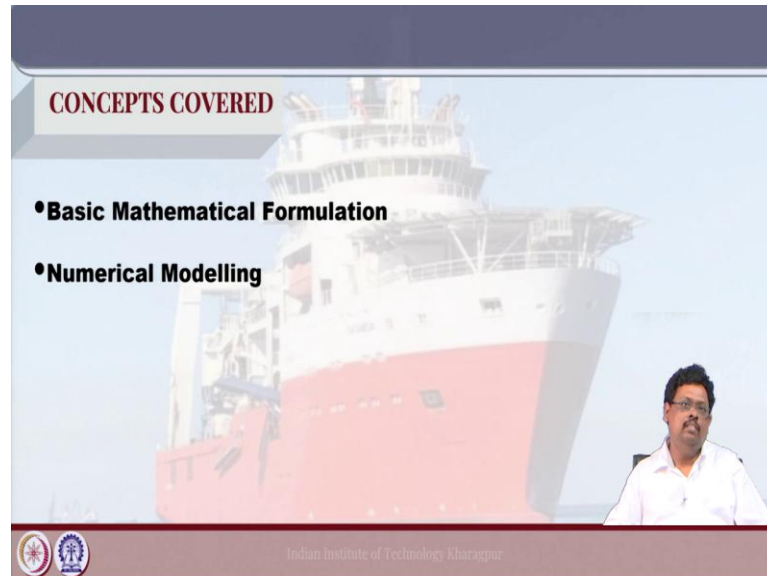


**Numerical Ship and Offshore Hydrodynamics**  
**Prof. Ranadev Datta**  
**Department of Ocean Engineering and Naval Architecture**  
**Indian Institute of Technology, Kharagpur**

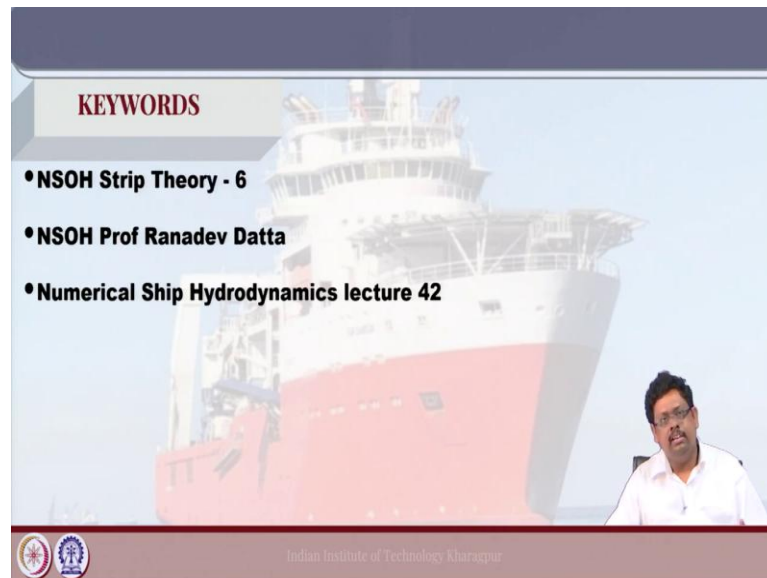
**Lecture - 42**  
**Strip Theory Part - 6**

(Refer Slide Time: 00:18)



Hello, welcome to Numerical Ship and Offshore Hydrodynamics today is the Lecture – 42, today we are going to discuss I mean continuation of our mathematical formulation as well as the numerical modeling. Today we are going to mostly focus on how to calculate the you know two dimensional added mass using several techniques one may be the Lewis curve technique another one is the Frank Close fit method ok.

(Refer Slide Time: 00:44)

