## Numerical Ship and Offshore Hydrodynamics Prof. Ranadev Datta Department of Ocean Engineering and Naval Architecture Indian Institute of Technology, Kharagpur

Lecture - 43 Strip Theory Part - 7

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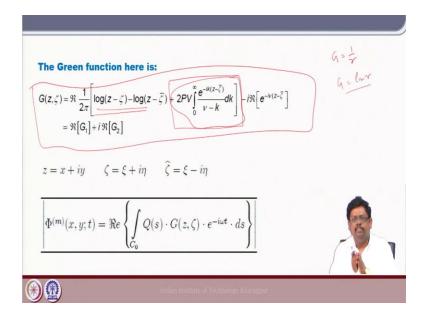


Hello, welcome to Numerical Ship and Offshore Hydrodynamics, today is the lecture 43. Today we are going to discuss about the Frank close fit method in last class we stopped somewhere we are start we are going to start from that point ok.

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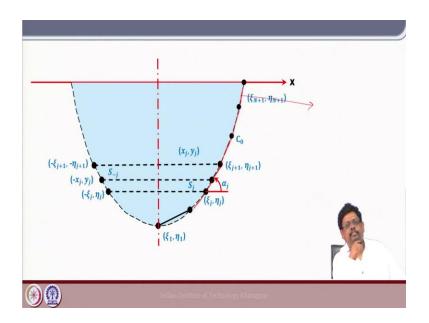


And this is the keywords that we are going to use to get this lecture ok. Now, if you remember in the previous class we said that we are going to use this Green's function to calculate the added mass and the damping right. Now, here if you see that the complex part involves in this expression which is the regular part and of course, I do not say that it is not complex, but somehow if you take G = 1/r is a strong singularity and if you take you know  $G = \ln r$ , it is little bit it is not that strong the singularity it is a weak singularity.

Now, but; however, still it is difficult and we have to deal with this also and how to deal with  $\ln r$  etcetera etcetera at least for 1/r we have discussed, but major difficulty involves in this calculation of this particular term ok and we called this as a regular term.

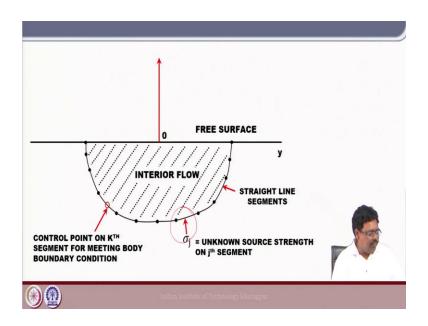
However, this is important because otherwise if you drop this term then we have to discretize the free surface and things become little bit more complex in terms of machine the computational effort etcetera it will it is better to use this Green's function so, that the domain of discretization becomes only the body right.

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And this is the setup that we have we are going to use that and this is the this is my section and this section actually I add with number of straight line and we said that if we use the number of straight line then if it is a straight line it is easier to find out what is the normal component it is analytical and it is trivial also.

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So, now here this is the unknown  $\sigma$  that also we discuss we first find out  $\sigma$  and then you have to put this  $\sigma$  into the source integral equation to get the value for the  $\phi$ .

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mputation of the Green function is non-trivial !	-pag
Involves angle computations : not as straightforward	as is thought!
Involves evaluation of complicated functions	X
$P(\int_{0}^{\infty} \frac{e^{k(y+\eta)}\cos k(x-\xi)}{\nu-k} dk = e^{\nu(y+\eta)} \left[ c(r,\theta)\cos \nu(x-\xi) + s(r,\theta)\sin \nu(x-\xi) \right]$	m=1 mm1 m
$PV\int_{0}^{\infty} \frac{e^{k(y+\eta)}\sin k(x-\frac{y}{\zeta})}{v-k}dk = e^{v(y+\eta)} [c(r,\theta)\sin v(x-\frac{y}{\zeta}) - s(r,\theta)\cos v(x-\frac{y}{\zeta})]$	$f(m) = \overline{Z}$ mm!
$\begin{aligned} \mathbf{c}(r,\theta) &= \gamma + \log r + \sum_{n=1}^{\infty} \frac{r^n \cos(n\theta)}{n n!} \\ \mathbf{s}(r,\theta) &= \theta + \sum_{n=1}^{\infty} \frac{r^n \sin(n\theta)}{n!} \end{aligned} \qquad $	
$\mathbf{s}(r,\theta) = \theta + \sum_{n=1}^{\infty} \frac{r^n \sin(n\theta)}{nn!} \qquad \theta = \tan^{-1} \frac{\Im[-i\nu(\mathbf{z}-\boldsymbol{\zeta})]}{\Re[-i\nu(\mathbf{z}-\boldsymbol{\zeta})]}$	

