Numerical Ship and Offshore Hydrodynamics
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Lecture - 05
Seakeeping - 4

Welcome to the numerical ship and offshore hydrodynamics. Today, we have the lecture 5.

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So, today, we are going to discuss this - the following topics. First we give a brief overview of the flow chart. If you remember in my last class, what we have discussed that if I have a differential equation, then how to get the velocity and displacement. Now, today, we just going to discuss a brief flow chart about the whole process ok; how to solve the equation of motion and from the scratch.

And then, we are going to discuss something about that frequency domain and time domain solution method ok and finally, we are going to discuss about the RAOs and natural time periods. So, here I would like to say that we touch only the very basic thing that is required for this particular course ok.

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So, let us see the flow chart ok.

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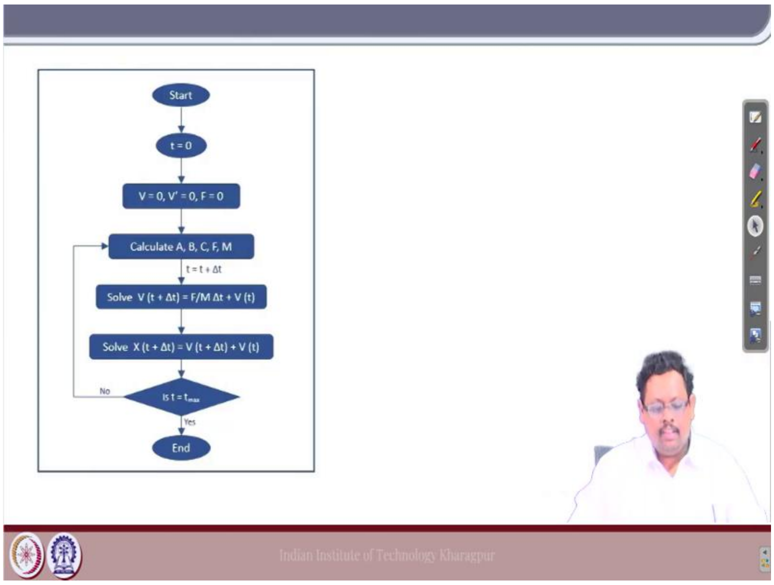

Of course, and these are the keywords that we are going to use to get this lecture. Now, let us get back to the flow chart. Now, at start, you can see that we assume the initial condition that all forces equal to 0 and then, all the velocity equal to 0 and also, the acceleration also equals to 0. So, $V = 0, V^{'} = 0, F = 0$. Now, in this position, what we are going to calculate the A which is the added mass the hydrodynamic force, the damping force B, the restoration force C.

In fact, this C actually you do not have to calculate, each steps unless it is in a non-linear problem or for A and B also. Essentially, what we are going to calculate is the exciting force at each time step F. And after that actually we can write the total force. We are going to discuss in a later on data stage of today that how we are writing the total forces and then, we solve this total force.

If you remember in the last class which is $M\ddot{x} = F$ and then, we are getting the velocity and the next time steps and then, we are getting the displacement or next time steps. So, then, we can increase the time level t and again, again you can go back and again, we can calculate the added mass B, C, F, M in case of a non-linear problem and in case of a linear problem, only the exciting forces ok.

So, today, let us do some work using a piece of paper. Now, we are going to discuss about the equation of motion.

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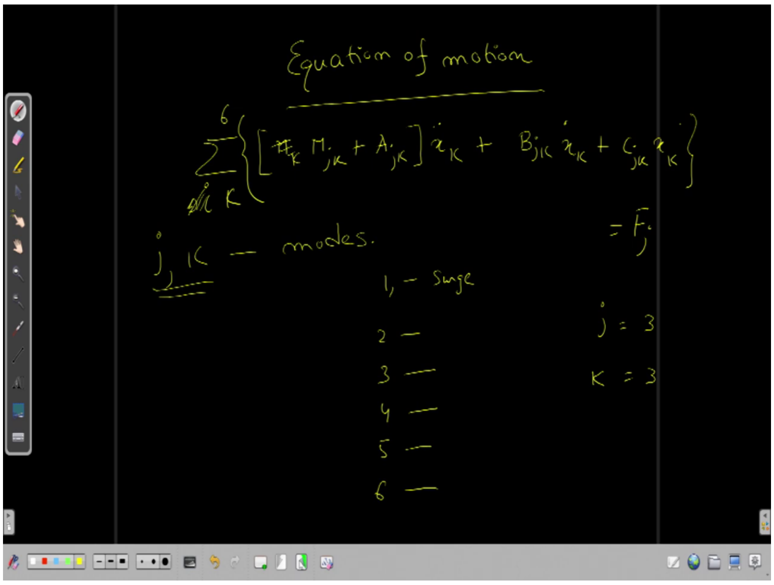

Now, if you remember I said just few minutes back, there is a two way of writing it; one is in time domain and one is a frequency domain. Before that let me write the equation of motion that actually we discussed in my first slide. It is a coupled equation motion. We have the summation. Let us say ok any indices, does not matter; you can just start with anything ok; k. I think this is the k, we are going to use there in the first slide.