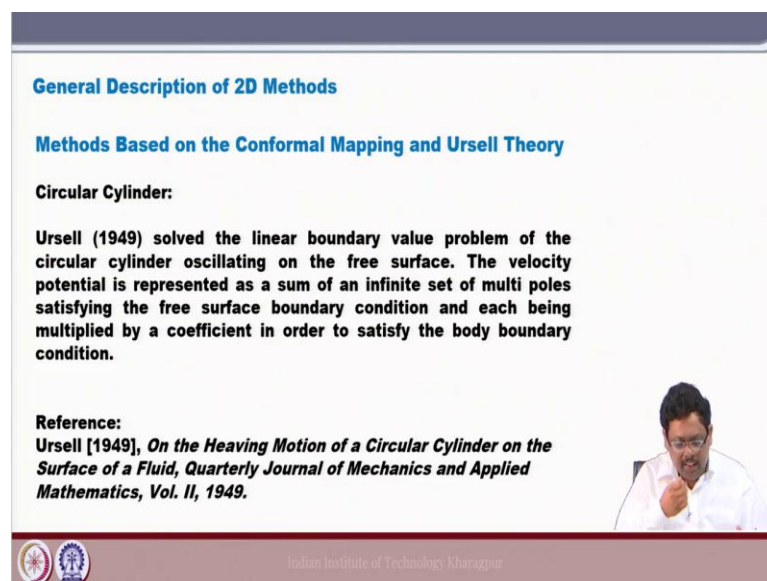


**KEYWORDS**

- NSOH Strip Theory - 6
- NSOH Prof Ranadev Datta
- Numerical Ship Hydrodynamics lecture 42

Indian Institute of Technology Kharagpur

(Refer Slide Time: 00:48)



**General Description of 2D Methods**

**Methods Based on the Conformal Mapping and Ursell Theory**

**Circular Cylinder:**

Ursell (1949) solved the linear boundary value problem of the circular cylinder oscillating on the free surface. The velocity potential is represented as a sum of an infinite set of multi poles satisfying the free surface boundary condition and each being multiplied by a coefficient in order to satisfy the body boundary condition.

**Reference:**  
Ursell [1949], *On the Heaving Motion of a Circular Cylinder on the Surface of a Fluid*, *Quarterly Journal of Mechanics and Applied Mathematics*, Vol. II, 1949.

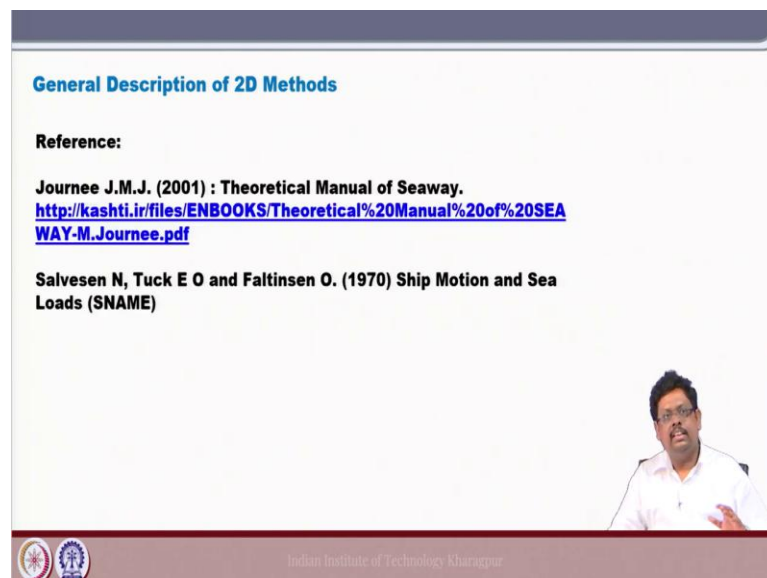
Indian Institute of Technology Kharagpur

And these are the keywords that you have to use to get this lecture well. So, now, we have you know first we discussed a very simplified method it is that constitutes are of course, not simplified, but at the end result is always very simple. So, I mean the first part of this lecture we are going to discuss that simplified method which is called the Lewis curve fitting. And then in the next phase of the lecture we are going to discuss something about the close fit solution which is equivalent to two dimensional boundary value problem.

Now, Professor Ursell in 1949 he has did one remarkable research, what he did is suppose if you take a two dimensional circular body ok he tried to find out he find out that how to analytically calculate the you know added mass and damping. So, this is the paper its references given over here you can see that on the heaving motion of a circular cylinder on a surface of a fluid.

So, it is a very pioneer paper and this leads to one of the you know simplest or greatest method to calculate the two dimensional added mass and damping for a arbitrary ship shape body ok.