Now, here you can see that this is the this is principal value that we are going to actually solve because it is in complex domain it has some cos component sine component and this is how actually you are going to get this value for this particular thing. Now, here if you see this is why it is complex, because it is has a improper integral 0 to  $\infty$  and in order to get this you have two series which is  $C(\mathbf{r}, \theta)$  and  $S(\mathbf{r}, \theta)$ .

Now, you see that  $C(\mathbf{r}, \theta)$  and  $S(\mathbf{r}, \theta)$  is given by this expression and of course, this  $\theta$  the argument is given by this expression. However, this theta you know theoretically it seems that this finding out this theta is very very easy I have you know this panel I have this the straight line and then if I do this so, this is my theta so it is easy.

However, if you do this coding in reality then you can you have to face so much of trouble because of this angle we are coming here, before that it is very interesting in even this suppose I find out somehow the theta somehow I find out the theta then how I compute this term ok. So, now computing this term also it is very interesting to see. Why, because if you look at this expression it is summation this n is equal to 1 to infinity.

So, it is  $\sum_{n=1}^{\infty} r^n \frac{\cos(n\theta)}{n.n!}$ . Now, we do not have we do not have n is say on  $\cos(n\theta)$  because this  $\cos(n\theta)$  is always oscillate between 0 and 1. So, it means that I have to multiply a factor which is between 0 to 1 with the other term. So, if I look at it that let us say  $f(n) = \sum_{n=1}^{\infty} \frac{r^n}{n.n!}$  right? Now, So, thing is that we need to multiply some factor between 0 to 1 with this.

Now, think of this r I mean this expression now n is going to infinity now you see like if I try to plot this graph ok. So, first let me erase this.

 Computation of the Green function is non-trivial !
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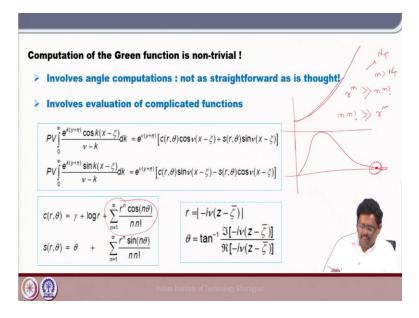
And now let us try to plot this graph which is like f n let me write again it is  $\frac{r^n}{n.n!}$ , now this one let us try to draw a try to draw the graph what will be the graph for this.

Now, you have to take some the value for r, now let us take this length is the in this normally the L is the length of the vessel now suppose you have here now ok sorry it is it is B in this case it is B and now this is let us say this is the body now at this point suppose you have one point over here and let us say you have this another point over here and this is the difference is r.

Now, you see for instance let us take some large value r = 10 let us say some realistic large value 10 now try to find out this value. Now when n = 1 then this value is 1 it is 1. So, now, I just plot f n versus n now when n = 1 this whole value is 1, when n = 2 then it is sorry 10 sorry the value is 10 sorry very sorry. Now, when n = 2 the value is 100 divided by 2 into factorial 2 so, it is 100 divided by 4 so it is 25.

Now, when n = 3 so, it is 10 cube which is 1000 and divided by *n.n*!, which is 3.3! = 6. So, essentially it is 1000 by 18. If you take 20 then it is 50. Now, you can see that more than 50 some let us say 55 let us say. So, the point is that if you look at it this series and this series has a infinite sum.

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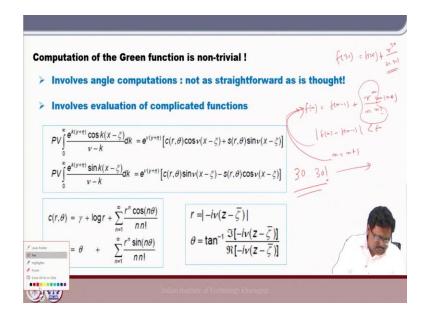
Now, it appears that this value starting from 10 and slowly progressing like this way right. Now, if I add this graph at you know infinitely because you see this sum is infinity.

