

**Numerical Ship and Offshore Hydrodynamics**  
**Prof. Ranadev Datta**  
**Department of Ocean Engineering and Naval Architecture**  
**Indian Institute of Technology, Kharagpur**

**Lecture - 44**  
**The Domain Panel Method**

(Refer Slide Time: 00:20)

**CONCEPTS COVERED**

- Discussion on Time Domain Panel Method part - 1

Indian Institute of Technology Kharagpur

(Refer Slide Time: 00:27)

**KEYWORDS**

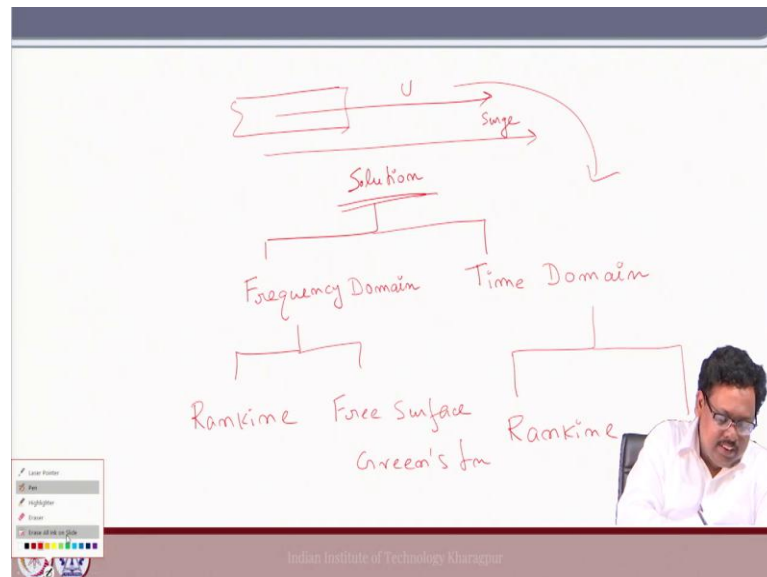
- NSOH Time Domain Panel Method part - 1
- NSOH Prof Ranadev Datta
- Numerical Ship Hydrodynamics lecture 44

Indian Institute of Technology Kharagpur

Hello, welcome to Numerical Ship and Offshore Hydrodynamics. Today is the lecture 44. Today we are going to discuss a new topic which is Time Domain Panel Method ok

and this is the keyword that you have to use to get this lecture. Now, in time domain panel method as you mention it is for the forward speed or you can say is the forward speed sea keeping problem. Now, in this case you know as I mentioned that always that there are several methods are available for each type of problem.

(Refer Slide Time: 00:55)



Now, what is the forward speed ship motion problem is as follows. Suppose, you have a ship and it is moving with a constant velocity  $U$ . Now, what is going to happen here apart from the you know when we discuss with the for this you know strip theory, we mentioned that the radiation force, diffraction force everything you have to find out for the zero speed.

And, then there are lot of theories that incorporate the zero speed into the how to incorporate the forward speed into the solution. However, in this three-dimensional panel method is more sophisticated compared to the two-dimensional panel method. Because, here we do not have to solve the radiation diffraction problem or radiation problem in particular in zero speed here, we solve all the problem in forward speed only.

Now, but remember here we do not consider the acceleration. So, therefore, the ship is moving only in horizontal location, it cannot take a turn. So, as you know that in order to take a turn it must accelerate, but we do not consider the acceleration of the ship. So, therefore, this ship is steadily moving forward. Now, it is steadily moving forward in the direction of the horizontal location or in the direction of surge.