(Refer Slide Time: 02:20)



**General Description of 2D Methods**

**Reference:**

**Journee J.M.J. (2001) : Theoretical Manual of Seaway.**  
<http://kashti.ir/files/ENBOOKS/Theoretical%20Manual%20of%20SEAWAY-M.Journee.pdf>

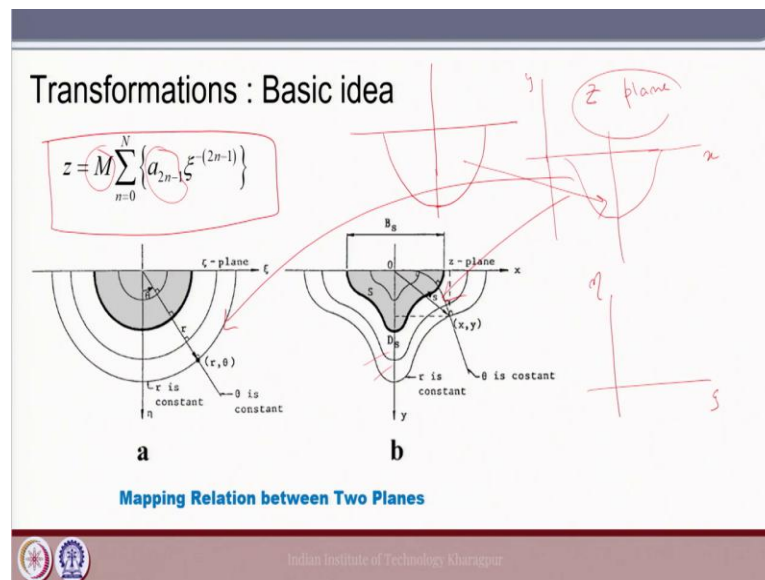
**Salvesen N, Tuck E O and Faltinsen O. (1970) Ship Motion and Sea Loads (SNAME)**

Indian Institute of Technology Kharagpur

So, now to discuss this we are going I mean I would like to refer these two paper it is not a paper the first one is a manual it is by Professor Journee and second was the classical STF paper it is you know most of the version of the strip theory mostly based on the theory given by this Professor Faltinsen, Tuck and Salvesen.

So, our lecture will limited to based on these two paper. So, if you want to know better because you know here in this course we really do not go much depth so, but if you want to interested I mean if you are interested to learn the you know in more detail. So, definitely you can see these papers ok anyway.

(Refer Slide Time: 03:16)



So, let us find out that what is the idea? Now, idea is that suppose I have a ship shape body which is here. So, let us say I have a ship shape body and now it is in basically my  $z$  plane. So, this is in  $z$  plane and I try to find out the added mass and damping of a ship shape body. So, so here I know that this there are some cross sections are like this and that you find out what is the added mass of this.

Now, I know about this Professor Ursell's theory that how to get the added mass for a circular cylindrical section ok. So, then what we could do is like what normally people are doing they try to find out a mapping that transform this ship shape body in  $z$  plane in they map back into the circular section which is in the let us say the it is in the  $\zeta, \eta$  plane.

So, it is  $x, y$  plane and we can call the  $z$  plane and it is we can call is the  $\zeta, \eta$  plane okay? So, idea is whatever there in this  $x, y$  plane; that means, the real plane they transform back into some  $\zeta, \eta$  some fictitious plane in  $\zeta, \eta$  and then it represent a circle and then I know how to calculate the added mass in circular section. So, eventually I get to know how I calculate for the real.

So, this is the idea of the you know conformal mapping and it is actually remarkable idea as I said and it is the it is based on this that paper by Ursell and later on people have picked up using the conformal mapping they use this theory to find out the added mass

of a any arbitrary ship shape body. Now, this is the in general this is the transformation of this you know conformal mapping.

Now, M is called the some scaling and then N is the is this is the something that we need to find out what is the it is the unknown to us and of course, I know that then this is the transformation law. Now, this is the general equation from which we can transform the you know the ship shape body into the circular section. So, we are preserving the angle theta here. So, all these theories are available in Journees that report.

(Refer Slide Time: 06:25)

**Lewis transformation**

$$x = M \left\{ (1+a_1) \sin \theta - a_3 \sin 3\theta \right\}$$

$$y = M \left\{ (1-a_1) \cos \theta - a_3 \cos 3\theta \right\}$$

$B_s = \text{Sectional Breadth}$  ✓  
 $D_s = \text{Sectional Draft}$  ✓

$M = \frac{\frac{1}{2} B_s}{1+a_1+a_3}$ ,  $M = \frac{D_s}{1-a_1+a_3}$

$N = 2$  ✓  
 $\frac{B}{T}$  ✓  
 $a_1, a_3$  ✓  
 $D_s$  ✓  
 $A_s$  ✓

Typical Lewis Forms

Indian Institute of Technology Kharagpur

So, therefore, we are not discussing this, now what we are going to discuss is as follows that how I mean we can use this Lewis transformation into it. Now if you now if you in the previous you know you have this N it is the number of I mean that how many terms you are going to take, now if you are going to take the 2 term and then actually this the general formulation is coming back to here.