So, definitely you know it is our what I said like assumptions it is blow up because it will going to blow up because it turns out to be divergent series interestingly it is not.

After some point you can see initially this  $r^n$  is increases much faster compared to n.n! up to some limit some N<sub>T</sub>. Now, when  $n > N_T$ , after that n.n! is growing much faster compared to  $r^n$ . So, therefore, the actual graph somehow it will blow up initially and then it is comes down and sometime asymptotically it goes to 0, that is the nature of this thing, but you need to find out that what would be the convergent limit.

Now, think of a you know computer computational thing and how we can using the integer we use integer 32 or whatever it has a limit. Now, if you keep doing this n.n! like.

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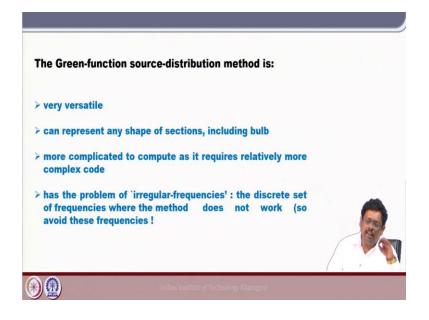
Suppose, if you write a very simple type of code like it is you know you can take that let us say  $f(n) = f(n-1) + \frac{r^n}{n \cdot n!} \sin(n\theta)$  and then you give a check if  $|f(n) - f(n-1)| < \varepsilon$ , you have to again you just n = n + 1 and again you just do this operation.

Now, this is how one should write simple way, but if you do that you have a you are going to get an error, why because you do not know the limit of n. Now in some situation let us say let us take n=30 let us try to. So, now, if you try to find out n, f(30) which is

nothing but 
$$f(30) = f(29) + \frac{r^{30}}{30.30!}$$
.

Now, I have very much doubt if 30 into factorial 30 will return your integer value or not right. So, once you start writing this code this kind of things you have to deal with. Now, here I am not going to tell you that what would be the best logical way to approach to find out this solution ok. So, you think and then you take this as your you know assignment problem some assignment and come up with some logic ok, if you fail then definitely we can discuss the same ok.

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Now, here what is the advantage, this is a very versatile approach the Frank close fit, it does not depend on the shape of the sections or even you can take the bulbous bow also right. And then that disadvantage is definitely it is more complicated if you compute the you know very accurately you try to find out is added mass all these things then radiation potential in fact, in a way.

It is much more complex because in other approach is really easy because the solution is developed we are having the chart we have used the chart to get this. And also as you know similarly like this three dimensional panel method code also there also we have the problem of a 'irregular frequency', in some frequency that solution becomes you know approach to infinity right. So, here also this is possibility.

So, that is also you have to understand that ok this result is not feasible, this is the unrealistic solution. So, you must have some kind of idea about the irregular frequency right.

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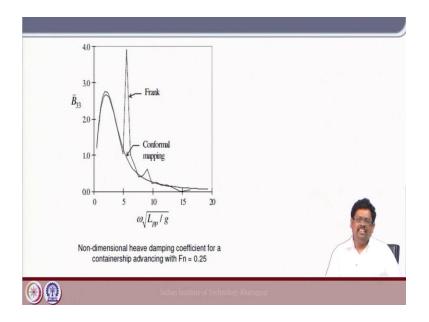
So, now I just listed upon some advantage and disadvantage of the conformal mapping methods and the Frank close fit method. So, it is a smooth solutions over all frequency range the conformal mapping. So, we do not have any irregular frequencies there. Now, sharp corners, but a sharp corners are not you know you cannot get very like if you take a rectangular barge exactly right and this is the section, it is very difficult to get using this conformal mapping ok.

And also sections with very low sectional area coefficient may not be well represented; that means, this  $\beta_n < 0.5$  so, maybe the results are not that good ok. So, and also you really cannot map you know map this bulbous bow sections those things are not possible.