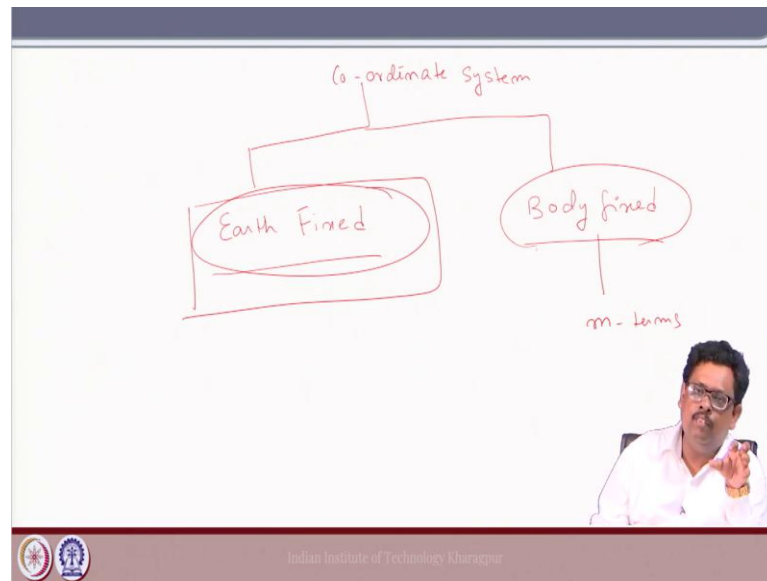
However, still ship can oscillate in all degrees of motion right so, degrees of freedom. Now, what I means it means that if this is the ship it is moving forward, but at the same time it can forward like this way. So, it is heaving and also it is moving forward and then you can take this also if you consider the both the things it is moving like this way right. So, this is how the ship is moving. So, even if it is forward moving, still it can have the 6 degrees of freedom motion.

Now, here if you look at the solution of this particular problem, what is the solution method? So, if you look at the solution then you know here there are lot of ways you can solve this and lot of theories are available, lot of numerical methods are available. So, it is not possible to discuss all sorts of methods, we are focusing on only one single. Now, but it is good to know that what other methods are available to address this problem.

Now, in solution you can define in two different thing. One is again the frequency domain or you can see the time domain right. Now, in frequency domain you can take the two type of Green's function. So, one is only the Rankine right and another is the free surface Green's function. Now, again in case of time domain also you can have again these two types. One of course, the Rankine and again of course, the free surface Green's function ok.

So, you can see there is a lot of theories are available right. I have frequency domain or I can have the time domain, again I have the free surface or Rankine. But, top of that now we are having some something we you can define as the we call this as coordinate, I mean the coordinate system.

(Refer Slide Time: 05:34)



Now, if you consider the coordinate system then one is there is a two reference. If you consider the coordinate system, then also we can have two coordinate systems. One is we can say is the earth fixed and one say the body fixed. Now, you see now you can see this lot of variation is available. Like this problem I can solve using the Rankine part, I can solve using the Rankine plus free surface part, I can solve in frequency domain.

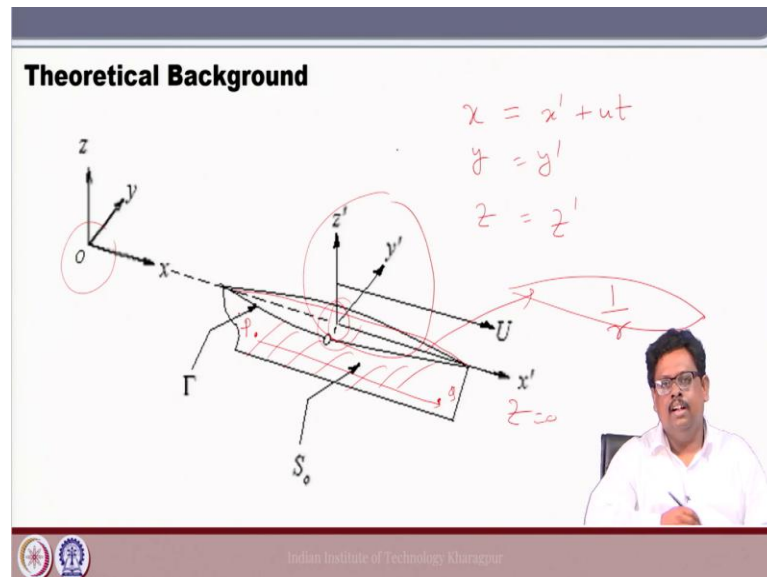
I can solve in the time domain or I can solve in the earth fixed system, I can solve in the body fixed system. Now, all other things we have discussed right, like the Rankine or free surface advantage, disadvantage everything. But, in case of a body fixed and earth fixed like the advantage of earth fixed system is that actually it is easier to incorporates kind of nonlinearity ok.