So, in Lewis transformation I am taking this the value of  $N = 2$  and then we can find out that x is given by this expression and y is given by that expression, now this M is nothing but the scaling. Now, of course, you know that the scaling is important right, because you have you know x, y physical domain and then you are transforming into the  $\zeta, \eta$  domain.

So, therefore, there must be a scaling and then how to get this value for M? I mean you

know see this formula is given over here. So,  $M = \frac{\frac{1}{2}B}{1+a_1+a_3}$  and or  $M = \frac{D_s}{1-a_1+a_3}$ . Now,

how now here this  $B_s$  is known as the sectional breadth and  $D_s$  is known as the sectional draft okay? And then what about the  $a_1$  and  $a_3$ ?

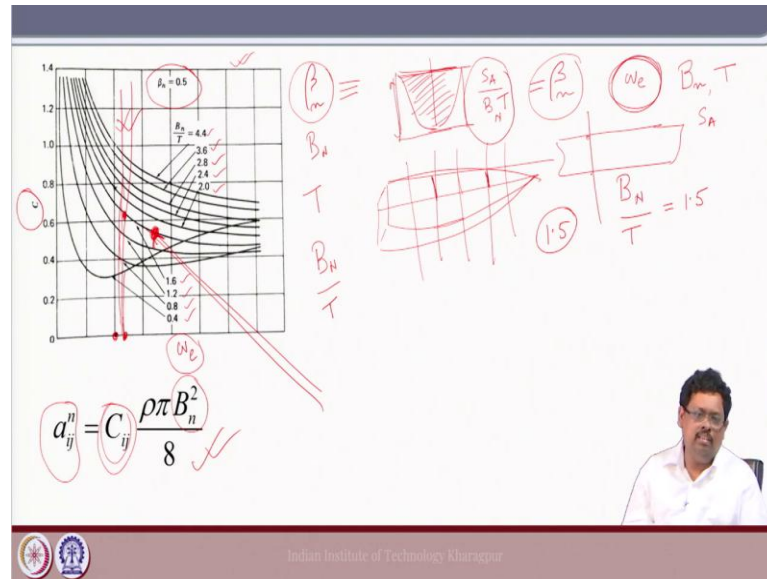
Now, remember this they want to preserve something like you have to choose this  $a_1$  and  $a_3$  in such a way so, that suppose this is the you know this is the actual cross section. So, this is the actual cross sections and then it must have some sectional area it must have some sectional draft so; that means, we have the sectional draft  $D_s$  maybe we can have the sectional area  $A_s$ .

Now, you have to choose  $a_1, a_3$  such a way this quantity will preserve ok. So, now, we have a constant like I in I have to design this sectional thing that using this transformation law I am getting to get this x and y and all these things the scaling that it should maintain this property; that means, that in original what is the sectional draft  $D_s$  and what is the sectional area  $A_s$  that must be preserved under this transformation.

So, based on this fact we have to calculate what is the  $a_1, a_2$  and correspondingly what is M? Now, these are the theory we really do not want to go into this theory because in the next slide we are going to discuss that what actually we are going to use. Now, here this in below I can some typical Lewis curve actually we are plotting like with respect to several you know  $B/T$  ratio and wearing some parameter just to show you that different kind of section is possible using the Lewis transformation ok.

So, this is the underlying theory that I am going I am just discussing right now, really we do not need to go deep into this theory, but what is the underlying theory that definitely we need to remember. Now, what actually a really going to use is something very interesting.

(Refer Slide Time: 10:08)



Now, you see people have use this formulation use this transformation and they are coming up with a set of chart right. Now, this is the chart it is very interesting chart that in this x axis we have this encountered frequency  $\omega_e$  and in the y axis we have some coefficient C, we are really going to discuss that what is this coefficient C.

Now, thing is that once you know the value for c from this chart I can calculate the added mass using this formulation. Now, what is the  $B_n$  is nothing but the sectional beam or sectional yeah sectional beam or breadth and then  $C_{ij}$  is the coefficient that I am getting from this chart.

So, that is what if you remember the beginning that is what we are going to discuss we discussed that mostly the most crucial part which is the finding out the added mass damping etcetera. We actually use some kind of chart right and this chart actually based definitely based on some very serious theory I mean we are just escape the theory and just we are into the application part ok.

