Numerical Ship and Offshore Hydrodynamics
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Lecture - 46
Time Domain Panel Method (Contd.)

Hello welcome to Numerical Ship and Offshore Hydrodynamics today is the lecture 46.

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So, today we are going to continue the discussion about the time domain panel method, in previous class we have discussed about the static how we compute the static hydrostatic components and other things. Today we are going to discuss how we can compute the hydrodynamic components ok.

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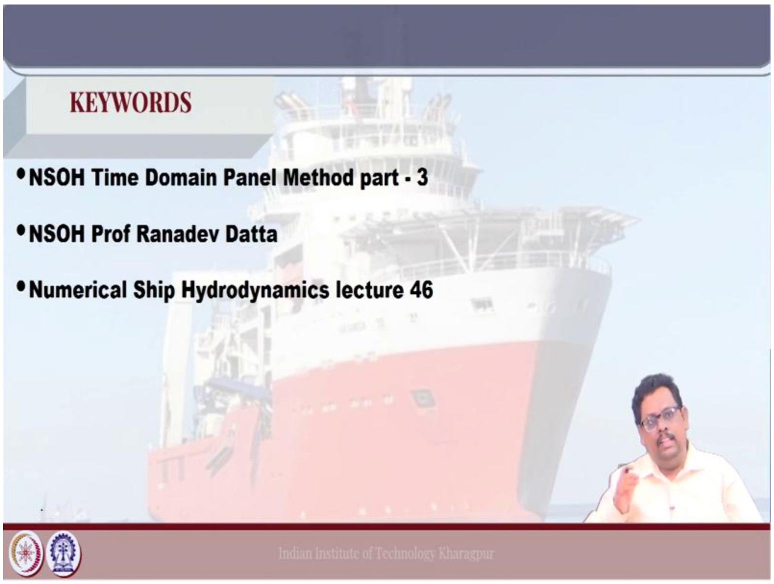

And these are the keywords that you have to use to get this lecture ok.

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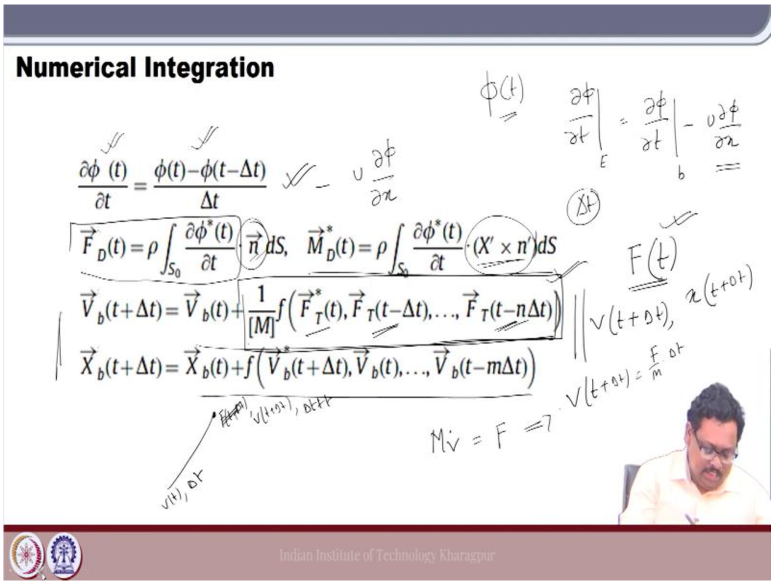

So, let us start now you know that this formulation is based on the Earth Fixed System and when we start applying this equation before that we have transferred the body coordinate system into the earth fixed system. If you remember that we said that when you compute the ϕ when you compute the $\phi(t)$.

We are actually finding out the ϕ with respect to the body coordinate system. Now then actually we have to find out this $\frac{\partial \phi}{\partial t}$ in the earth fixed system. So, you can call it E, $\left.\frac{\partial \phi}{\partial t}\right|_E = \left.\frac{\partial \phi}{\partial t}\right|_b - U\frac{\partial \phi}{\partial x}$ Now this is actually in case of a zero speed when you when you do not have this the forward speed part, it is going to 0 because equal to 0.

But when if you want to do it in case of a forward speed, so along with this we have to add this $U\frac{\partial \phi}{\partial x}$ star ok. So, this is how I can get the pressure term. So, which is it is simply you are using the forward difference scheme. Now once we get this $\left.\frac{\partial \phi}{\partial t}\right|$ term then we have to integrate over the body to get the forces. So, we are integrating it is $\rho\int_{S_0}\frac{\partial \phi(t)}{\partial t}.\vec{n}$, so then we are getting the force.

And also you can take the moment this also, if you take this normal you can get the moment. We have we discussed about all this normal n and what is x cross n. So, today we really do not take much time or any time to define I mean to tell you what is the $(X'\times n')$ or what is the n it is you know very well. Now this actually is a forward marching scheme.

Thing is that we are getting all the force and momentum about the time step t. So, our idea of any time domain good idea is I am getting it at time step t and then we are using some kind of numerical technique to get the velocity in the next time step which is $(V+\Delta t)$ and also we can get the displacement also in the next time step $(t+\Delta t)$. Now I did not mention I do not mention here any particularly what kind of scheme you know you have to use in this particular case, because there are many right.

So, you can use here I mean if you ask me what to use I will prefer to make delta t small I prefer to make delta t small and I can use the Euler scheme that actually what we use in when you use the impulse response code when he wrote that table also use the Euler scheme. So, it is pretty robust if you make Δt little bit smaller; however, there is a Adam Morton predictor character scheme is available or maybe that Runge Kutta that is also available right.

So, you can use any kind of such scheme to find out the velocity and the displacement in the next time step right. And now at this point after having lot of I mean almost the 45 class we should know I mean we should not ask the question about what actually I wrote here. Now this is nothing but I am solving this $M\dot{v}$ which is $M\ddot{x} = F$ and from there I just try to figure out my $V\left(t+\Delta t\right) = \frac{F}{M}\Delta t$.

Now, if you are using a Euler scheme you really only need the 2 component right. If you use the Euler scheme you only need about here that you know F(t+ Δt) or maybe that $V\left(t+\partial t\right)$ and $\left(\Delta t+\partial t\right)$ and here you need that V (t+ Δt) and then actually you can use any linear scheme right. So, this is how we can do that; however, if you want to use some higher order scheme.

So, then you need this $F\left(t-\Delta t\right)$ and $F\left(t-n\Delta\right)$ depending on that what scheme we are going to use right. So, everybody aware of all these things. How to solve the ordinary differential equation and get the velocity and the distance spin the next time step? So, this is the scheme that people are going to use to get the velocity and the displacement in the next time step right.

Here I really do not mention about the schemes right, this is the many schemes available you can take anyone. If you ask me I would take Euler scheme which is most easy it only has a only have the linear approximation or we can take the high order approximation using the higher order method you can take predictor character scheme. Now here actually you are using the predictor character scheme, why because, if you look at this so first let me rub everything.

