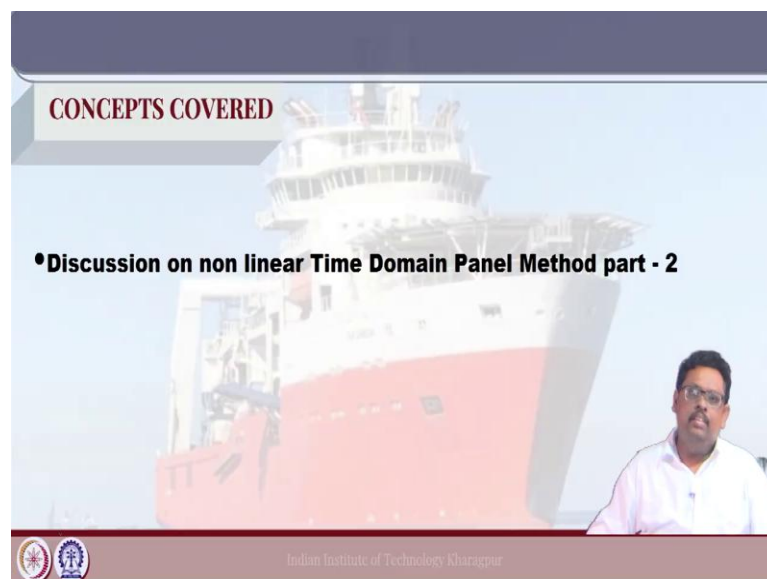


Numerical Ship and Offshore Hydrodynamics
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Lecture - 48
Time Domain Panel Method (Contd.)

Hello welcome to Numerical Ship and Offshore Hydrodynamics. Today is the lecture 48.

(Refer Slide Time: 00:20)



Today we are going to discussion on the non-linear Time Domain Panel Method ok.

(Refer Slide Time: 00:27)

KEYWORDS

- NSOH Time Domain Panel Method NL - 2
- NSOH Prof Ranadev Datta
- Numerical Ship Hydrodynamics lecture 48

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So, these are the key word that you have to use to get this lecture.

(Refer Slide Time: 00:33)

Brief Mathematical Formulations

Co-ordinate system

$(x_1, 0, 0)$
 S_b
 $z=0$

U

Γ

S_0

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Now, let us jump into the method now here of course, we need to first discuss about the coordinate system. Now, as you know that you are going to you know all this formulation is based on earth fixed system. Now in earth fixed system you have a coordinate system which is fixed in earth and you can call is a at $T = 0$ when there is no wave is a mean location; that means, it is as $Z = 0$. And then we have another coordinate system which is o dash x' y' and z' dashed right this is actually fixed in the body.

Now it is up to you that where you are going to fix your body fixed coordinate system somebody place the origin at the you know c g location. But here we are putting it at the at water line just above the c g so; that means, here it is basically at $x_g = 0, 0$ in this location actually we are putting our origin ok.

And then there is a forward speed and the ship is in advancing in a forward speed U and this S_0 is nothing but the mean weighted surface. That means, at $Z = 0$ now if you have a I am drawing the two dimensional picture like this right. So, this part we can call is a mean weighted surface. Now if you consider the wave if you consider the wave then this is called the exact weighted surface and that we denote as S body ok. So, this is about the coordinate system.

(Refer Slide Time: 02:32)

Brief Mathematical Formulations cont...

$$\phi_t(\vec{X}; t) = \phi_s(\vec{X}; t) + \phi(\vec{X}; t)$$

$$\nabla^2 \phi(\vec{X}; t) = 0; \vec{X} \in \Omega$$

$$\frac{\partial^2 \phi}{\partial t^2} + g \frac{\partial \phi}{\partial z} = 0 \quad \text{on } z=0$$

$$\frac{\partial \phi}{\partial n} = V_n - \frac{\partial \phi_t}{\partial n} \quad \text{on } S_0$$

$$\nabla \phi \rightarrow 0 \quad \text{as } R_H \rightarrow \infty, \text{ on } z=0$$

$$\phi, \nabla \phi \rightarrow 0 \quad \text{as } t \rightarrow 0$$

Handwritten notes and diagrams:

- $\phi_s + \phi_D + \phi_R$
- $\nabla^2 \phi^T = 0$
- $\nabla^2 \phi^I + \nabla^2 \phi = 0$
- $\nabla^2 \phi^I = 0$
- $\nabla \phi = 0$

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Now, this is the brief mathematical formulation we have discussed a lot. So, here I am just refreshing you like here this total potential total velocity potential. Here it is composed of two parameters one is wave instant ϕ_t and another we can call this is called the disturb potential. Now under this disturb potential as you know we have that ϕ steady then plus ϕ diffraction and then plus ϕ radiation ok. So, all this ϕ it is included in this ϕ , fine.

Now, here so therefore, under the assumptions of the linearity; so, $\nabla^2 \phi^T = 0$. So, definitely it is $\nabla^2 \phi^I + \nabla^2 \phi = 0$ now as you know that it is under the linearity again the

assumptions is each of the component should be is equal to 0. So, then I can find further split up $\nabla^2\phi' = 0$ and $\nabla^2\phi = 0$ right.

So, you know so this is how actually proceed the total potential should be 0. So, therefore, $\nabla^2\phi' + \nabla^2\phi = 0$ each of the component should be equal to 0. So, therefore, $\nabla^2\phi = 0$ now this is what we are going to do over here because the solution for $\nabla^2\phi'$ is known to me right is the analytical solution.

So, therefore, we really do not have to worry about $\nabla^2\phi'$; however, we have to worry about what about the $\nabla^2\phi$. So, all these boundary value problems are based on $\nabla^2\phi = 0$ right ok. Now here this is the governing differential equation we know that it is the Laplacian right and then together with you have this linearized free surface boundary condition right.

And then we have this body boundary condition remember that all are having in the linear. Now you can ask me that we have the non-linear time domain framework that is what we are discussing that is why this all these boundary value problems based on the linearity we are going to discuss this later on. Now but; however, when we talked about the problem the major problem that it is actually linear; what we are going to do is we are going to incorporate some of the non-linearity in some forces right.

However the overall structure is linear and that is what we are talking about the non-linearity. If you remember we discuss in previous classes that it is not purely non-linear right. We are again discretizing the several component and then each several component some of the component we are considering some part non-linear; however, the overall framework is linear.

And therefore, the solution also in absolutely the boundary conditions are the linear ok now this is nothing but V_n is nothing but the normal velocity of the body right. So, and then $\frac{\partial\phi'}{\partial n}$ is nothing but the normal incident potential ok now we have already discussed how it is coming just for sake of completeness we know that.

(Refer Slide Time: 06:43)

Brief Mathematical Formulations cont...

$$\phi_r(\vec{X}; t) = \phi_i(\vec{X}; t) + \phi(\vec{X}; t)$$

$$\nabla^2 \phi(\vec{X}; t) = 0; \vec{X} \in \Omega \quad \checkmark$$

$$\frac{\partial^2 \phi}{\partial t^2} + g \frac{\partial \phi}{\partial z} = 0 \quad \text{on } z=0$$

$$\frac{\partial \phi}{\partial n} = V_n - \frac{\partial \phi^I}{\partial n} \quad \text{on } S_0$$