Now, how to select the point C that is actually one it is much more interesting or meaningful at this moment because this theory is already developed some 56 years back. So, therefore, and already these charts are very much exist. So, it is better to know little bit about the theory and more about that how it applied to get the added mass for a cross section ok.

Now, you can see in this chart there are some parameter one parameter is called the  $\beta_n$ , one is the  $B_n$ , one is the  $T$  and then it is the encountered frequency of course,  $\omega_e$  and based on that I have to find out what is the value for  $C$ . Now, what is the  $\beta_n$ ?  $\beta_n$  is the sectional area coefficient. So, it means that suppose in some section suppose this is the section and then let us say this is the sectional area.

So, I just this  $\frac{S_A}{B_n T}$  or  $B_n$  is the sectional beam so, this one by  $T$  the draft. So, this ratio actually we called that is equal to  $\beta_n$ . So, now, suppose I try to find out let us say we have some encounter  $\omega_e$  right suppose we try to figure out what is the added mass for some encountered frequency  $\omega_e$ .

So, now let us say here is this or maybe here is this value, now how to get this coefficient? So, what I do is so, I know that this is you know I just draw it point over here and then I need to check that you know how I can get the value for  $C$ . So, first check is I have to I have to calculate the  $\beta_n$ , now you see here I just have only one picture where the  $\beta_n$  value is 0.5 ok.

However, you know  $\beta_n$  is very much different because you see if I let us take that ok like let us take this some kind of shape from the top something like this. Now, if you cut in each section like this way  $y - z$  plane not you know this all the time you can see that this  $B$  is different, now if  $B$  is different so, definitely the area also different then value of  $\beta$  and also different right.

So, first thing that I take now if I take in a in this plane I take a section and then I find out this coefficient beta n because at this section I know what is my the sectional breadth  $B_n$  and also I know what is my the draft  $T$  at this particular section all I mean it is remain same and also I know what is the sectional area of this particular section. So, definitely I can calculate the  $\beta_n$ .

So, first thing I fixed my  $\omega_e$  and then I calculate what is my  $\beta_n$ , let us say the  $\beta_n$  is equal to 0.5; and then I need to calculate the ratio  $\frac{\beta_n}{T}$ . Now, here I would like to mention that now ultimately one should understand that you have to interpolate something. Now, at which level you are going to interpolate?



Now, see here now  $\frac{\beta_n}{T}$  might not fall into all these values it may not be 4.4, it may not be 3.6, it may not be 2.8, it may not be 2.4, it may not be 2 or 1.6 1.2 0.8 0.4 suppose these are the available to you ok and then let us take that some value here coming around it is let us say 1.5.

So, you will get your  $\frac{\beta_n}{T} = 1.5$ , now you have no data for 1.5. So, then how you then can you interpolate? So, answer is you know, no, absolutely not. So, then it is your judicial decision that if  $\frac{\beta_n}{T} = 1.5$  then which graph you need to take.

So, let us take in case of one point 1.5 I am going to take the curve for 1.6. So, I select the graph. So, this is the graph actually I have selected this one ok and now after doing this. So, I first select this chart it is 0.5, then I select the graph that I am going to discuss fine and after that only I can interpolate the value.

So; that means, let us say now this is the graph I am going to take. So, this is the graph. So, now let us this is the graph and now I have to calculate this omega e I try to find out this point and this point now I can interpolate between the consecutive two data I can interpolate what is the value.

So, once I so you see that before you start doing the interpolation you have to take you know lot of decision by your own like, which graph we need to take like, what is the beta now  $\beta/N$  it might not be 0.5 it could be 0.4 0.6 then what to do. So, suppose may be that all the ratio is not available to you. So, you have the chart for  $\beta_N = 0.5$ , then 0.6, then 0.7, 0.8, 0.9, 1.4.

Now, suppose this ratio comes 0.55 what you are going to do? So, at that time can you interpolate the chart, I cannot, right? So, at that point again you need to find out like what you which chart it best to take, if it is 0.55 I should go with 0.5 or I should go with 0.6 that is up to you ok.

Now, once you understand these things then getting this value for C is really easy right. So, this is something we called the Lewis curve and name of this curve is Lewis curve and it is first proposed by Lewis and now it is very popular nowadays. In fact, you know if time permits at the end maybe or maybe the you I can take one more class to find out

like numerically how do I write very simple way a strip theory code, we are not going very much you know into theory and very sophistic strip theory code let us at in this class let us not write that.

But; however, I can give you or we can discuss very simple way with very simple understanding how could I write a strip theory code may be it you know roughly work or, but at least can give a some realistic data ok we will see that.