And now on the other hand, we have the Frank close fit method so, it is does not depend on that what is yours the cross sections, you can apply everywhere even you can use this the sharp corner also, you can use this for the Frank close fit method.

However, we have the problem of the corners sorry the irregular frequency some frequencies we this solution may have blow up getting some numerical instability etcetera ok.

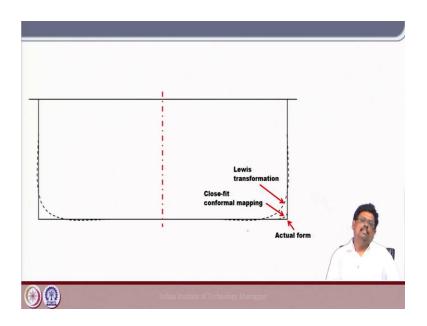
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Now, this is a classical the graph it is now here this is taken from a paper by Nano Fonseca I am in reference definitely we are going to say this. Now, here you can see that in the Frank close fit method we can have this irregular frequency; however, in a conformal mapping we do not have anything right ok. So, this is the problem now it is only one frequency blow up and remaining. So, it has more advantage because you can take any type of cross sections.

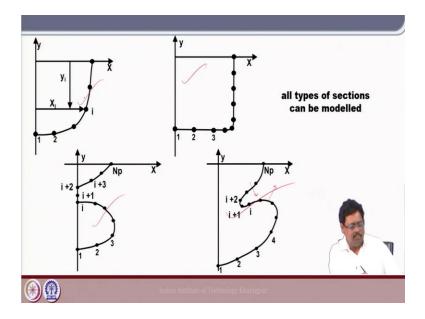
Now, the main advantage of the conformal mapping is it is easy to apply nowadays because theory is developed already and we know exactly what to do right. So, it is now depending on the choice of Lewis coding to use this Frank close fit or conformal mapping ok.

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Now, and again this is the disadvantage as I said that in case of a you know even if you try with that will very you know this Lewis curve fitting can go up to this and if you really use you know many terms like as you know that we can use only two term for the Frank close fit method if you take more than two terms and all still you do not get the sharp corner.

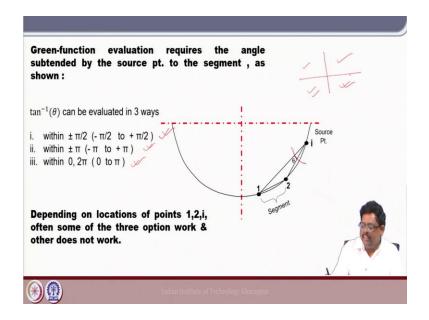
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Now, also; however, if you take any kind of shape in Frank close fit this, this, this, this, this, this, this, this all are possible ok. Now, you can see that here you can see that it is in this

shape is very critical because you see that abruptly the direction of normal is changing right. So, it might have the some coding complexity, but it is possible and this is not possible for Frank close, I mean this is not possible for your conformal mapping techniques or Lewis curve this is not possible, this is of course, possible for both ok.

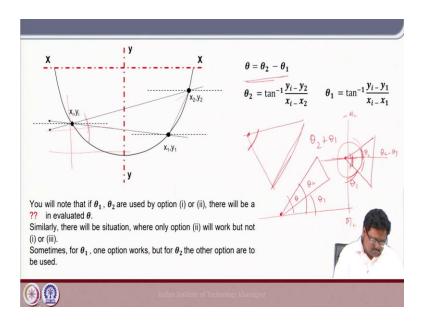
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And now this is the problem main problem we mentioned before also if you look at my previous slide somebody mentioned that this inverse angle of competition that is not very straight forward. Now, you see what is happening over here this angle you can see if this is the source point and this is your angle. Now, this angle might lying in either from  $-\pi/2$  to plus  $\pi/2$  or it is in  $-\pi$  to  $\pi$  or it is 0 to  $\pi$  all three are possibility.

Now, if you really do not understand and carefully address this the range where it is then because you are you know in a  $\tan \theta$  if you remember we have a  $\tan \theta$ . So, here all positive then here you know this sine sin positive cos negative right here and this is direction the cos positive and tan sin negative and here both sin cos both are negative.