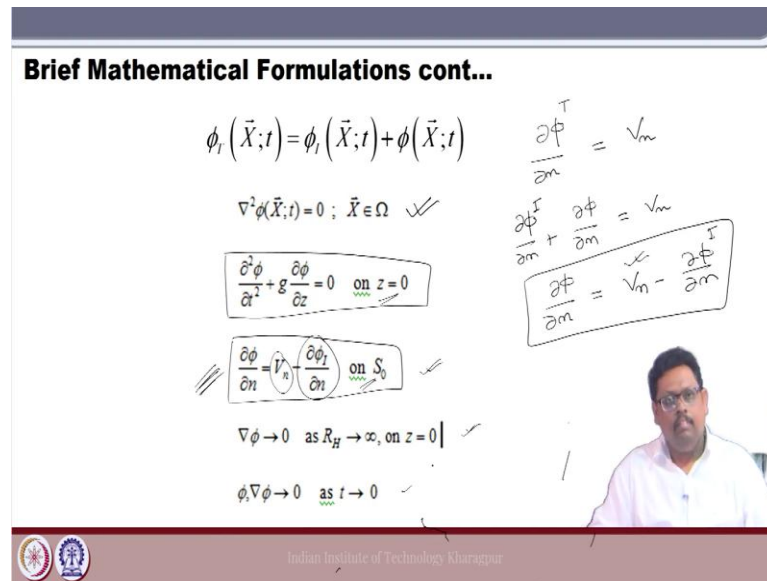
$$\nabla \phi \rightarrow 0 \quad \text{as } R_H \rightarrow \infty, \text{ on } z=0 \quad \checkmark$$

$$\phi, \nabla \phi \rightarrow 0 \quad \text{as } t \rightarrow 0 \quad \checkmark$$

$$\frac{\partial \phi^I}{\partial n} = V_n$$

$$\frac{\partial \phi^I}{\partial n} + \frac{\partial \phi}{\partial n} = V_n$$

$$\frac{\partial \phi}{\partial n} = V_n - \frac{\partial \phi^I}{\partial n}$$



The total $\partial \phi$ the total potential actually $\frac{\partial \phi^T}{\partial n}$ that should be is equal to the normal velocity of the body that is the idea. Now, this total velocity is splitted into two component one is $\frac{\partial \phi^I}{\partial n} + \frac{\partial \phi}{\partial n} = V_n$ right and. So, therefore, I know that my $\frac{\partial \phi}{\partial n} = V_n - \frac{\partial \phi^I}{\partial n}$. So, that is how this formulation has come.

Now under this how we compute the normal velocity everybody knows that you have to find out V_1 which is the horizontal component of the velocity you have to find out V_2 we have to find out the V_3 and then of course, that V_4, V_5, V_6 and then we have to find out the V_n doing $V \cdot n$ ok.

So, this definitely when we are actually some less some in later stage actually I am thinking of where some algorithm again we are going to discuss that time we will come we will see that how we can do that numerically in computer coding anyway.

So, but this is the main equation which is the that is we are going to use to solve for ϕ or σ . And then we have the radiation potential and as well as the incident wave potential. So, this is the framework right.

(Refer Slide Time: 08:31)

Brief Mathematical Formulations cont...

$$\phi(p,t) = -\frac{1}{4\pi} \left\{ \iint_{S_0(t)} \sigma(q,t) G^o(p,q) dS + \int_0^t d\tau \left[\iint_{S_0(\tau)} \sigma(q,t) G'_t(p,q;t-\tau) dS - \frac{1}{g} \int_{\Gamma(\tau)} \sigma(q,t) G'_t(p,q;t-\tau) V_n V_n dL \right] \right\}$$

$$\frac{\partial \phi(p,t)}{\partial n_p} = -\frac{1}{4\pi} \left\{ \iint_{S_0(t)} \sigma(q,t) \frac{\partial G^o(p,q)}{\partial n_p} dS + \int_0^t d\tau \left[\iint_{S_0(\tau)} \sigma(q,t) \frac{\partial G'_t(p,q;t-\tau)}{\partial n_p} dS - \frac{1}{g} \int_{\Gamma(\tau)} \sigma(q,t) \frac{\partial G'_t(p,q;t-\tau)}{\partial n_p} V_n V_n dL \right] \right\}$$

$\sigma = [A]^{-1} \left[\frac{\partial \phi}{\partial n_p} \right]$

So, and also you know that we have to use these two integral equation we are going to solve right and this is called the from the. So, you know the you know that idea is the from the source formulation. So, this is the main integral equation which is the source method right. And you have the ϕ equal to we can solve in terms of a source.

Now here this sigma is unknown to me and also this ϕ is also unknown to me right we have discussed this before and then we differentiate this with respect to n right and now this is my now I apply the body boundary condition over here right. And now here everything is known apart from the value for sigma right.