However, there are lot of theories are available or lot of methods are available in the body fixed system. Here, we have the concept of m terms and, but this here we do not have the I mean we do not need this m terms for the earth fixed system. So, now, as I said everything is so, deep and each of this method require like tedious mathematics involved, theories involved and it is not possible to cover all sort of things right.

So, let us pick one of them, like what I would like to pick which is the earth fixed system, as I said the reason behind this is here this comparatively little bit easier to incorporate the non-linearities compared to body fixed system; because we do not have to deal with the m terms. So, in case of a non-linear motions, m terms are really difficult.

In case of a linear it is very easy, you know that it is the first three terms is 0, fourth term is  $n_3$  and fifth term is  $n_2$  the normal. But, in three-dimensional is really a difficult task to get. So, now, on let us focus on the earth fixed system and try to figure out what is the boundary value problem for the earth fixed coordinate system.

(Refer Slide Time: 08:20)



Now, as a theoretical development, here we have defined two coordinate system, then  $x, y, z$  is nothing the earth fixed system. And then of course, we need to fix another coordinate system which is the body fixed coordinate system. So, here we are defined this body fixed coordinate system is  $x', y'$  and  $z'$  right. So, now, it is very elementary to see that both the coordinate system are related with the simple equation. So, I can define this,  $x = x' + ut$ , then  $y = y'$  and  $z = z'$ .

So, right so, very simple way these two can connect. Now, the question is why I need the body fixed system here? Now, the main reason is the Green's function that we are actually computing that is we compute with respect to the body fixed system, because that is easier. Why it is easier, because you see if you remember that in body fixed system or earth fixed whatever you have to deal with something called  $1/r$ . Now, with  $1/r$  is nothing but the distance between the field point and the source point.

Now, if I define this field point  $P$  here with respect to the source point  $Q$  over in the reference frame of  $x', y', z'$ , then this distance remain invariant right; because I am moving with this velocity and then this  $P - Q$  remain invariant right. However, in case of

a earth fixed system, it actually translating right. So, therefore, this distance I mean know you know this now field point; it could be the any point. So, then this invariance may be not there.

However, in present situation that we have the very stable code of the Green's function evolution based on the earth fixed system. But however, this computation of the Green's function we are getting in earth fixed system and then we connect the equation of motion with respect to the sorry that Green's function do in the body fixed system and then the pressure and all again we come back in the earth fixed system ok anyway.

So, this is the and also this  $S_0$  is nothing but the mean weighted surface as you know that when you solve the linear problem, we say it is  $S_0$  and  $\Gamma$  is nothing but the we call is the water line. So, this the top most which is at  $z = 0$ . So, at  $z = 0$ ; so, this top part we can call this as a, this one is called the water plane, a water plane we call as a  $\gamma$  ok, fine.

(Refer Slide Time: 11:34)

**Theoretical Background**

All with respect to earth fixed system,

**Governing Equation**  
 $\nabla^2 \phi(\vec{X}; t) = 0 \quad \vec{X} = (x, y, z) \in \Omega$

**Pressure Equation**  
 $\frac{1}{\rho} P(\vec{X}; t) + \phi_t + \frac{1}{2} |\nabla \phi|^2 + gz = 0$

**On the rigid surfaces**  
 $\phi_n = V_n$

**On the free surfaces**  
 $\eta_t = \phi_z - \phi_x \eta_x - \phi_y \eta_y$   
 $\phi_t = -g\eta + \frac{1}{2} |\nabla \phi|^2$

Indian Institute of Technology Kharagpur

Now, this is the governing equation based on the earth fixed system. So, we are using that Laplacian of course, the  $\nabla^2 \phi = 0$  right? And, on the rigid surface which is  $\phi_n = V_n$ . Now, this is the body boundary condition right and of course, this is the free surface boundary condition right. Now, I am writing here the non-linear free surface condition, why I am writing this we are definitely going to discuss later on.

And, then this is how I obtain the pressure. Now, if you remember that when you discuss with the linear system, we normally ignore the second order quadratic term. However, if I define the whole problem in earth fixed system, then we really do not ignore the second order term over here. So, therefore, we could say that this method is you know neither linear and nor non-linear, you can say it is a quasi linear formulation.