So, eventually tan is positive now in which range actually whether it is from here whether the range is here or whether the range is here that is very much important to calculate exactly the value of this. I am you know it is only when you start coding using Frank close fit method you can only realize at that particular point.

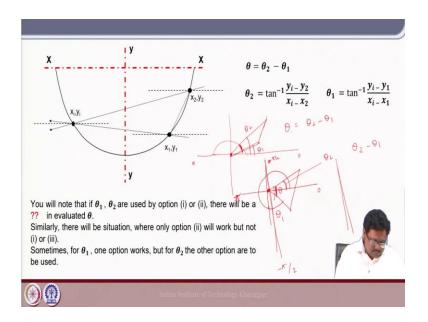


Now, here for instance I am just giving the two points here right so,  $x_1$  and  $x_2$ . So, now, here is straight forward this is the theta it is now you know right it is in which angle it is. So, it is  $\theta_2 - \theta_1$  right? Because here if it is  $\tan \theta_2 - \tan \theta_1$  a also this is the your  $\theta$ . So, this  $\theta$  is from this axis you know it is or this axis  $\theta_2 - \theta_1$ .

Now, suppose this is not this situation now here it is here now here this is the section, now here is the point. So, now what about this  $\theta$ ? Still  $\theta_2 - \theta_1$  or in this case it should be  $\theta_2 + \theta_1$  now it is depending on how you are measuring, are you getting my point? Like if this is your ok, here point and this is it so, this theta now I know this  $\theta$  is nothing but my  $\theta_2 - \theta_1$  fine.

Now, suppose your section is like this way now tell me here it should be if I do again  $\theta_2 - \theta_1$  you know I do not get it because suppose this is my axis. So, it is you know your  $\theta_2$  and if you take the same orientation so, this is your  $\theta_1$ . So, then if you take in a same reference then  $\theta_2 - \theta_1$  would not give you this angle.

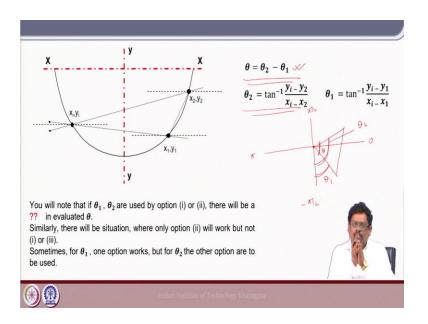
However, if you take this section which is you know  $-\pi/2$  to  $\pi/2$  whatever in this region if you take this way then you can take it is  $\theta_2 - \theta_1$  are you getting ok.



Let me explain again, now I take two situations here this is your source point and the cross section is let us say this is your cross section. So, I am lying here in this panel which is you know it is 0 to  $\pi$  in this region. So, then this angle is  $\theta_2$ , this angle is  $\theta_1$ . So, your  $\theta$  is this  $\theta$  this  $\theta$  is nothing but  $\theta_2 - \theta_1$ , this is the picture number 1 absolutely fine.

Now, what I am doing in the second situation. So, let us take the same point over here now this section is here like this and then this is your  $\theta$  right. Now, if I try to go over here so, this is my as you know it is my  $\theta_2$  and if I go this way it is my  $\theta_1$ . Now, my question is can this  $\theta_2 - \theta_1$  gives you this angle in 0 to  $\pi$  in this reference frame? No, right; however, if you take this reference frame and you define this as  $-\pi/2$  to sorry it is  $\pi/2$  to  $-\pi/2$ .

Let us take this reference frame. Now, now you see now I change my reference frame ok. So, let me do this erase.

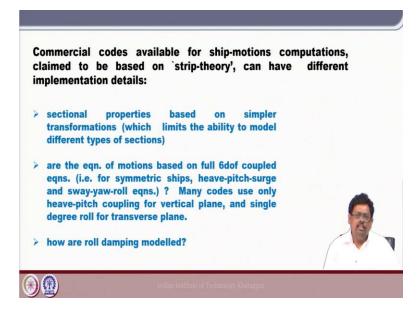


Now, in this second location now what I am doing is I am not doing in this reference frame which is 0 to  $\pi$  I am doing it from  $-\pi/2$  to  $\pi/2$ , now this is my point and this is your axis. So, this is the  $\theta$ . Now if I measure from here so, this becomes my  $\theta_2$  and this becomes my  $\theta_1$ .