(Refer Slide Time: 19:26)

**Boundary-integral method based or Frank close-fit method**

**This is essentially a Green-function based source distribution method for the solution of the 2D boundary value hydrodynamic problem.**

*$G = k_0 r$*

**The boundary value problem**

**Laplace equation:**  $\nabla^2 \psi_j = 0$

**Linear free-surface condition:**  $-\omega_e^2 \psi_j + g \frac{\partial \psi_j}{\partial z} = 0$  on  $z = 0$

**Bottom condition:**  $\nabla \psi_j = 0$  as  $z \rightarrow -\infty$

**Body condition:**  $\frac{\partial \psi_j}{\partial n} = i\omega_e N_j$  on  $C_x$

**And a radiation condition enforcing waves to be outgoing.**

*$\nabla^2 \psi_j = 0$*

*$\frac{\partial \psi_j}{\partial n} = i\omega_e N_j$*

So, now so, this is all about the Lewis curve and then next we are going to discuss something called the Frank Close fit method. And this is also one pioneer work by Frank laws I mean they have come up with this solution to calculate the two dimensional added mass and damping. Now, here you can see that from this equation we are going to solve the two dimensional boundary value problem right. So, I mean since we have already discussed three dimensional boundary value problem.

So, I hope that this will be you know it is would not be much trouble for you. So, now, here the  $\psi$  is the velocity radiated potential definitely right and therefore the  $\psi$  must satisfy the Laplace equation  $\nabla^2 \psi_j = 0$ . So, this is the step 1, second step is we need to write the free surface boundary condition, body boundary condition etcetera etcetera and what and then condition infinity etcetera etcetera.

So, now you know that this is the expression for your free surface boundary condition right and this is the radiation condition or sorry it is the bottom condition, we do not have the radiation condition here and then this is more important that we have the body boundary condition. So, mainly when you try to find out the value for  $\psi$ .

So, mostly we are using this equation  $\nabla^2\psi_j=0$  with respect to the body boundary condition  $\frac{\partial\psi_j}{\partial n}=i\omega_e N_j$ . Now, we have discussed that how I can get this  $i\omega$  in the right hand side. So, I am not going to discuss anymore here right and then why we do not discuss much about this it is depends on the what is the Green's function we are taking.

Now, if it is a two dimensional problem then if I use the Rankine panel method so, the Green's function definitely I am going to take something called  $\ln r$ . Now, if I take  $\ln r$  as a Green's function then definitely we are going we need to discuss about this free surface boundary condition. However, if you remember if this Green function  $G = \ln r + H$  with satisfy the free surface boundary condition.

So, that free surface free surface boundary vanishes then we do not need to discuss about this right. So, you have to be very careful when you are doing it because suppose I have a let us take a two dimensional domain here and then I have this one the wave and then I have a body over here.

Now, if I use this Green's function I only need to discretize this weighted surface by some straight line and here in here only this body boundary condition satisfied we can call this as a kinematic body boundary condition. However, if we take only  $\ln r$  so, therefore, you need to discretize the free surface also and in the free surface you need to use this free surface boundary condition right.

So, that is why I said that based on that what actually the Green's function you are taking based on that we need to find out which are the boundary conditions are essential for me to solve this problem ok.

(Refer Slide Time: 23:32)

The 2D sectional added mass/damping (and the exciting force/moment components) are given by the 2D hydrodynamic 'radiation' potential, i.e. the pressure-field created by forced oscillation of 2D sections

OUTGOING WAVES TO INFINITY

WAVES

$\frac{\partial \psi_k}{\partial N} = i\omega_s N_k$

$-\omega_s^2 \psi_k + g \frac{\partial \psi_k}{\partial z} = 0 \mid z = 0$

$\nabla^2 \psi_k = 0$

$\lim_{z \rightarrow -\infty} \nabla \psi_k = 0$

$k = 1$  SURGE  
 $k = 2$  SWAY  
 $k = 2$  HEAVE  
 $k = 2$  ROLL

Indian Institute of Technology Kharagpur

So, now, here I you know in a graphical way I said the same thing, now here you have in this body you must have this body boundary condition right and here in the free surface you have this free surface boundary condition right and then at the bottom definitely the bottom boundary condition right. So, this is the whole picture.

Now, based on your choice of your Green's function you really need to find out whether you need to discretize only this weighted surface we call this as weighted surface or we need to discretize disweighted surface with the free surface, it is depending on the choice of your Green's function ok.