Now, here we ignore this non-linear term this thing, we ignore this quadratic term to get the linearized free surface condition, because Green's function does not work for  $Z = \eta$ . It always work for  $z = 0$ . So, we have to use this, but when we use the pressure term that term, I am using the non-linear contribution ok. So, in that way we can say that this formulation is not purely linear or purely non-linear. It is we call is a quasi linear formulation.

(Refer Slide Time: 13:33)

**The forward speed diffraction problem**

In the forward speed diffraction problem, the body has a steady forward motion  $\vec{U} = (U, 0, 0)$  and the body boundary condition

$\phi_n = Un_x$  on  $S_B(\vec{X}; t)$

$\phi_s = \phi_d + \phi_I$

$\phi_m = (V_n) U \cdot n_x$

If the total potential  $\phi$  could be the decomposed as  $\phi = \phi_I + \phi^*$ , then the body kinematic condition for  $\phi^*$  becomes:

$\phi_n^* = Un_x - \phi_{I_n}$

Indian Institute of Technology Kharagpur

Now, there is certain advantages of this formulation. Now, if you look at this rigid body equation  $\phi_n = V_n$ , now I can change the  $V_n$  and then I can get you know some component of the force. Now, if I try to figure out what is the steady component of the force so, then actually this would be the boundary condition. Why? Now, here I can say that this  $\phi_n = V_n$ . Now, if I only consider the steady part of it so; that means, it is moving steadily in forward.

So, therefore, this  $V_n$  I can simply replace by  $U$  multiplied by the normal  $x$  component,  $U_x$ . So, then we can so, you know with this setting of the boundary condition, we can get

actually you know that is steadily moving diffraction problem right. So, this is one and second like or you can sorry is it is for the wave resistance problem. And, then if I consider this one, it means that you know that now here it is there is no there is no waves right. I am moving just steadily the forward that is my  $V_n$ .

Now, if I say that body is moving and also you know wave is hitting. So, the scattered potential as you know it is basically the scattered potential  $\phi_s$ , I can call that a diffraction potential  $\phi_d$  plus incident potential  $\phi_I$  right. So, in that case in this expression if I say that it is  $\frac{\partial}{\partial n}$  okay? So, before that let me that clear everything.

(Refer Slide Time: 15:53)

**The forward speed diffraction problem**

In the forward speed diffraction problem, the body has a steady forward motion  $\vec{U} = (U, 0, 0)$  and the body boundary condition

$$\phi_n = Un_x \text{ on } S_B(\vec{X}; t) \quad \frac{\partial}{\partial n} (\phi_D + \phi_I) = Un_x$$

If the total potential  $\phi$  could be decomposed as  $\phi = \phi_I + \phi^*$ , then the body kinematic condition for  $\phi^*$  becomes:

$$\phi_n^* = Un_x - \phi_{I_n}$$

*Handwritten notes on the slide:*  
 $\frac{\partial \phi^D}{\partial n} = Un_x - \frac{\partial \phi_I}{\partial n}$

The slide also features a video feed of a lecturer in the bottom right corner and logos of the Indian Institute of Technology Kharagpur at the bottom.

So, now in this here instead of  $\phi_n$ , if I take this with respect to the scattered potential  $\phi_D$  is equal to  $\phi_n$  I mean  $\phi_n = \phi_D + \phi_I = Un_x$  because it is moving steadily forward, it is not oscillating. Because the ship, the right hand side is the body, body is moving steadily in the forward speed.

So, then I can have del  $\frac{\partial \phi^D}{\partial n} = Un_x - \frac{\partial \phi^I}{\partial n}$ . Now, this is nothing but it is the forward speed diffraction problem right ok. Now, here actually here it is not the diffraction, it is the wave resistance; it is just moving forward in absence of waves. Now, if you incorporate the absence, then the second equation will come ok anyway.