Now, if I measure I am always doing  $\theta_2 - \theta_1$ , but now instead of 0 to  $\pi$  I am changing it is from  $-\pi/2$  to  $\pi/2$ , the moment I change is this again a valid. See this is how you have to set the thing, you know that is what like if you blindly always do  $\theta_2 - \theta_1$  and if you always taking the limit 0 to you know 0 to  $\pi$  this is your thing. So, you know you really do not get the correct answer many I am it is it is possibility right.

So, therefore, one has to be very much careful on all this aspect ok and this is the. So, once you do this once you find out this theta and if you try to figure out that what is the algorithm what is the algorithm to get this value of  $C(r, \theta)$  and  $S(r, \theta)$  then you can able to efficiently you can find out the value of the PV this complicated term and then you are done with your Frank close fit method ok.

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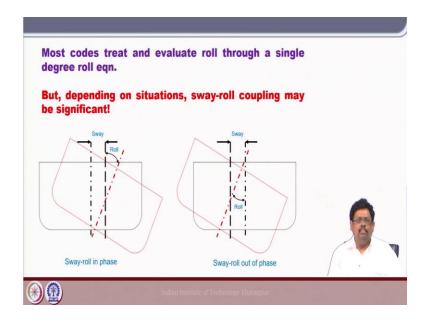


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~	II 6 dof eqns. of motion $r^2(M \to A)$ is $R$	+ $C_{jk}$ $\overline{\eta}_k = F_j^{EX} = F_j^l + F_j^D$ ; $j = 1, 2,, 6$	
k=1		$C_{33} = \rho g A_{WP}$ $C_{35} = C_{53} = -\rho g A_{WP} LCF \approx 0$ $C_{44} = \rho g \nabla GM_{T}$ $C_{55} = \rho g \nabla \left(GM_{L} + \frac{LCF^{2}}{\nabla} A_{WP}\right) \approx \rho g \nabla GM_{L}$	

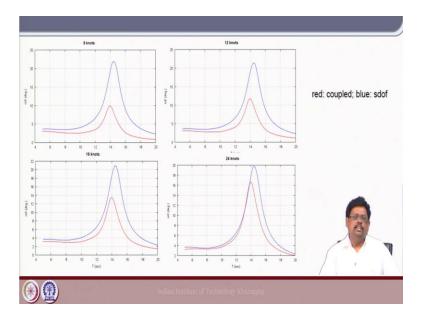
So, now I am just going to finish this by you know showing some kind of result this is all we have discussed. So, we are not going to discuss any more.

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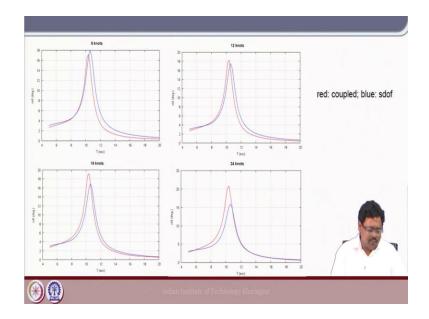
Now, we are talking about the coupling as I mentioned that most of the time we are using that roll is uncoupled. However, in reality, roll is really not uncoupled it is always coupled with you know the sway. Now, how you see like if you consider that this is your rolling right. So, if this is the rolling then it can be roll like this way or it could be roll as in a opposite way this way and this way right, you see it is it can roll this way or it can roll this way right. So, now, here this one should use the coupling between sway and roll ok.

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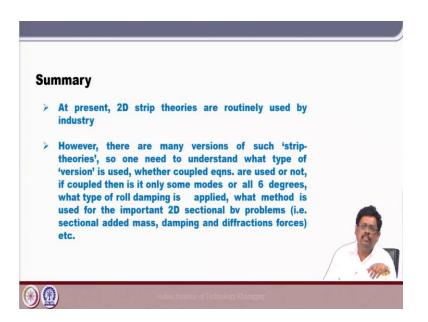


So, now that is what just the last part I just going to show you this one that the red one is the coupled one and then blue one is I just use the single degrees of freedom roll motion. Now, you can see that in the in this here that there is a difference right? So, that is the one key point people normally think that roll should be should not be it is a single degree of motion, but it is not definitely if you want to solve the role you have to again use the coupled equation motion with the sway.