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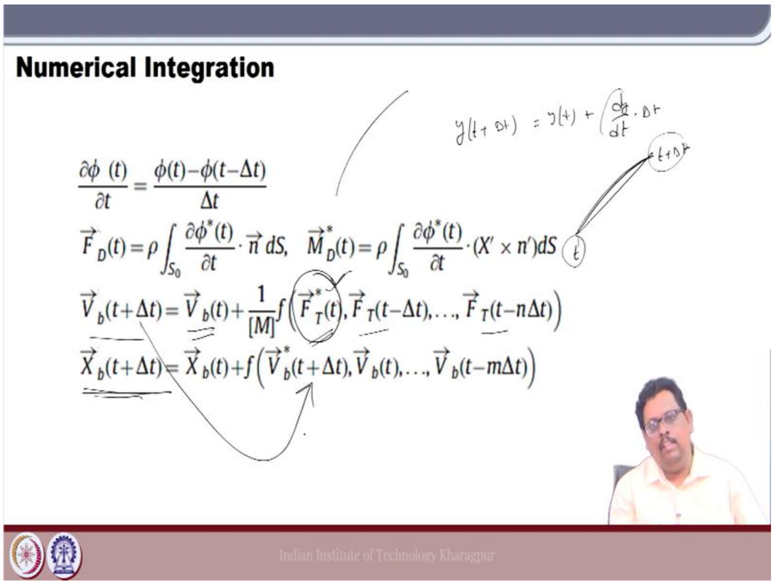

Now, if you look at here when you get the velocity, we are taking the force at the time T right, because if you remember I just tried the Euler scheme it is nothing but some value $Y(t+\Delta t)$ it is nothing but $y(t)+\frac{dy}{dt}\Delta t$, so this is the Euler scheme. Now $\frac{dy}{dt}$ you can take at now if it is t and if it is $(t+\Delta t)$.

So, this slope you can take either this point or that point based on you are going to use the implicit scheme or explicit scheme. Now here the same thing here unless you have single you can approximate this solution maybe in some in a curvilinear fashion. So, in that case you need more points. So therefore, you can take you know many parameters $F_T(t-\Delta t)$ $F_T(t-2\Delta t)$.

So, many parameters to fit a higher curve right. However, the point is that I am actually using the when you try to figure out the velocity I am using the force at time t right. But however, when you calculate the displacement I am using this updated velocity right. So, this is what you can say it is a predictor character scheme ok. So, now it is your choice you can take anything. In fact, you can use Euler's and you can use the same thing right. In fact, that is what we did in our code when you develop the impulse response function right anyway.

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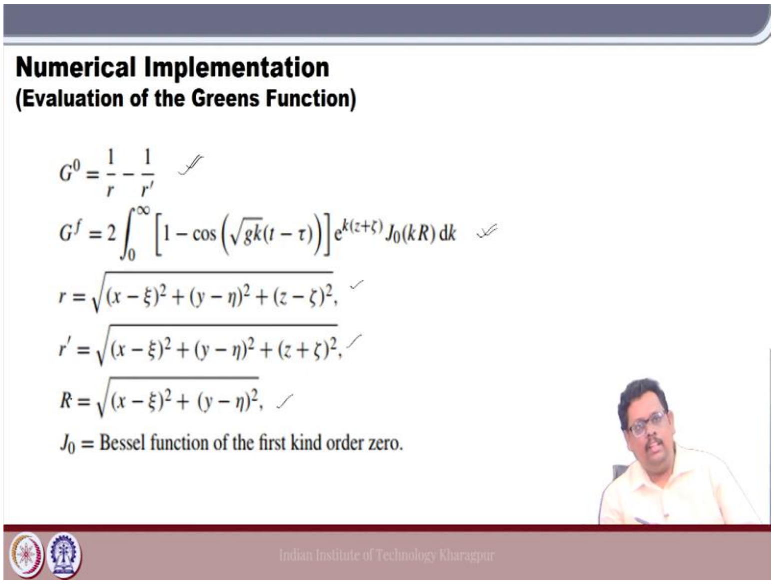

So, now second part is how we are going to evaluate the Greens function. Now here you have 2 part 1 is the G_0 and another is the G_f right and this is the expression for r it is the r’ and it is the R and J_0 is nothing but the Bessel function. Now, what is this r, r’ and R let me see in the graphical manner I mean it is better to see this graphical manner.

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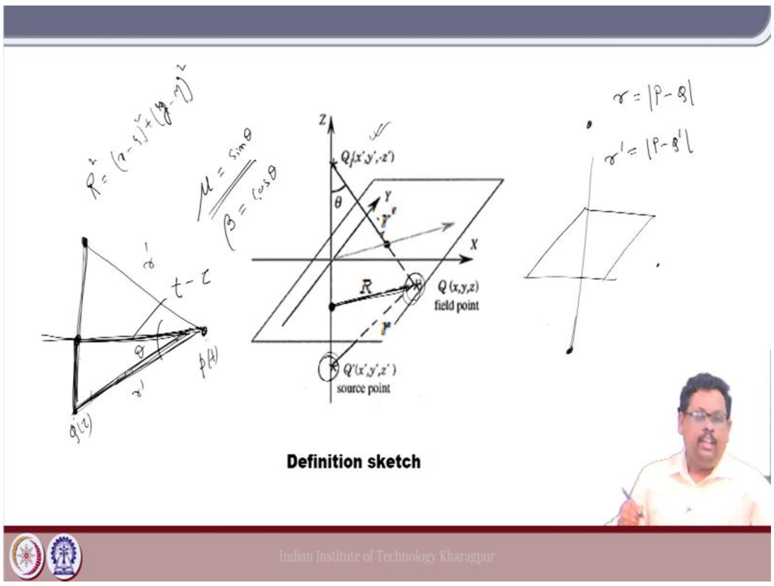

Now, suppose this is a point a source point P and let us take this is another point the field point it is let us say Q dot or anything and here it is called the field point and then if you take the image about the horizontal plane

So that means, if I draw a horizontal plane and now if I have in the below, I have the source point and maybe I have some field point here. So, if I take the image about this horizontal plane on the top. So, that is nothing but the image point or you can call the mirror about the mirror point about the xi plane horizontal plane. Now here if I add this point P to Q. So, source point to the field point that we defined as r. So, this is nothing but the difference between source points to the Q.

And then you have another point which is r', now r' is nothing but the distance between the source point to the image point. So, r' is nothing but $|P-Q'|$ or Q' is the image point. So, this is the definition for r and r'. Now what is R, is nothing but the projection right. So, that is why we write this R is nothing but $R^2 = (x-\xi)^2 + (y-\eta)^2$, so this is the distance or you can call it projection.

So, this is the definition of r, r' and the R ok and this all things are used here. So, here you can see the r you can see the image point it is $z+\xi$ right.