(Refer Slide Time: 24:31)


**Applying the Green theorem to the fluid volume (see Newman, 1977):**

The velocity potential ' $\Phi(P,t)$ ' at a point ' $P(y,z)$ ' in the fluid can be represented in terms of a dipole distribution of moment ' $\Phi(Q,t)$ ' and a source distribution of strength  $-\frac{\partial\Phi}{\partial n}$  distributed over the boundary surfaces:

$$\Phi(P,t) = -\frac{1}{2\pi} \iint_{S_0 + S_F + S_B} \left[ G(\overline{QP}) \frac{\partial\Phi(Q,t)}{\partial n} - \Phi(Q,t) \frac{\partial G(\overline{QP})}{\partial n} \right] ds$$

*Handwritten notes:*  
 $S_0 + S_F + S_R + S_B$   
 $G = \frac{1}{r} + H$

- $Q(y,z)$  is a point on the surface of the body
- $G(QP)$  is the Green function representing a source potential which satisfy the Laplace equation, the free surface boundary condition, and the radiation condition:



Indian Institute of Technology Kharagpur

And now one when you choose the Green's function so, then this is only a integral equation that you going to solve now, this integral equation is very well known to you is it not, because this actually you have used from the day one right. Now, here you can see that in general we have you know the three surface. So, we called one is the  $S_0$  which is the weighted surface plus we have the you know the free surface and also we have the radiated surface.

And you know we should consider the bottom also, but this bottom is 0. So, normal derivative 0 so, therefore, we are not taking it right. So, it is also so, this is also fair enough that you know that how to discretize the surface now if you take  $G=1/r$  then you have this surfaces. However, if you take  $G=1/r+H$  inside the surface, so, sum of the surface you can actually delete right? Okay?

(Refer Slide Time: 25:41)

The Green function here is:

$$G(z, \zeta) = \Re \left[ \frac{1}{2\pi} \left[ \log(z - \zeta) - \log(z - \hat{\zeta}) \right] + 2PV \int_0^{\infty} \frac{e^{-k(z - \zeta)}}{v - \gamma k} dk \right] - i \Im \left[ e^{-i\nu(z - \zeta)} \right]$$

$$= \Re[G_1] + i \Im[G_2]$$

$z = x + iy \quad \zeta = \xi + i\eta \quad \hat{\zeta} = \xi - i\eta$

$$\Phi^{(m)}(x, y, t) = \Re e \left\{ \int_{C_0} Q(s) \cdot G(z, \zeta) \cdot e^{-i\omega t} \cdot ds \right\}$$

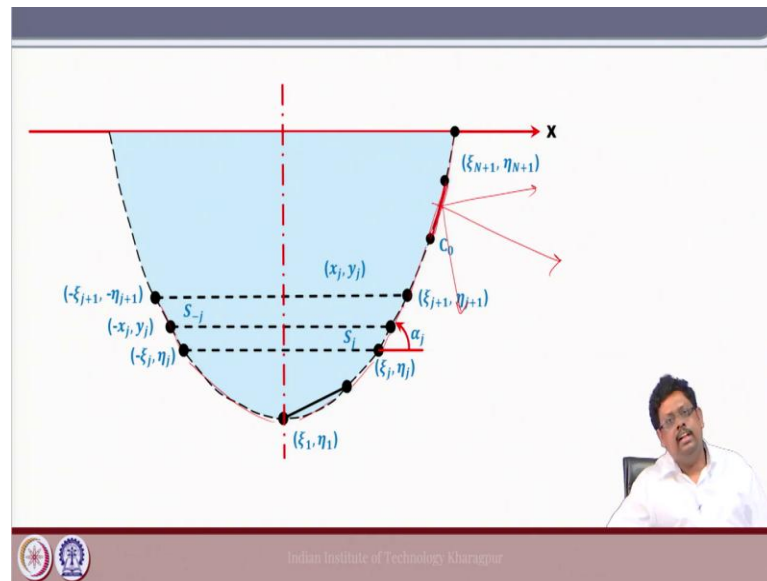
Indian Institute of Technology Kharagpur

Now, third is the now this is the Green's function I called the free surface Green's function. Now, here you can see we have this log term right so; that means, that this is the Rankine part this is the called the Rankine part of the Green's function and this is called the regular part right.

So; that means, if you use this Green's function then actually only the body surface you know it lead turns out on the body surface you do not need to discretize anything else. And this expression also you know very well and this is nothing but your, it is source panel method right this is a Green's function and this is the you know strength of your source right.