(Refer Slide Time: 17:05)

**The wave resistance problem**

While the above two are unsteady problems, the wave resistance problem is the steady problem. This steady wave resistance problem, a problem of great practical interest which has kept ship hydrodynamicists and naval architects occupied for many decades, can also be considered a sub part of the forward speed diffraction problem where the body boundary condition is same as before but there are no incident waves. Alternatively one may think the applicable body condition same with incident wave potential set to zero. From a physical point, here a hull is steadily moving forward in an otherwise quiescent fluid.

Indian Institute of Technology Kharagpur

So, now here it is I just wanted to write this because, this shows you some kind of you know easiness like how this earth fixed formulation from this how we could get the forward speed diffraction problem, I can formulate the wave resistance problem. So, we can see in a way it is a versatile ok. So, now, these two things actually what I said is if I go back over here this  $\phi_n$  equal to  $V_n$ , it is the my original radiation problem.

Now, since here I am not decomposing, you know I am not decomposing here with all the  $\phi$  s right, that  $\phi$  equal to you know  $\phi$  diffraction plus  $\phi$  incident plus  $\phi$  mode of  $\phi$  radiation; I am not doing it. What I am doing is slowly, slowly, slowly I am showing you how to change the boundary condition little bit. And, you can take all sort of problem you can find.

You can find out the wave resistance problem, you can find out the forward speed diffraction problem right. In case of a zero speed problem also it is easy, because if you just drop this term because  $U = 0$ , you will get the zero speed diffraction problem right ok.

(Refer Slide Time: 18:33)

**Continue....**

$$\phi_t(\vec{X};t) = \phi_0(\vec{X};t) + \phi(\vec{X};t) \rightarrow \text{Disturbed Potential}$$

$$\nabla^2 \phi(\vec{X};t) = 0 \text{ on } \vec{X} = (x, y, z) \in \Omega$$

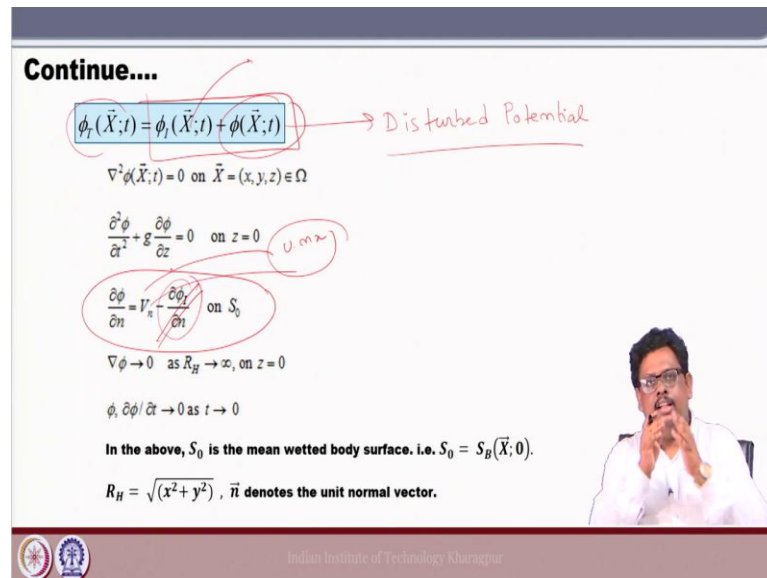
$$\frac{\partial^2 \phi}{\partial x^2} + g \frac{\partial \phi}{\partial z} = 0 \text{ on } z=0 \quad (U, m \sim)$$

$$\frac{\partial \phi}{\partial n} = V_n - \frac{\partial \phi_0}{\partial n} \text{ on } S_0$$

$$\nabla \phi \rightarrow 0 \text{ as } R_H \rightarrow \infty, \text{ on } z=0$$

$$\phi, \partial \phi / \partial t \rightarrow 0 \text{ as } t \rightarrow 0$$

In the above,  $S_0$  is the mean wetted body surface, i.e.  $S_0 = S_B(\vec{X}, 0)$ .

$$R_H = \sqrt{(x^2 + y^2)}, \quad \vec{n} \text{ denotes the unit normal vector.}$$


Now, here this is the major component of the  $\phi$  that I am using to address the classical ship motion problem. Now, carefully look at this phi. Here, in this  $\phi$  you can see here I decompose the total  $\phi$  only two component see. Now, in case of a linear problem, you know if you look at the strip theory or even in the zero speed radiation diffraction problem, we split the  $\phi$  into  $\phi'$  plus rigid body  $\phi$  mode and then diffraction.