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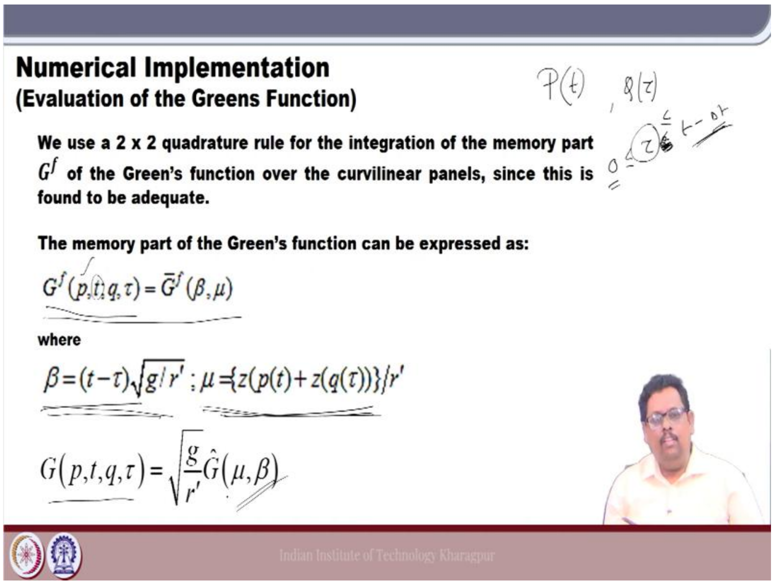

Now, how we compute this actually we do a transformation? So, what the transformation we are doing here we are actually we were using a 2 point gauss - quadrature rule to do that.

Now, here we do not have the sufficient time to tell you about this numerical scheme of this how I do this beta and mu. I mean of course I am going to tell you that what is the a numerical scheme, but in detail in coding we do not get the time to do the coding. But here I can promise you one thing like you know at the end I can provide you at least the 'Matlab' code of this the evaluation of the free surface greens function ok; of course at the you know end of this course ok.

Now here we can non dimensionalize actually what we are doing is that we are non dimensionalize the Greens function. Now remember this Greens function actually have four component 1 is p, p is nothing but the source point and t is nothing but the present time.

Now it is actually called the Impulsive Greens function or you can cause the memory function it involves a memory function. What is that like? Suppose at time at present time I try to compute that contribution of the source potential, but it is not sufficient to only consider the impulse that is I am providing right now at let us say some let us say some t equal to let us say 10 second.

I am giving a impulse and I am trying to figure out that what is happening because I am putting this disturbance in the wave field. Now it is not sufficient right already we have shown a picture like in a glass I put a ice I can see there is a memory effect.

So, it is not enough only to consider the existing the impulse, also we need to consider some impulse that I actually I am giving the impulse starting from the 0^{th} second. So, at the 0 second the first time I put impulse and again in the first second also I put another one. So, in each second actually I am going to give 1 impulse. Now at 10 second it is not sufficient only consider the impulse I mean the effect because of the 10 second impulse.

We need to consider the 9 second 8 second 7 second 6 second all this combined effect also we need to consider, so that is where I say here that p is only the function of t. That means the, that my source point sorry the field point actually at it is there at the present time step. However, my source point it is actually function of the Γ, Γ actually runs from the tau is actually runs from 0 to t it is strictly less than t.

Of course it does not take the present time step, but actually it is it should be $0 \le \Gamma \le t - \Delta t$ till that point. So, this is the limit for Γ. So, in that way I am considering the effect from the 0 till the $t-\Delta t$ second ok. So, now so this is how actually I that is why we see that Greens functions taken care of all aspect is taken care P at time t and Q at time Γ.

So, now we need to non-dimensional the whole thing. So, this is how we non dimensional t in this form and also we non dimensionalize the p and q in this form ok. Now let us see in the picture what actually it is? So, eventually this G(p, t, q, Γ) turns into this g/r'G (μ, β). So, I am shifting the coordinate now here actually I have to deal with the 4 parameters right.

And this is actually very difficult how can I how I how can I handle this p and then Q then t, Γ 4 dimensional variable. So, then I use this transformation law to make this 4 dimension into the 2 dimension variable, now we are comfortable of dealing with the greens function in the domain of mu beta. And of course, once we get the value $G(\mu, \beta)$ definitely we need to go back to the original (p, t, q, Γ) in this domain right. Definitely we are going to do that.

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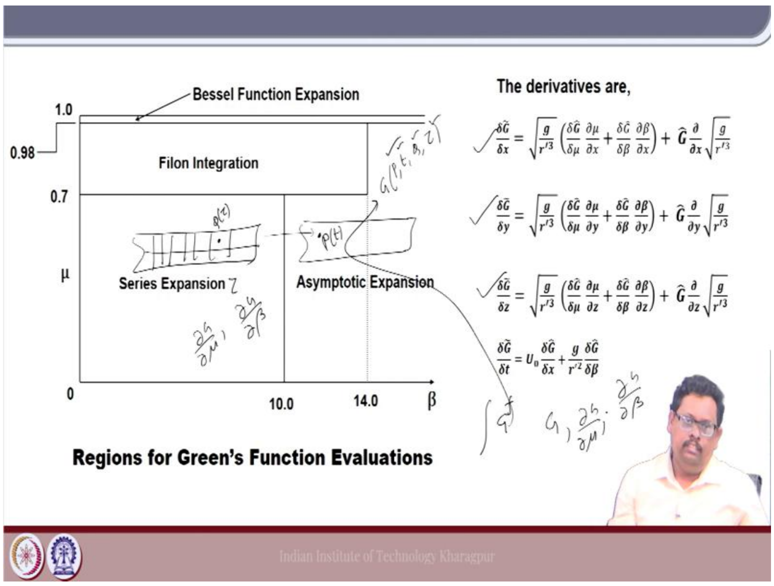

Now, if again this is actually the integration scheme in the mu beta coordinate system right and before that also I would like to see here that about this transformation that is $\mu = \frac{zp(t) + zq(\tau)}{r'}$. Now what is the meaning of this? Now you see here from this picture I can draw a nice thing, so if this is the time scale so I am here I have this p (t).

So, then I have this image point over here sorry source point over here and at some previous time step tau right and then I have image over here and then actually this is my r' and then this is somehow it is my the image point. So, this is my and now if I do this then you can see here that my transformation is nothing but z it is the z coordinate of the present time step plus it is the z coordinate of the previous time step right so and divided by r'.

So now how I can idealize this? So, it is very important to see this idealization. So, if this is the r dash, so somehow we can think this is the z. Now if I project here, so it may be this is the thing and then this r dash or you can image you can take this r dash. So, ideally we can think this is something a coordinate system.

Now, if you make a theta, so basically mu I can think of in terms of some you know $\sin\theta$ because this is the high hypotenuse and this is the height. So, this I it is not actually like this, but somehow actually we can idealize the μ as it is behave like a kind of $\sin\theta$ ok. Similarly this beta you can it is the β if you look at this idealize the beta it is the time scale.

So, this is your this is your nothing but $t-\Gamma$ this is the length and this is the hypotenuse. So, then β I can idealize the β as you know $\cos\theta$. It is really you know we cannot say $\sin\theta$ $\cos\theta$, but you know you can thought of that this mu and beta actually it playing the this role by $\sin\theta$ $\cos\theta$, because in that way you can at least you know try to see the thing ok.

Now, once we do that in mu beta system, then actually I divide the whole thing in some domain right. In the (μ,β) if you look at it that only it should be lie between 0 to 1 right? Because if you look at this idealization this by r'. So, it is the maximum value it could take actually 1 and when you get this value 1 can you think of yeah. Let me just rub the whole thing.