If you try to find out the heave it should be coupled with the pitch definitely and if you try to figure out the role it should be coupled with the sway right ok.



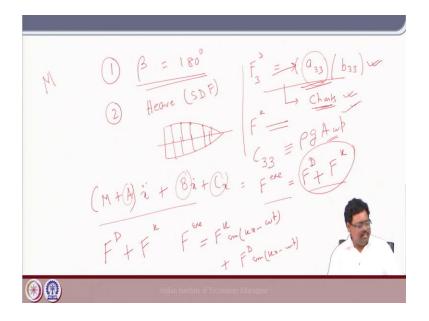
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And again this is another picture. So, like another and I mean this coupled and uncoupled ok. So, with that and of course, this is for the different speed right. As a summary that at present the two dimensional strip theory are routinely used by the industry reason is very simple because this added mass damping all these things actually you can use using some kind of chart that is coming from the Lewis curve and also we can simplify the exciting force also.

We mentioned that using the buoyancy that extra buoyancy or in fact, in case of a head waves we can figure out that using that simple added mass damping can represent the diffraction force. So, in many ways most of the industrial people they are having this strip theory ok and also there are many version of strip theories is available based on several theories, here in this class we may mostly focus on the two approach one is by STF paper and second one is the that Journees lecture not lecture the report ok.

So, this is all for the strip theory. So, now, before we finish this strip theory part I would like to give you a small assignment that you can try in your home to write a strip theory code with lot of simplified assumptions. So, the assumptions are as follows.



So, number 1 you know you can take that  $\beta = 180^{\circ}$  ok. The moment you take this  $\beta = 180^{\circ}$  degree then you can write your this F<sup>D</sup> in terms of some added mass some  $a_{33}$  now you see that ok. So, second point is that you are only the heave. So, it is single degree of freedom only the heave, okay? So, then in that case F<sup>D</sup><sub>3</sub> you can figure out in terms of  $a_{33}$  and  $b_{33}$ .

Then, how you get this  $a_{33}$  and  $b_{33}$ ? You can get it through some kind of chart right. So, this chart maybe I we will try to prepare this chart and give to all the students so, that we can get it. So, the  $a_{33}$   $b_{33}$  get from the chart and then this Froude-Krylov force F K force it is analytical. So, you know how to do that.

And then after doing all these things and then this you figure out how to get this  $c_{33}$  which is nothing but  $\rho g A_{wp}$  right? And you also know how to get the water plane area right, if I if you know this the y values. So, you can discretize it right you can discretize it and you can take the area. So, you can get the water plane area.

So, you get all these three terms right and also you have this data what is the mass right. So, therefore, after that use this strip theory that for that what we discussed throughout. So, if you get the sectional added mass from the chart so, how to get the by integrating, how to get the added mass for the body that also you know and then you solve this equation  $(M + A)\ddot{x} + B\dot{x} + Cx = F^{exc}$ , exciting force which is nothing but addition of diffraction force plus addition of Froude-Krylov force.

So, everything you can do with time domain it will be easy for you because you know the added mass in real number, you know the damping is a real number, you know C is a real number. So, then you need to find out the amplitude of everybody using this analytical method that we have discussed, not analytical like that diffraction force in terms of added mass and damping that is also we have discussed right.

So, using that you will getting this the diffraction force with amplitude and then you will get the Froude-Krylov force amplitude. So, everything you can you are actually you can generate the right hand side is a harmonic motion right because you are getting this  $F^D$  by this method and plus you are getting the  $F^K$  by this method. So, therefore, you can generate your exciting force  $F^{exc} = F^k \sin(kx - \omega t) + F^D \sin(kx - \omega t)$  in that way you can get it ok.

So, it is a very you know it is not very accurate, but what I wanted that you use this and then at least you just write a code finally, using the motion of the body ok. You have all the prerequisite, you know this how to solve this equation in time domain using Euler equation, you know how to get this in time domain, how to write the exciting force, you get the damping from this chart it will be provided and then you have to solve the equation of motion.

So, that is actually your some kind of we can say the home assignment and just for your try let us see how many of you can do this ok.

Thank you very much.