Now, here we are simply not using any such you know decomposition of the  $\phi$  further. We have only two component, it is called the wave instant wave potential and this we can call this as my disturb potential. So, in this disturb potential, actually all sort of radiation, diffraction, steady effect all are incorporated. So, therefore, you know that is why I showed you in previous slide that how actually I could you know get back.

If I suppose, here now if you solve this problem, you really do not understand that what is the component of the diffraction potential here, what is the component of the radiation potential here, what is the component of the you know the steady wave part here. However, if I change this body boundary condition, actually I can get all sort of component. Now, can you tell me that how I can get the from this the wave resistance right, that we have done. Like I just drop this right and I just use this equal to  $U n_x$ .

So, therefore, I can get the wave resistance problem. Now, how I can get the forward speed diffraction problem? So, in forward speed diffraction problem, I need to change  $V_n$

as this  $Un_x$  and then it is  $\frac{\partial \phi^I}{\partial n}$  right? Now, suppose, if I ask you to find out what is the you know how I get the radiation problem right? We discuss how I get the wave resistance problem, we discussed how I get the diffraction problem, even you can get the zero speed also, just simply make this equal to 0. Now, my question is how we can get the radiation problem?

(Refer Slide Time: 21:35)

**Continue....**

$$\phi_T(\vec{X}; t) = \phi_I(\vec{X}; t) + \phi(\vec{X}; t)$$

$\nabla^2 \phi(\vec{X}; t) = 0$  on  $\vec{X} = (x, y, z) \in \Omega$

$\frac{\partial^2 \phi}{\partial t^2} + g \frac{\partial \phi}{\partial z} = 0$  on  $z = 0$

$\frac{\partial \phi}{\partial n} = \vec{n} \cdot \frac{\partial \phi}{\partial \vec{x}}$  on  $S_0$


$\nabla \phi \rightarrow 0$  as  $R_H \rightarrow \infty$ , on  $z = 0$

$\phi, \partial \phi / \partial t \rightarrow 0$  as  $t \rightarrow 0$

**In the above,  $S_0$  is the mean wetted body surface. i.e.  $S_0 = S_B(\vec{X}; 0)$ .**

$R_H = \sqrt{(x^2 + y^2)}$ ,  $\vec{n}$  denotes the unit normal vector.

$x = 1. \cos \omega t$   
 $v = -\omega \sin \omega t$   
 $\frac{\partial \phi}{\partial m_i} = \sqrt{g} (-\omega \sin \omega t) \cdot m_i$   
 $\frac{\partial \phi}{\partial m_i} = U m_x - (\omega \sin \omega t) \cdot m_i$



Now, here it is  $\frac{\partial \phi}{\partial n}$  equal to now in case of a you know radiation problem there is no waves. So, therefore, this term goes to 0. So, definitely it is equals to  $V_n$ . And, now if we assume, if we assume this is a harmonic function. So, you can take your displacement,  $x = \cos \omega t$ . So, I can get the  $V = -\omega \sin \omega t$ . So, this you can change as  $-\omega \sin \omega t \cdot n_i$  and that the more you are interested to figure out this one.

Now, if I say the forward speed diffraction problem, it is a zero speed diffraction problem right. Because, you know thus in forward speed diffraction what is happening, you have to consider ship is moving forward also.

So, it is so, in case of a forward speed diffraction problem, this becomes  $\frac{\partial \phi}{\partial n} = Un_x - \omega \sin \omega t \cdot n_i$  okay? So, this is the beauty of this formulation. We really, when

you solve the classical problem, the ship motion problem; we do not split up any of this radiation, diffraction for or the steady part.

So, but with this general formulation, anytime you need you can always figure out what is the component of the diffraction, what is the component of the radiation and what is the component of the steady part. So, this is my boundary condition which is very important to me right. And of course, this is the linearized as I said; we are solving the linearized free surface condition because otherwise I cannot use the Green's function. And, this is the initial condition, then or could the continuation of infinity everything is there.