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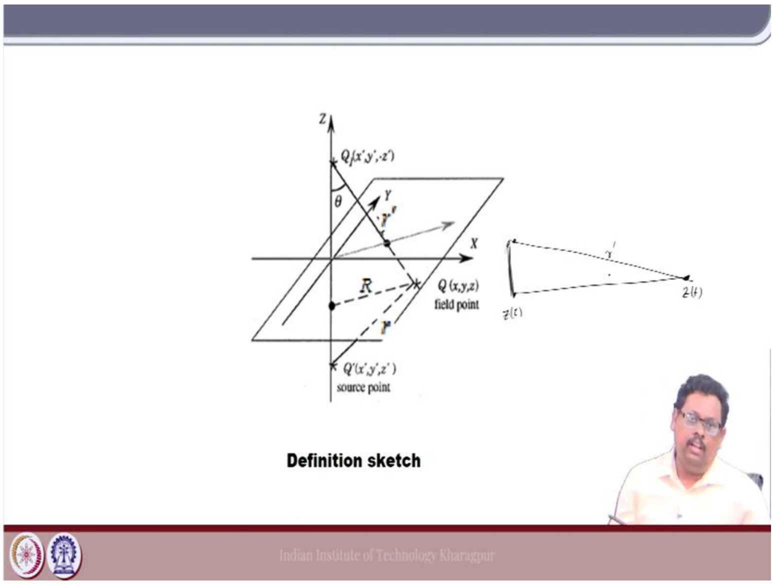

I will show you that when this it could get the value 1 like in this domain, now if I take a point here; now this is my $z(t)$ and if this is my you know $z(\tau)$. So, if the both point are lie in the same horizontal line at that time actually your r dash and this difference become the same. So, at that point, I mean the only when it could be a one otherwise in any other situation this is your r'.

And actually this is the difference right? $z(t)+z(\tau)$. So, this is you can think of this is the different z values. So, this cannot be more than 1, I mean more than r', right? Okay? Because this defines the high pertinence this is just a 1 length or height. So, we understand that value of mu it should be live in 0 to 1, but the $t-\Gamma$ can go to ∞ right?

So, in the horizontal axis the β axis it run from 0 to ∞ and the vertical axis mean should 0 to 1. Now here this scheme is very popular scheme and it is given in many books, you can see the Newman's book or some very thesis you know early not early that like late 80's thesis all the thesis have this scheme. Nowadays you know again I am telling you that solution of the Greens function is not unique also.

This is how we solved it and in fact in late 80's or early 90s people use this. But however, you know later stage this can be I mean people actually transform this into a ordinary differential equation and can and they are getting some analytical solution also. But you know all discussions, is a very long discussion. So, let us stick with one which is nowadays if you are doing a 'matlab' code these things are very easy.

Because in 'matlab' the inbuilt Bessel function, then ε integration series expansion asymptotic expansion everything all this component you get into the matlab. So, writing this code in 'matlab' is nowadays is not that difficult ok. So, anyways the only thing you need to do is you need to figure out that when you do the integration. Now suppose I am doing a gauss quadrature rule to figure out the integration ok. Now maybe not today, but definitely in one class I am going to show you that how we are doing the integration thing.

So, that time again we can come back to this and we are just showing you that how to use this chart ok, how to use this chart that time we are going to show you. Today we are just telling you one thing that when you do this integration of the so now when you do the integration of Greens function there is a_2 component, one is done with the G_0 another is running with the free surface G function G_f.

Now, when you do this integration numerically what you are going to do is from here you are transferring this into the that G (p(t) ,q (τ)) you know all four values is it not. Suppose you are running it let us say at 10 seconds, so now let me here only I just draw this picture ok it is easy. So now, this is your panel this is your panel at times tau and now let us take you are move the ship at some point.

Now, here this is your the updated position of the ship and you are taking some value over here P and this P is at the time t. So, what we are going to do is now with respect to this t you are figure out the this is your source point q. Now you know right that how I do this panel that panel method how it works, we have to have a P and then we have to have a point Q right. So now this q it is at the time tau.

Now, when you are doing the integration you know the information about the P, you know the information about the t, you know the information about the Q, you know the information about the tau. So, definitely if you apply that you know this previous thing that if you apply these 2 equation then easily you can transfer this (P, t, q, τ) into the (μ,β) system and then you know the corresponding value of (μ,β) also.

Now, we are going to check that in which range you are going to get the value it is fall into this Bessel function region or ε integration or series or asymptotic anything and then you apply those formulation and you get the value for G. Now once you get the value for G also you should you can again very easily you can get the value for $\frac{\partial G}{\partial \mu}$ and also $\frac{\partial G}{\partial \beta}$ that also very easy you can get.

Now, using these this technique actually you can get all the component you can now G already you can get then $\frac{\partial G}{\partial x}$ also you can get, $\frac{\partial G}{\partial y}$ $\frac{\partial G}{\partial y}$ $\frac{\partial G}{\partial t}$ l everything you can get from this expression. Once you know the value of g and then $\frac{\partial G}{\partial \mu}$ and $\frac{\partial G}{\partial \beta}$ once you have this value you can get all this value over here ok.

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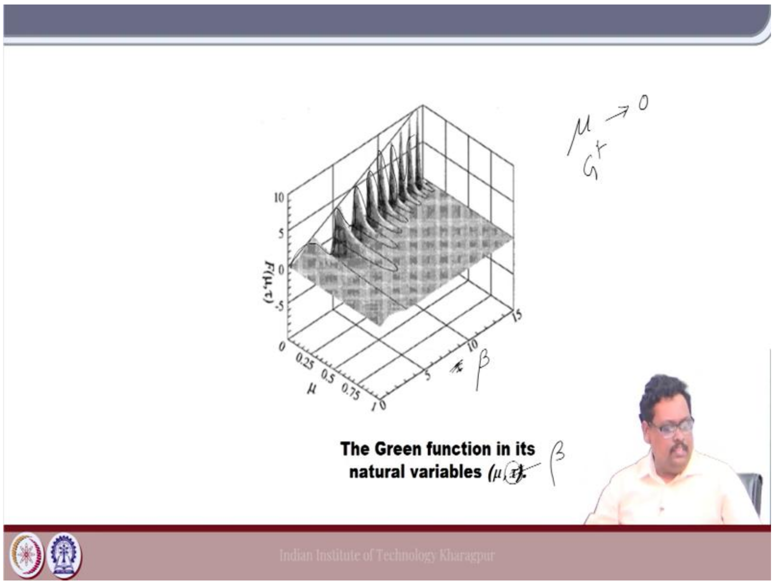

We are going to discuss about the numerical instability. Very quickly I will tell you this here we can see there is a graph. So now, the τ is nothing but your you know the β ok. So, do not confuse the τ it is τ you know sometimes we can use tau also it is a (μ, β) let us say now you have this axis μ you have this axis β. So, I can see that when this μ is 0 the Greens function is become very oscillatory in nature.