So, now the it is the exactly the way you solve the panel method right sorry if you solve the frequency domain panel method in three dimensional floating body case, this is the exactly the similar thing, but now you are doing for the 2 D ok.

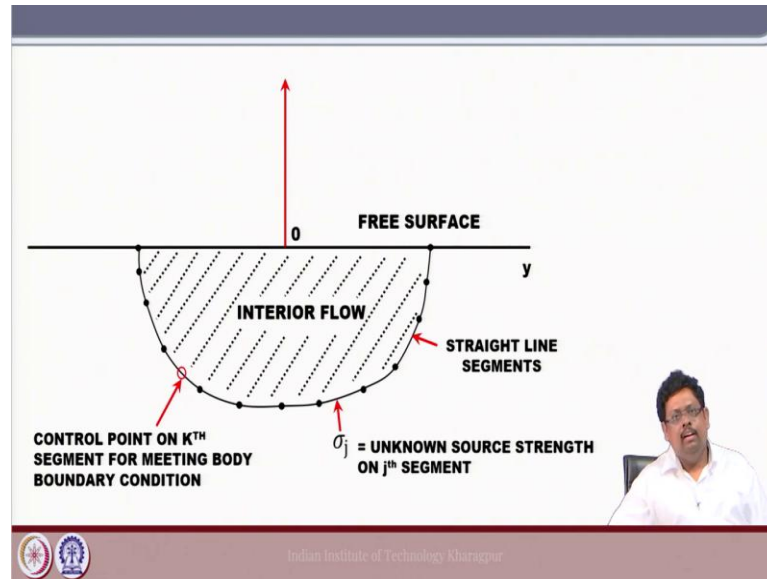
(Refer Slide Time: 26:54)



And now as you as you know that in case of a 3 D you have to have a panel so, in case of a 2 D you have a straight line. So, these are the point and then you joining these point as a you know as a straight line right. So, this is the thing. So, this is you see it is exactly the same like three dimensional panel it is little bit easier maybe because instead of a panel you are try to figure out that straight line and if you know the straight line because, if since it is a straight line it is very easy to find out the normal that and also the component of the normal.

So, in that way geometrically dealing with this two dimensional strip may be easier because once we know the straight line we know the equation of the straight line. So, definitely know what is the equation of the normal which is passing through some particular point it is elementary mathematics, right so, in that way maybe, it is little bit of easier compared to the three dimensional.

(Refer Slide Time: 27:57)



And now again it is the same it is a source panel method. So, initially this is the this is the strength is unknown and once you find out the strength and then you can from the this from this equation you can get the expression for the phi also right. So, again exactly the same right so, initially this we can call the strength and normally we define this as sigma. So, once we know this sigma and then we apply this sigma over here and then we know the value for the  $\phi$  right ok.

(Refer Slide Time: 28:41)

**Computation of the Green function is non-trivial !**

- > Involves angle computations : not as straightforward as is thought!
- > Involves evaluation of complicated functions

$$PV \int_0^{\infty} \frac{e^{k(y+\eta)} \cos k(x-\xi)}{v-k} dk = e^{v(y+\eta)} [c(r,\theta) \cos v(x-\xi) + s(r,\theta) \sin v(x-\xi)]$$

$$PV \int_0^{\infty} \frac{e^{k(y+\eta)} \sin k(x-\xi)}{v-k} dk = e^{v(y+\eta)} [c(r,\theta) \sin v(x-\xi) - s(r,\theta) \cos v(x-\xi)]$$

$$c(r,\theta) = \gamma + \log r + \sum_{n=1}^{\infty} \frac{r^n \cos(n\theta)}{n n!}$$

$$s(r,\theta) = \theta + \sum_{n=1}^{\infty} \frac{r^n \sin(n\theta)}{n n!}$$

$$r = |-iv(z - \bar{\zeta})|$$

$$\theta = \tan^{-1} \frac{\Im[-iv(z - \bar{\zeta})]}{\Re[-iv(z - \bar{\zeta})]}$$

The Indian Institute of Technology Kharagpur logo is visible in the bottom left corner.



And now you know as I said that it is a it is not a straight forward because you need to calculate all this term and very efficient you need to calculate all this term right. So, therefore, definitely it is a non-trivial task ok.

So, today actually where let us stop at this point and in the coming class let us try to discuss that where, is the complex part involved in this equation and how actually we could address this problem and we can solve this problem ok. So, today we are stop here fine.

Thank you.