Now here we have to find out the sigma right. Definitely you know later on when we are going to discuss the algorithm that you will get to know that it looks little bit I mean you think that this is the expression this is the unknown that I do in the vomit test. So, when you do that this is the unknown.

And then right hand side basically a matrix equation right and then you are definitely get something about that sigma equal to some kind of influence matrix A inverse into this $\frac{\partial \phi}{\partial n_p}$. You know if you apply this state for it is actually not like that here when we

when you do that definitely we are going to discuss that how one should do this ok.

Here I mean, but I just wanted to say only one thing that here we cannot straight away do that we cannot do this way because it is the involvement of this parameter θ to t ok. Now here because of presence of this actually this term actually should go into the left hand side this term should go into the left hand side ok. So, therefore, these things actually cannot be the part of the matrix A ok.

Now, why it is not the part of the matrix A why this is the part of this matrix here. That means, actually the sigma is equals to some A inverse and then this matrix actually here you have the not only the $\frac{\partial \phi}{\partial n}$ component which is coming from your body boundary condition also contribution from this also go here contribution of this also goes here.

So, this is it is not that trivial we are going to discuss the whole algorithm ok. At the end of this theoretical session one or two session with default only to discuss this small small algorithm so that you could able to write this code ok yeah. So, after this when you solve this definitely I get to know what about the ϕ right this ϕ where we can get.

(Refer Slide Time: 12:12)

Brief Mathematical Formulations cont...

$$P(\vec{X}, t) = -\rho \left(\frac{\partial \phi_t}{\partial t} + \frac{1}{2} (\nabla \phi_t)^2 + gz \right)$$

$$P^L = -\rho \left(\frac{\partial \phi}{\partial t} + \frac{1}{2} (\nabla \phi)(\nabla \phi) + \frac{1}{2} (\nabla \phi)(\nabla \phi) \right)$$

$$P^{NL} = -\rho \left(\frac{\partial \phi_t}{\partial t} + \frac{1}{2} (\nabla \phi_t)(\nabla \phi_t) + gz \right)$$

$$F_i = \iint_{S_0} P^L n_i dS_0 + \iint_{S_b} P^{NL} n_i dS_b$$

$\frac{\partial \phi}{\partial t} \Big|_E = \frac{\partial \phi}{\partial t} \Big|_\theta - v \frac{\partial \phi}{\partial z}$
 $\phi^F = \phi^E + \phi$
 $F = K \text{ pr.}$

Now, after getting this ϕ we need to get the pressure right. Now you know that here also again the first part this first component is my total pressure right. Now you can see here it is $\left(\frac{\partial \phi_t}{\partial t} + \frac{1}{2} (\nabla \phi_t)^2 + gz \right)$ right and definitely this is going to you know we are

getting this actually in earth fixed system. Now here I can split this ϕ^T into two component right one is ϕ_l and another is the ϕ .

Now, remember that before you apply over here we have to apply this equation that $\left. \frac{\partial \phi}{\partial t} \right|_E = \left. \frac{\partial \phi}{\partial t} \right|_B - U \frac{\partial \phi}{\partial x}$. We have already discussed that when you find out the ϕ , the ϕ actually we find out in the body fixed reference frame, but my problem is defined on the earth fixed reference frame.

So, therefore, we have to change the coordinate system of $\frac{\partial \phi}{\partial t}$ from the body fixed system to the earth fixed system. And once we do that then everything in a earth fixed system then only we can apply the whole that ϕ total then only you are able to write the total ϕ^T on this earth fixed system ok. So, here what we are going to do is as follows.

This total ϕ^T now you know that this is ϕ^T s combination of $\phi^l + \phi$ only ϕ we can call the ϕ disturb. Now if we put into here then actually you know you can do that by your own. Then because you know that what is alpha plus beta whole square right all these things you know. So, actually I can get the two component actually splitted into two component.

As I said that when you talk in terms of non-linearity we really do not talk about the non-linear fully. It is some force component actually we consider some kind of non-linear thing. Now what is the component that you are considering in the non-linear part that is written in the second pressure expression. So, this total pressure also distributed into two component.