So that means, when μ actually is very close to 0, then we are getting that Greens function is very highly oscillatory in nature.

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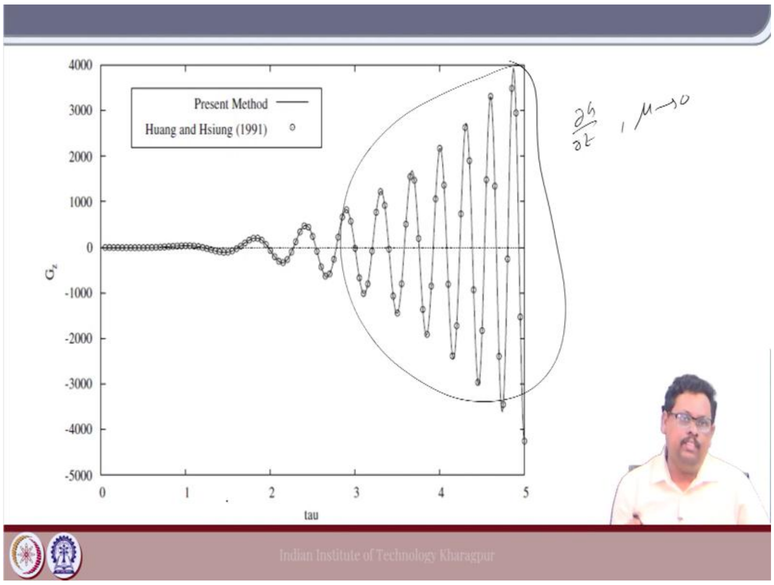

Now, you see that in the next figure also I can see that this is the value for $\frac{\partial G}{\partial z}$. So, when that mu is close to 0 we can see that how divergence it is. Now it is actually that is the typical problem of this time domain panel method, the solution sometimes become the divergence. The solution does not come correctly numerical instability is occurring because of this these things.

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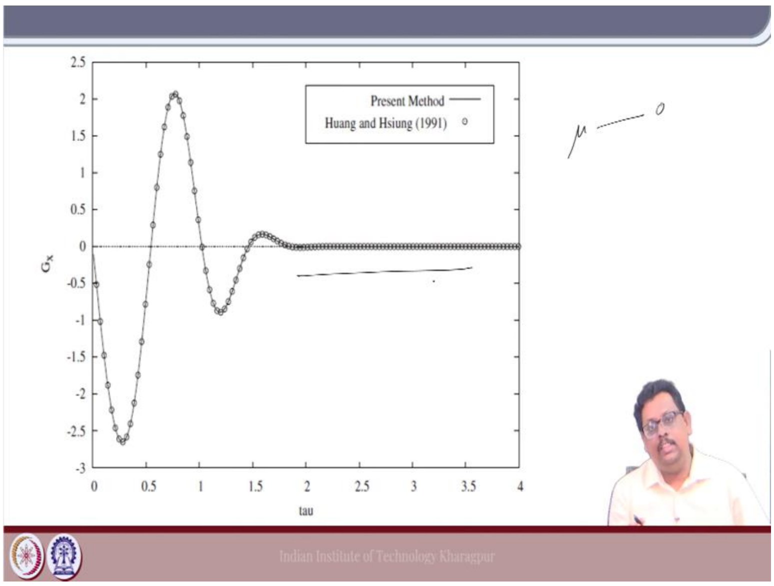

Now, you see here otherwise if you just μ is somewhere away from 0 ok. So, then you can see very nice convergent solution. So, now you have to remember this sometimes leads to some kind of instability ok.

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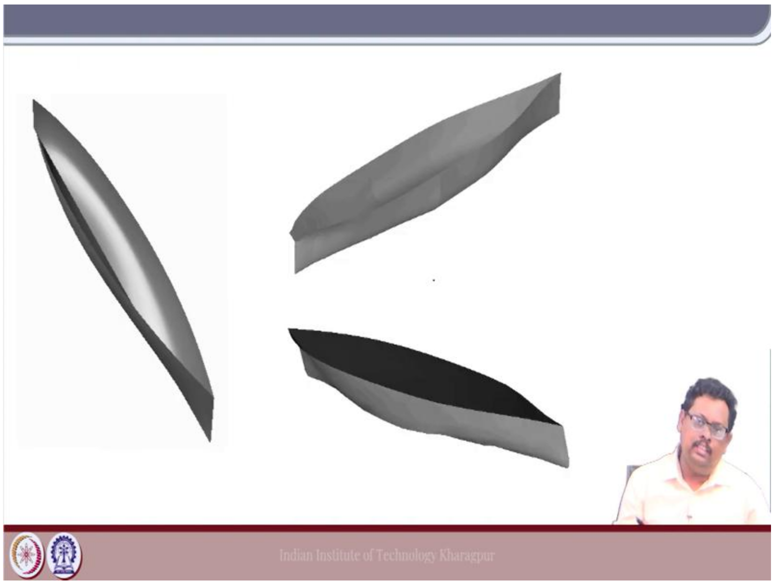

Otherwise the result normally is pretty good, now I just have some kind of Bessel. So, these are the 3 different types of ship.

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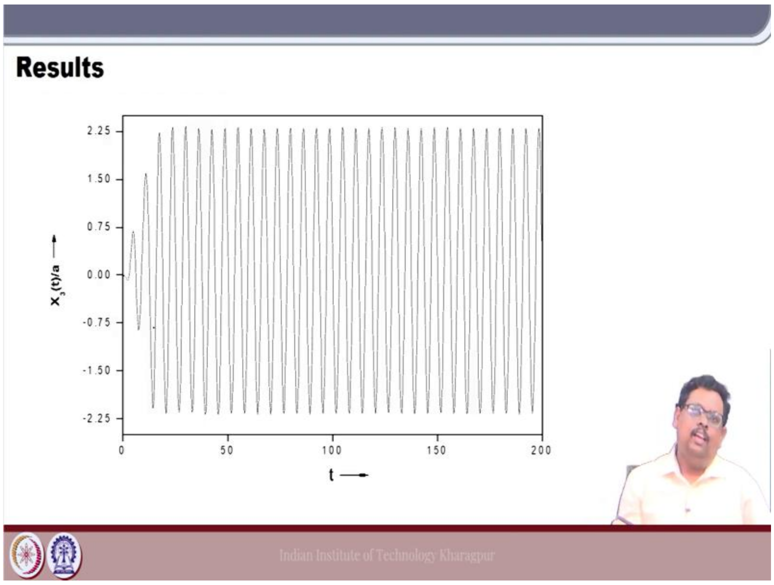

And then you can see the results are normally this is very stable.