So, I am just having the uniformity. So, it can be anything i j k does not matter right. So, it is if you remember this $\sum_{k}^{6}\left\{\left[M_{jk}+A_{jk}\right]\ddot{x}_k + B_{jk}\dot{x}_k + C_{jk}x_k\right\} = F_j$ which is the exciting force. Now, this is actually how we are writing the equation of motion. Now, here it is a coupled equation.

As I said, we did not discuss about what is the meaning of this j and what is the meaning of the k; I mean why it is here in this equation right. We are not discussed. Today also we do not discuss this very much; but we understand these are something called the modes right and now, you remember that we have six modes right. One then 1 refers for the surge right; then 2 refers for the sway; 3 refers for the heave; 4 refers for the roll; 5 refers for the pitch and 6 refers for the (Refer Time: 05:30) we know that.

Now, let us do one thing here; let us not go into all six modes, let us pick anything. Let us pick let us say you know $j=3$ and then, $k=3$. Now, if you use only $j=3$ and $k=3$ in this equation, let us see what you are getting. Now, remember I am using $j=3$ and then, $k=3$.

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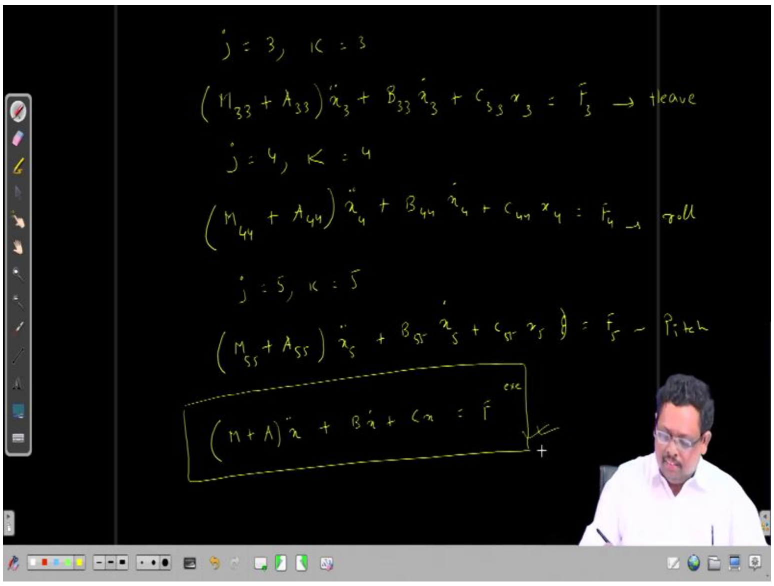

So, there is no summation form because we have only using j and k. So, then this equation takes the form $(M_{33}+A_{33})\ddot{x}_3 + B_{33}\dot{x}_3 + C_{33}x_3 = F_3$ right and then, actually this we can say is the equation for heave.

Now, similarly, if you put $j = 4$ and then, $k = 4$, then also you can have an equation $(M_{44} + A_{44})\ddot{x}_4 + B_{44}\dot{x}_4 + C_{44}x_4 = F_4$. This is equation for roll and then, if you put j equal to 5 and k is equal to 5, now why I am saying this 3, 4 and 5 because these two are most important in sea keeping. So, then we have $(M_{55} + A_{55})\ddot{x}_5 + B_{55}\dot{x}_5 + C_{55}x_5 = F_5$ right.

In general, now let us start you know and this is of course for the pitch. Now, let us not use any heave, roll and pitch in general. So, if I omit these modes and then, in general, we can write the equation motion in single degrees of freedom right. See here, I am using single degrees of freedom that is why using j and k both same 3 for heave and then, for the roll, it is 4 and for the pitch it is I take 5.

So, in general, you know we can call these as single degrees of freedom equation of motion and just you can write in this form. We just we are not going to write any kind of subscripts; only just simple equation $(M + A)\ddot{x} + B\dot{x} + Cx = F^{exe}$ and we can call F^{exe} exciting force. Now on, we are just unless we do the coupled equation of motion, we are going with this simple expression of the equation of motion ok.

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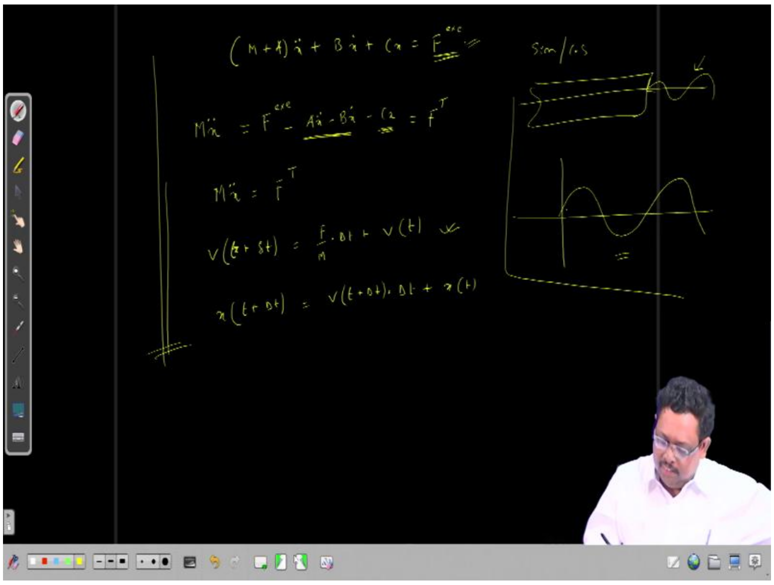