One is the linear component and the second one is the non-linear component very compressed way definitely I am writing here, but it is your task to break this $\phi \cdot \phi_l + \phi$ and please get these two expression. Now anything that involves ϕ anything that involves ϕ we solve in linearly ok. Now you see if you look at this expression now here when you do this, $\frac{\partial \phi}{\partial t}$ square as a plus b whole square; so, $a^2 + 2ab + b^2$.

So, you can see that that a b part and then let us say the b square part it is actually we are here in the linear part. So, because here we have this ϕ now if you see our the mathematical formulation here everything we solved in terms of a linear right everything is linear right. So, therefore, wherever we have this ϕ definitely it should be in under the assumption of the linearity.

So, therefore, wherever I have the ϕ any pressure related to that ϕ we solve it linearly; however, there is another component which is the incident. Now what is $\frac{\partial\phi}{\partial t}$ it is nothing but the Froude Krylov pressure. So, this is Froude Krylov pressure term right now when you calculate this Froude Krylov pressure and this pressure I can calculate analytically now, when I calculate this analytically so I can take the exact weighted surface.

So, now when we calculate the pressure because of the wave remember when you do this pressure because of the wave definitely I can do that with the exact free surface right. And here also you know there is one catch like which surface you call the exact free surface I am coming here I am coming this concept is really very you know when we apply it is easy; easy means we are taking the most simplest situation.

But it is very important to visualize the whole aspect and where we are actually compromising and where we are applying the approximation we have to be very clear about that right. Now this line is not at all a very difficult task right. So, we can for a moment we just we just keep it right now. Let us focus on when I say that it is the exact free surface what I mean.

(Refer Slide Time: 18:32)

Brief Mathematical Formulations cont...

$$P(\vec{X}, t) = -\rho \left(\frac{\partial \phi_t}{\partial t} + \frac{1}{2} (\nabla \phi_t)^2 + gz \right)$$

$$P^L = -\rho \left(\frac{\partial \phi}{\partial t} + \frac{1}{2} (\nabla \phi)(\nabla \phi_t) + \frac{1}{2} (\nabla \phi)(\nabla \phi) \right)$$

$$P^{NL} = -\rho \left(\frac{\partial \phi_t}{\partial t} + \frac{1}{2} (\nabla \phi_t)(\nabla \phi_t) + gz \right)$$

$$F_i = \iint_{S_0} P^L n_i dS_0 + \iint_{S_b} P^{NL} n_i dS_b$$

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Now, what is the meaning of exact free surface? Now suppose I have I am writing in the 2D only right. So, we have let us say I have this ship with me. Now I have a wave interacting with this ship ok and because of that this ship start oscillating. And because this ships start oscillating then I have the flow everywhere this side also, this side also everywhere I have a flow.

So, initially I have initially I have only incident wave potential I we can call this is ϕ^I . Then actually this interaction is with then this wave is interacted with the ship. When it interacted with the ship then the that waves actually I am getting it is it here you have the component of the ϕ^I definitely. But also remember one should not we you know forget that we have the component of ϕ also.

Because this body is oscillating then the wave I am looking at this wave is superposition of instant wave potential and as well as the disturbed wave potential, which is again the combination of the radiation, the diffraction, the steady everything. Now, my question is like after some time like initially everything is fine.

(Refer Slide Time: 20:14)

Brief Mathematical Formulations cont...

$$P(\vec{X}, t) = -\rho \left(\frac{\partial \phi_I}{\partial t} + \frac{1}{2} (\nabla \phi_I)^2 + gz \right)$$

$$\cancel{\nabla} P^L = -\rho \left(\frac{\partial \phi}{\partial t} + \frac{1}{2} (\nabla \phi) (\nabla \phi_I) + \frac{1}{2} (\nabla \phi) (\nabla \phi) \right)$$

$$\cancel{\nabla} P^{NL} = -\rho \left(\frac{\partial \phi_I}{\partial t} + \frac{1}{2} (\nabla \phi_I) (\nabla \phi_I) + gz \right)$$