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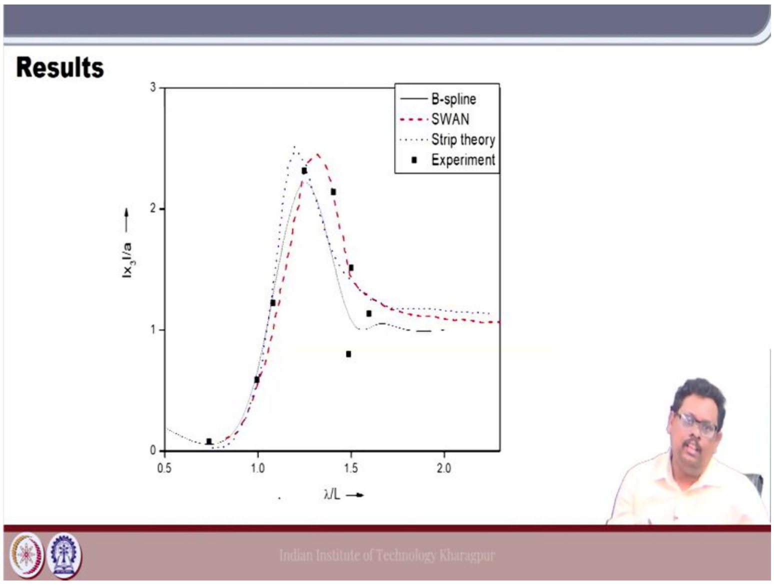

And then it is matches nicely with the other experiment results or other thing ok.

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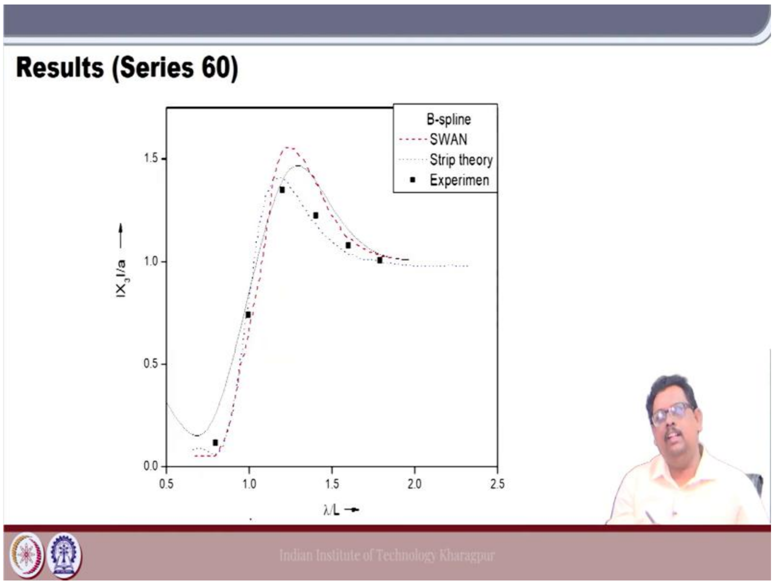

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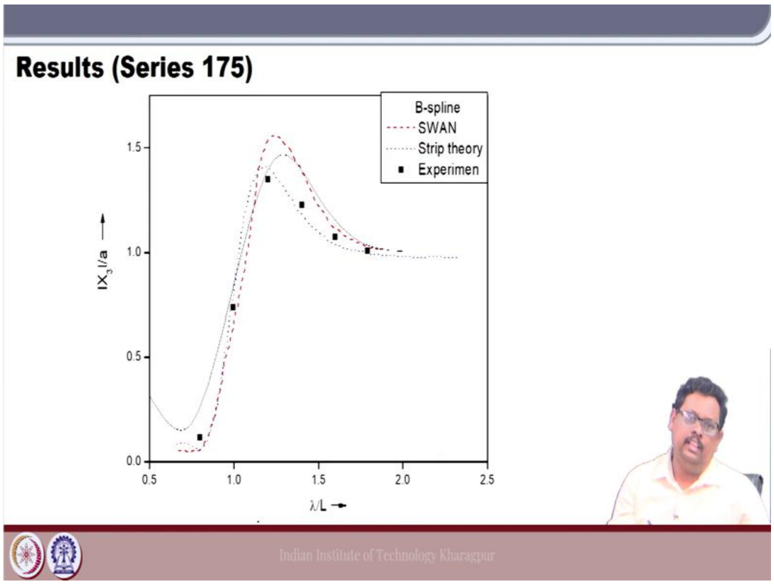

So, we can see the results are pretty good.

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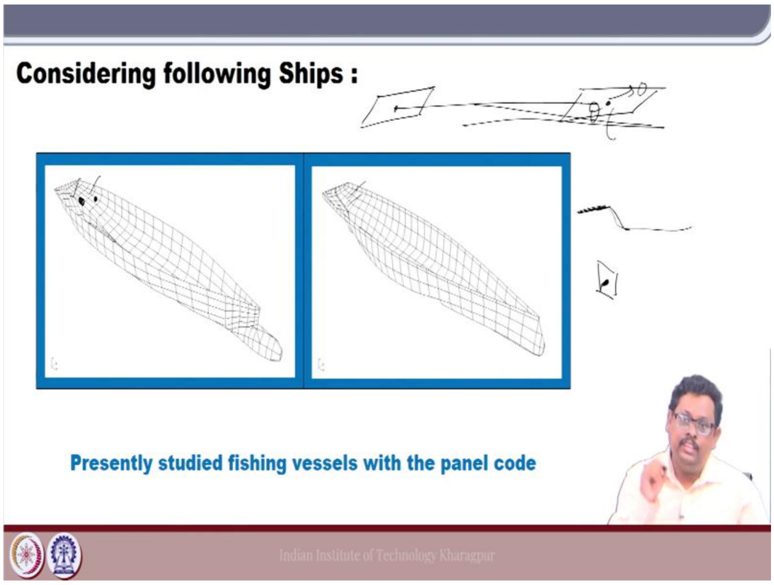

However if I take some kind of vessel which is having is a flare. So that means, if I look at the profile view some ship sometimes have some flare here turns down. So, once you have this sort of thing now you can see over here and also you can see here also you can see some kind of you know the flare is there.

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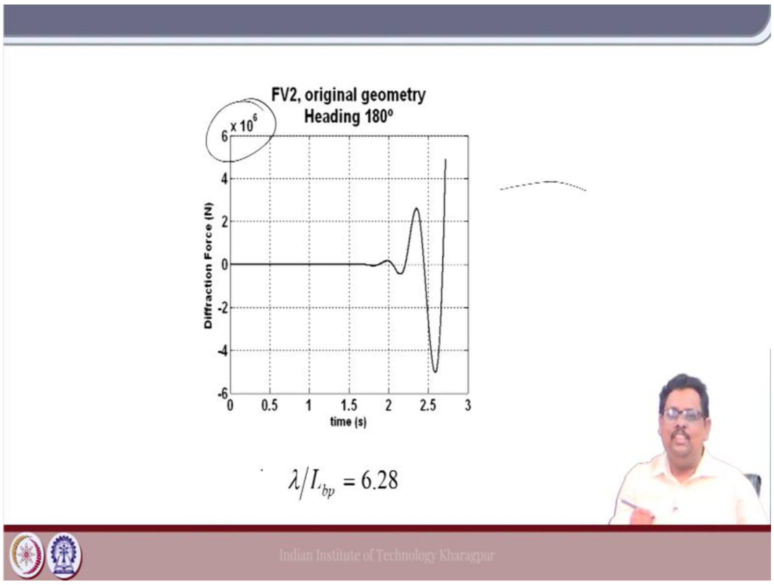

Now, what is happening mostly in the fishing vessel will get this. We can see the results are actually instable. Now what will be the reason for this instability and this is the classical problem of order speed time domain. Then it means that some geometry it will work nicely, but some geometry when they have some kind of layer at top like. So, it do not work it is actually it is that is what happening. Now how to you know handle this in numerical instability.