(Refer Slide Time: 23:51)

**Continue....**  $G(p,t;q,\tau) = G^0 + G^f$  with  $t \geq \tau$

with

$$G^0 = \frac{1}{r} - \frac{1}{r'}$$

$$G^f = 2 \int_0^\infty [1 - \cos(\sqrt{g\gamma}(t-\tau))] e^{\gamma(z+\zeta)} J_0(\gamma R) d\gamma$$

$$r = |p-q| = \sqrt{(x-\xi)^2 + (y-\eta)^2 + (z-\zeta)^2} \quad \checkmark$$

$$r' = |p-q'| = \sqrt{(x-\xi)^2 + (y-\eta)^2 + (z+\zeta)^2} \quad \checkmark$$

$$R = \sqrt{(x-\xi)^2 + (y-\eta)^2} \quad \checkmark$$

$J_0$  = Bessel function of the first kind of order zero.  $\checkmark$

Indian Institute of Technology Kharagpur

So, now the next part is how what is my Green's function right? Now, you can see over here we have this we call is the impulsive Green's function, it is not pulsative Green's function. Now, what is the difference between the pulsative and impulsive Green's function? Pulsative Green's function is that it is harmonically you are giving the disturbance.

Now, here we are not harmonically giving this one, I just impact something, impulse though that is called impulsive Green's function; I put it and then I allow it to carry out. So, that is called the impulsive Green's function ok. Now, in this impulsive Green's function, you can see this is the formulation right. And, here you can see this the definition of the r, definition of the r' which is the image of r.

And, then you know this is how the R which is the projection and then  $J_0$  is called the Bessel function. So, this is the format of the Green's function. So, definitely we are going to discuss how we can get this impulsive Green's function, the solution of the impulsive in the later classes right.

(Refer Slide Time: 25:19)

**Continue....**

$$2\pi\phi(p,t) + \iint_{\tilde{S}(t)} \left( \phi \frac{\partial G^0(p,q)}{\partial n_q} - \frac{\partial \phi}{\partial n_q} G(p,q) \right) dS$$

$$= \int_0^t d\tau \left[ \iint_{\tilde{S}(t)} \left( \phi \frac{\partial G_t^f(p,q,t-\tau)}{\partial n_q} - \frac{\partial \phi}{\partial n_q} G_t^f(p,q,t-\tau) \right) dS \right]$$

$$+ \frac{1}{g} \int_{\tilde{S}(t)} \left( \phi \frac{\partial G_t^f}{\partial \tau} - \frac{\partial \phi}{\partial \tau} G_t^f \right) V_n dL$$

$$\phi(p,t) = -\frac{1}{4\pi} \left[ \iint_{\tilde{S}(t)} \sigma(q,t) G^0(p,q) dS + \int_0^t d\tau \left[ \iint_{\tilde{S}(t)} \sigma(q,t) G_t^f(p,q,t-\tau) dS \right] \right]$$

$$- \frac{1}{g} \int_{\tilde{S}(t)} \sigma(q,t) G_t^f(p,q,t-\tau) V_n dL$$

Now, we are using the source formulation, now you see that I really do not want to go much detail into the formulation. We are only focusing on that equation and lot of theories are available, in reference I will put at the end all the journal available, we discuss this problem. So, all the details are there. Here, we can we are only taking that what is the important to us.

The important here to solve the  $\phi$  and we are using the again the source method. We are not using the source dipole distribution. You can see here, this is the you know the final expression for the  $\phi$  and this final expression of for the phi involves the  $\sigma$  right. And, now how this comes and how you know we finally, coming back to here that is not important. But you see here finally, what where we are arriving is as follows.

We are having the surface integral over the mean weighted surface which is the body and also some integral over the water line. And, this only involve that the forward speed which is  $V$ ; that means, the body is moving forward with the velocity  $V$ . So, in case of there is no velocity, then you do not have this term. This term you are only having when

you have the forward speed. Second thing this important is this is called the convolution integral.

So, we have to integration having 0 to the present time step with that  $t - \Gamma$ . Now, if you now if you remember our impulsive response function, that formulation also we are having the term  $t - \Gamma$ . So, here also we are having the term  $t - \Gamma$ . So, at least you can guess now why we call this as a impulsive Green's function right. Because, in Green's function again we are having the memory effect, that is what we call the memory effect.