Free Surface $\rightarrow \phi^I + \phi$

$\phi^I \rightarrow \eta = \int \eta_0 e^{i(kx - \omega t)}$

$\phi \rightarrow \eta = \int \eta_0 e^{i(kx - \omega t)}$

K.F.B.-C } $\eta = \int \eta_0 e^{i(kx - \omega t)}$

B.F.B.-C } $\eta = \int \eta_0 e^{i(kx - \omega t)}$

$$F_i = \iint_{S_0} P^L n_i dS_0 + \iint_{S_0} P^{NL} n_i dS_0$$

Wetted surface Unseen Incident wave

Initially I know that there is only ϕ^I initially when there is not much interaction I know everything is coming under the ϕ^I . But after some time when actually we started that ship started interacting with the wave when ship started interacting with the wave after let us say some t equal to let us say 10 second later on. Then this free surface, then this free surface is not anymore with respect to ϕ^I .

This free surface is actually combination of the instant wave potential plus the disturb wave potential. However, the assumptions is that when I say that I am going to find out the Froude Krylov pressure on exact free surface what I mean is I found I am getting it on the exact incident free surface. Again I am ignoring the component of the diffraction force you are getting it right.

Like so; that means, physically if I look at it let us take I have this I have a mobile with me right now ok I have this. So, now, initially when there is no mobile or no ship let us say that we have the wave. Now the moment we start oscillating then the surrounded wave is not anymore only because of the incident wave. It is because of the ship is oscillating because of that some component is there.

So, that free surface is now complicated free surface right, but we are not going into that complicated free surface. When you are taking that exact weighted surface I am taking it on exact incident wave free surface. Because it is easy to find out ok otherwise it is not I

am not saying that it is impossible to get the non-linear free surface. It is easy I mean it is not that easy, but it is possible right.

Because we have the now when we have the expression for ϕ' am just telling theoretically how it is possible. When you have this expression for ϕ then I can use the kinematic you know free surface boundary condition and then dynamic free surface boundary condition. If I apply this to free surface condition then from this ϕ' get to know what is my eta that is the wave elevation for the diffracted potential.

So, the moment I get η from the ϕ . So, I know my what is η and also I know my $\eta = \eta_d \cos(kx - \omega t)$. I already I know my eta is something on $\eta_d \cos(kx - \omega t)$ this is already I know and this from η also. I can get this η also after solving this kinematic free surface condition and dynamic free surface condition once I get the solution for ϕ right.

When I get it from this ϕ also I can get some eta I can say $\eta = \eta_d \cos(kx - \omega t)$ and if you superpose these two. I can get the somehow the exact weighted surface this is complicated so we are avoiding it. So, so here when I talked about the non-linear pressure this S_b is not based on the actual weighted surface it is the weighted surface under the incident wave potential.

So, S_b is nothing but the weighted surface under incident wave ok. So, this is how we actually get the S_b right, fine. Now if you understand this one that how I can obtain the P L that linear pressure term and how I can obtain the P^{NL} ; P^{NL} is basically that. So, I what I am getting that I am finding out the pressure at each point ok fine.

So, when I get these two things. So, this P linear should be integrated on the weighted surface the linear weighted surface is 0 and this P^{NL} should be integrated on the non-linear you know it is the it is the exact weighted surface under incident wave potential both are integration. So, see you know that P^{NL} is nothing like you have the panel ok.

How numerically we do that we are definitely going to discuss in this algorithm later on. But at least now you should know that that how we get the P^L ; P^{NL} it is if this is my ship and if I discretize in the panels. So, this pressure should be at the centroid of the each

panel. So, that is how we can get the P^L . Now this panelling should be if it is under the $Z=0$.

So, then I can call it is coming over here and now if you take some this also I am just making a little bit bolder and over this we can get it is the on the P^{NL} ok. So, now, today let us stop at this point ok. And from the from next class onwards how we are try to figure out how to find out this even if we need to find out this exact non-linear weighted free surface under instant wave. And how to split the panels right it is a panel method so, how to deal with this all these things we are going to discuss in the next class.

Thank you.