You can see here this value is to 10^6 which is actually not reasonable; I mean it is absurd I would say. So, now actually you know we have to take care 3 things, so let us see.

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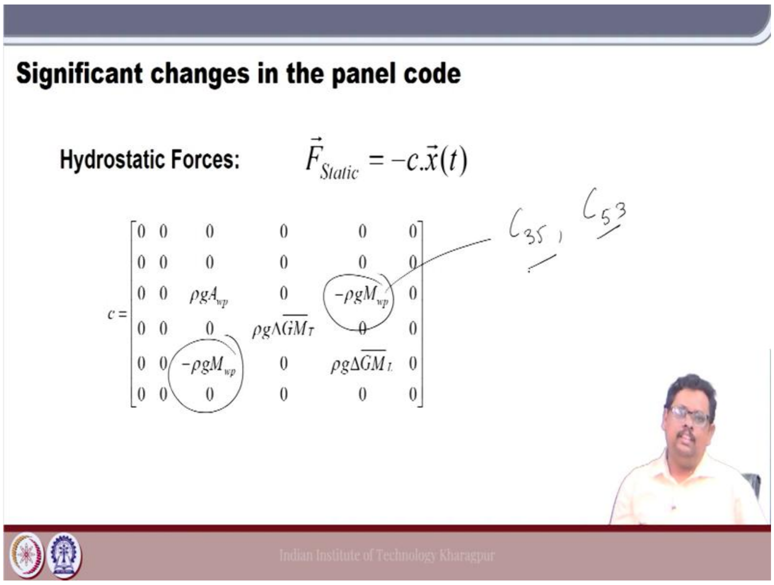

Now, the first thing is that when normally most of the time we ignore this cross coupling c_{35} and c_{53} term. So, sometime we ignore this c_{35} and c_{53} term. So, this is one wrong thing we do we should not do that.

So, this is the first thing you should be you know very careful when you write the code. So, do not drop these 2 terms so keep this is the first thing.

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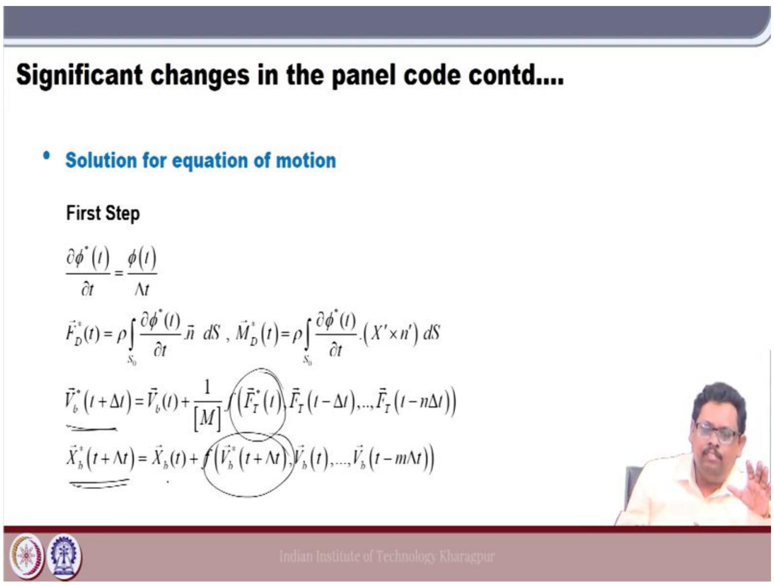

The second thing is that in the equation of motion whenever you are normally what you are using is the Euler scheme or predictor scheme. So, what we are using that when do the velocity we are using the present time step and you are approximating the future time step.

But in case of a displacement we are using the $V\left(t+\Delta t\right)$; that means, the implicit scheme for the displacement. Here actually you know what you know it is I mean I would say that there is a I do not see the solution But it works most of the time.

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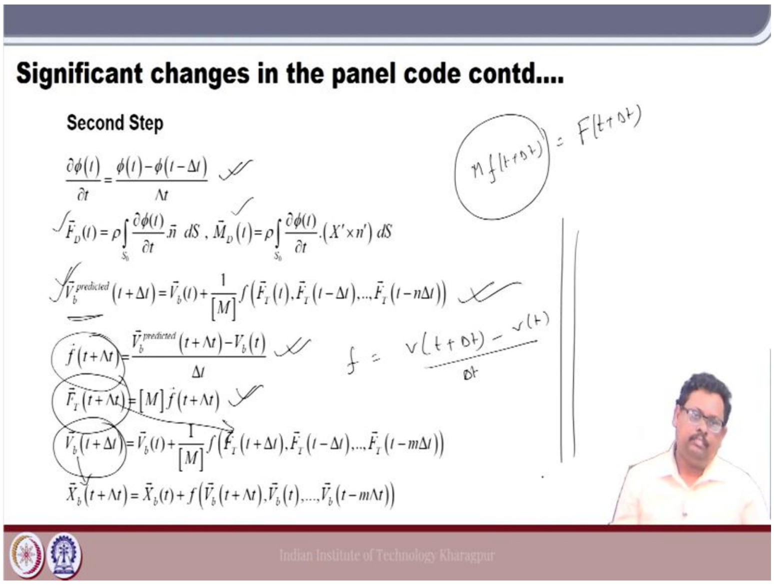

So, what actually we are using that? Now first we are trying to get $\frac{\partial\phi}{\partial t}$, the forward marching scheme.

And then we are getting the force and the moment and then we are using the predicted velocity using the any kind of Euler scheme or whatever the scheme And then we approximate the acceleration, $f=\frac{V\left(t+\Delta t\right)-v(t)}{\Delta t}$. Now I know that the dynamic force balance equation tells you $Mf\left(t+\Delta t\right)=F\left(t+\Delta t\right)$.

So, what I will do this we are using this technique to approximate the force in the next time step, it is an approximate force. Now we are using this approximate force here to get the corrected velocity. So, actually earlier we did not use this technique to approximate the force in the next time step, but now we are using it ok. So, we are getting the force in the next time step and similarly from here we can get the displacement in the next time step.

So, in you know you can modify your numerical scheme in this way, you are getting the velocity then using again forward different scheme to get the acceleration. Then mass into acceleration you predict that is the total force in the next time step and with that you try to figure out what is your the predicted velocity or corrected velocity. So, this is one scheme that we are we use and it works nicely.