So, I am impulsing at this point and maybe some point and then actually I can see the memory effect also. Because, when the second time I am making the impulse, I consider that not only impulse also the memory effect which is coming from my previous impulse right. So, this is all the all theory is very very you know well discussed theory for many journal, paper everywhere it is there. So, let us not focusing on how this comes, let us focus on what is the final expression of the phi and how we are going to solve this  $\phi$  okay?

(Refer Slide Time: 28:10)

**Continue....**

$$\frac{\partial \phi(p,t)}{\partial n_p} = -\frac{1}{4\pi} \left\{ \iint_{S_0(t)} \sigma(q,t) \frac{\partial G^0(p,q)}{\partial n_p} dS + \int_0^t d\tau \left[ \iint_{S_0(\tau)} \sigma(q,\tau) \frac{\partial G'_\tau(p,q,t-\tau)}{\partial n_p} dS - \frac{1}{g} \int_{\Gamma(\tau)} \sigma(q,\tau) \frac{\partial G'_\tau(p,q,t-\tau)}{\partial n_p} V_x V_n dL \right] \right\}$$

$$P(\vec{X},t) = -\rho \left( \frac{\partial \phi}{\partial t} + \frac{1}{2} (\nabla \phi)^2 + \mathcal{E} \right)$$

$$\vec{F} = \iint P \vec{n} ds, \quad \vec{M} = \iint P(\vec{X}' \times \vec{n}) ds$$

$$\frac{\partial}{\partial t}(\phi) = \frac{\partial}{\partial t_m}(\phi) - U \cdot \frac{\partial}{\partial x}(\phi)$$

And, then once you solve this  $\phi$ , now as you know that we have to in order to get the expression of  $\phi'$  need the  $\sigma$ . So, therefore, I do  $\frac{\partial \phi}{\partial n}$ . So, in this expression, in this expression first I solve this  $\sigma$ . We discussed so many times so, really do not need to repeat all these things and then we substitute this  $\sigma$  in this  $\phi$  to get the  $\phi$ .

And, then when you get the  $\phi$ , I can get the pressure, when I get the pressure I can get the force. And, this is actually I am making the transformation, we already discussed in our previous classes in strip theory that how the now; this all formulates in the body fixed system. So, have to return back to the earth fixed system. So, I do this operation to change the pressure from the body fixed system to the earth fixed system.

(Refer Slide Time: 29:05)

**Equations of Motion**

$$\vec{F}_T = \vec{F}_{dynamic} + \vec{F}_{static} \quad \vec{M}_T = \vec{M}_{dynamic} + \vec{M}_{static}$$

$$\vec{F}_{static} = (0, 0, -\rho g A_{wp} z); \quad \vec{M}_{static} = (-\rho g \Delta G M_T \phi, -\rho g \Delta G M_L \theta, 0)$$

$$\vec{V}_b = \dot{\vec{X}} = \int_t \left( \frac{\vec{F}_T}{M_s} \right) dt \quad \vec{X}_b = \int_t \vec{V}_b dt \quad \text{Linear motions}$$

$$\dot{\vec{\Theta}} = \int_t \left( \frac{\vec{M}_T}{I} \right) dt \quad \vec{\Theta} = \int_t \dot{\vec{\Theta}} dt \quad \text{Rotational motions}$$

Indian Institute of Technology Kharagpur

And, then when it is done then I use the equation of motion, the total force I write the dynamic component which is for the  $\phi$ ,  $\phi$  disturbance, and, then the static component as you know is a hydrostatic component and this is the expression for the hydrostatic component. These are also we discussed a lot so; really we do not need to discuss over here. And finally, we have to solve using the time marching algorithm.

Now, you see this is the basic setup of this the whole thing. So, from the next class onward, we slowly slowly we are going to touch each of the component; how I solve this  $\phi$ , how to get this  $\sigma$  value, how to compute the Green's function. And, how I converted the earth fixed system to body fixed system, everything we are going to discuss from the next class onward. Today is the overall the structure, I we discussed fine. So, let us stop today.

Thank you.