So, for the pitch now it is for the F_3 then for $F_5(t)$ you just need to take this M_0 which is nothing but if this is your the cg then you have to take the moment about the cg to get the M_0 right.

And, then again it is $\cos(\omega t)$ and now you can see like how you incorporate the phase, now if you for the low frequency limit is really work because at that time there is no phase between the wave and the you know this wave and the response. So, therefore, you can take this phase is 0, and then very simplistic way you can calculate the exciting force in this expression and then you can write a strip theory code.

So, the idea is you get this added mass a and b from the chart and then you calculate this as an exciting force you know and then you write the strip theory code. So, this will get from the chart this is available for you and then you are ready to write the strip theory code ok.

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Combining these forces with the force and moments due to the ship weight results in the restoring forces:

$$F_1^B = F_2^B = F_6^B = 0$$

Heave restoring force $F_3^B = -C_{33}\xi_3 - C_{35}\xi_5$

Roll restoring moment $F_4^B = -C_{44}\xi_4$

Pitch restoring moment $F_5^B = -C_{55}\xi_5 - C_{53}\xi_3$

Or in a more compact format the restoring force in the K direction is:


$$[F_k^B] = [C_{ij}]\xi_j, \quad k,j = 3,4,5$$

where C_{ij} are the restoring coefficients:

$$C_{35} = C_{53} = -\rho g \iint_{A_w} x ds \quad C_{33} = \rho g A_w$$

$$C_{55} = \rho g V_0 \overline{GM}_L \quad C_{44} = \rho g V_0 \overline{GM}_T$$

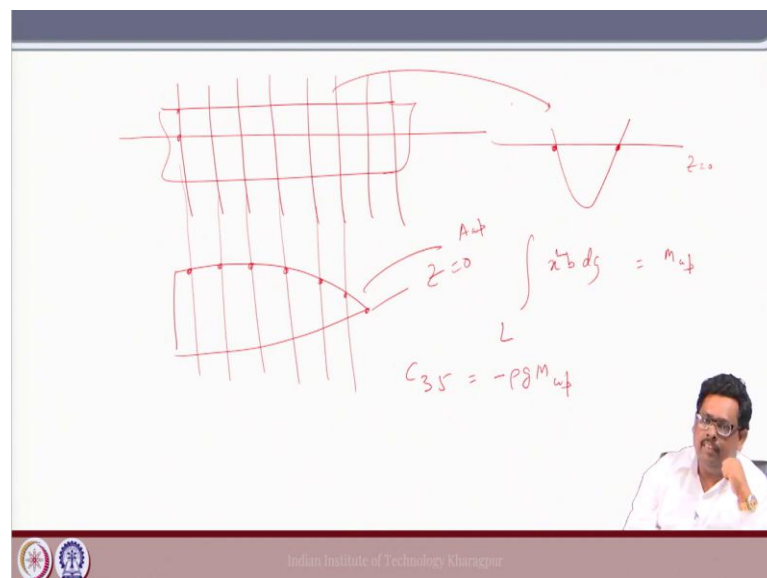
and $\overline{GM}_T, \overline{GM}_L$ represent the transversal and longitudinal metacentric heights



Now, let us discuss that what actually we are trying to figure out like before that the ok before we go to the radiation force let us discuss about the restoring force also. Now, in case of a strip theory you know the it is same is a geometric property and this is the this is how we can get the restoring force. So, therefore, there is no big deal we know how to get it.

So, we really do not discuss on this because we have already discuss this for you know panel method for omit type solution. So, only thing we would like to discuss as follows. Now, in case of a strip theory what the data you are having is basically the sectional data.

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So, that is why how to get this coefficient just I just give some kind of a very quickly how to get this coefficient, let us take heave or something else. Now, here what you are getting the data set you are receiving is nothing but the in each section you are receiving the sectional section sectional point like sectional curve.

So, maybe if you take this then you will have a sectional curve like this, and let us say this is your $Z = 0$ line. So, this is this curve we are getting, then from this data how you can get the you know the this coefficient that like C_{33} , C_{35} or C_{55} . Now, it is very easy. Now, if you what you need to do is at each section you need to collect these two point, at $Z = 0$, what is your values.

Now, if you plot this value then you can get this shape that from the water plane area. So, it is at $Z = 0$. So, now, in $Z = 0$ we have this curve because this is nothing but you take this point in all sections, the y values at $Z = 0$. So, if you make it then definitely if you project that into this plane you are getting these values. So, you have this value now if you integrate it then if you integrate it you get the water plane area right, if you integrate you know $x b d \zeta_i$ along L . So, it is nothing but your moment about the water plane area.

So, therefore, your $C_{35} = -\rho g M_{wp}$ and if you take the second moment, so, you will get the C_{55} also right. So, those things are very known to us and, so I mean you know we really do not discuss on this is very trivial and, at any point of time that you can use either trapezoidal rule or Simpson rule to get this coefficients right.

So, today we are going to stop here, and in the next class we are going to discuss on how we can get the two dimensional radiation potential or may be the two dimensional added mass damping, what are the methods are available. So, those discussions, we are going to discuss in the next class ok till now.

Thank you.