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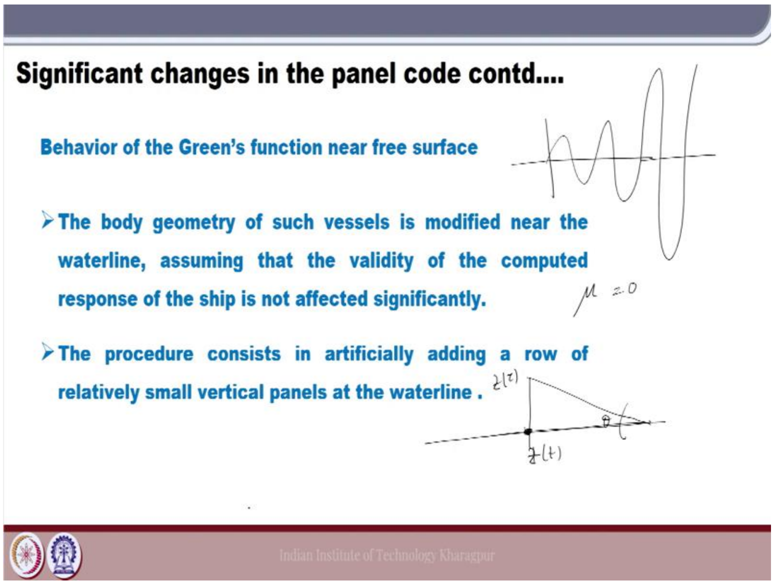

But the mostly that most important part we had here the behavior of the Greens function near free surface, as I you know as I told you it goes to infinity in this way. So, we try to avoid the situation when μ=0 now when the μ=0 is possible. Now if I if you go back to our previous idea, so we can call this is the your θ. So, this $\sin\theta$ should be 0.

It means that this now this means that it is that z(t) and $z(\tau)$, if both are lying in the same horizontal plane and which is happening in case of a flare ship. Now you see in case of a flare ship what is happening? I will show you in case of a flare ship. Now if you have this panel if you have this panel and if you have this panel both are in same horizontal plane. Now since both are in same horizontal plane, then this angle θ is tending to 0. So, that is what happening.

Now if you have a vertical panel if you have a vertical panel, now this vertical panel cannot be 0 it is impossible it only happens for the horizontal panel. If the panel is horizontal this panel also horizontal that time only this $\sin\theta$ can be or μ can go to 0, so we understand this. So, this is the problem with the flared ship. So, what actually here we do as a solution it is very interesting solution.

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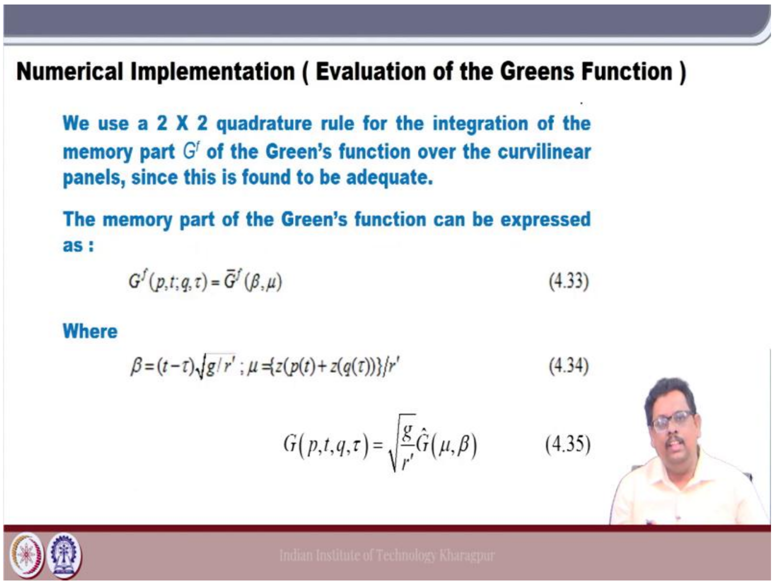

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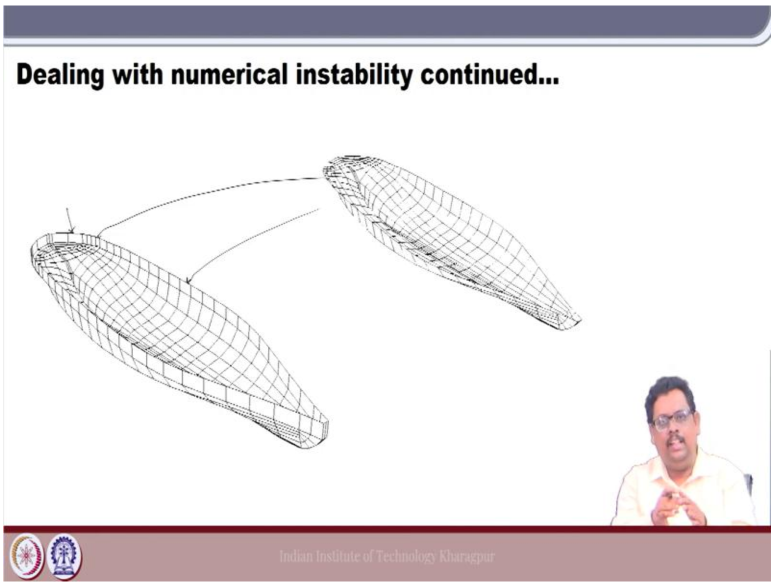

What we do is we actually use some kind of artificial paneling, you see here I have this flare ship and here I can use the artificial paneling to make some small vertical panel. It does not increase much the hydrostatic thing or other, very tiny very small a vertical strip actually I attach to the original ship.

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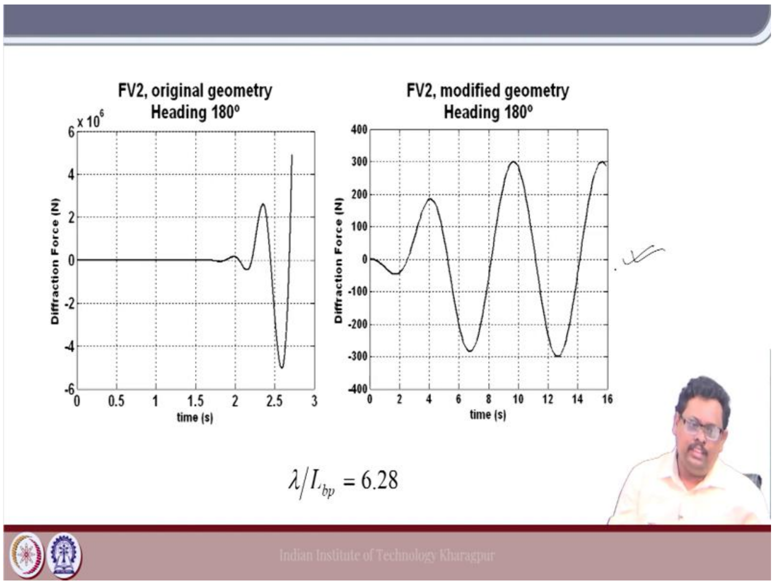

Once we do that you can see we can get a remarkable result, earlier it was earlier the force is going to 10^6 and now you can see we can get very regular very nicely the results which is very sinusoidal right.

Now see this is the thing that we would like to tell you about this the panel method thing, this is the strategy you have to use ok. And now we are now ok with this linear formulation of this time domain panel method and from the next class we are going to study or discuss about the time domain non-linear panel method.

Thank you.