Now, now let us write it again. Now, what we are going to do is I do not assume this to be a time harmonic; that means, what I am trying to tell you that this exciting for, now just let us discuss little bit of physics like you know if you have a ship over here and you know initially, it is perfectly fine. Like it is statically very much stable like weight is equal to buoyancy and it is stands still and suddenly, you know the wave hit the structure and because of this disturbance, this ship start oscillating or offshore structure start oscillating.

So, therefore, this wave forcing, you can call the exciting force right. Lot of things can happen. Again, we are going to discuss later on, but not right now. But what I am trying to tell you if you look at this nature, you know it is you can understand that wave that disturb the structure, I am assuming it to be harmonic function right. So, sometimes explicitly we assume this function to be harmonic; sometimes we really do not explicitly use the exciting force or the term which is disturbing the whole phenomena is a time harmonic ok.

Now, let us take that we do not assume explicitly this is time harmonic right. So, we are not writing this in terms of any sin function or any cos function. We simply say it is a F^{exe}, a exciting force; it could be harmonic, may not be, we do not know. Then, how I solve this problem? Then, what we do is we take this $\mathrm{M}\ddot{\mathrm{x}}$ which is the inertial force, this side and you take all other force in the other side of the equation. So, here I can write it is $\mathrm{M}\ddot{\mathrm{x}} = F^{exe} - A\ddot{x} - B\dot{x} - Cx$.

Though we understand this, simply again this is a linear because otherwise you really cannot write C into x right. Anyways now here you know this, we call as a radiation force that we discussed right and this is a restoring force. So, this you can call this as your total force safety. So, that is what I say in this the flow chart, so this is actually I mean to say that time the $\mathrm{M}\ddot{\mathrm{x}} = F^{T}$.

Now, once we have this, so at that time, then we can get that velocity at next time step, $V(t+\delta t) = \frac{F}{m}\nabla t + V(t)$ and then, we can use the $\mathrm{x}(t+\delta t) = V(t+\delta t).\nabla t$. As you know that I said this is we do for the explicit, but we can do here for implicit scheme plus $x(t)$.

Now, this is how actually we are going to do deal with the time domain method. But in case of a frequency domain, what we do is we explicitly assume that the forcing function which is the right hand side is a time harmonic function.

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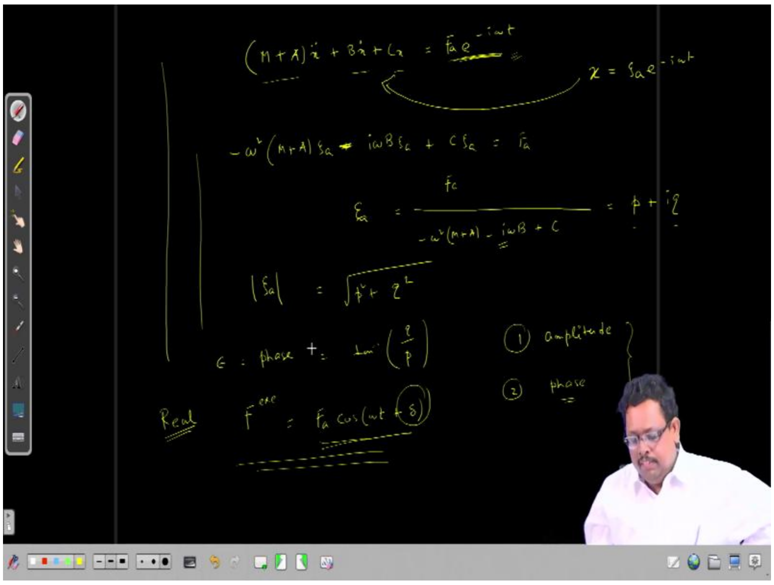

Now, what is happening in that case, we assume that the expression is same right? the left hand side is still remains same $(M+A)\ddot{x}+B\dot{x}+Cx$. Now, in the right hand side, I am assume it is a; it is a harmonic function. So, it has some amplitude $F_a e^{-i\omega t}$; you can take it is $F_a e^{i\omega t}$ also and I take it minus i; it does not matter, you can take anything.

Now, if you assume this you are forcing term to be harmonic, we expect the response also harmonic. So, in that case, we assume that my response x that is also a harmonic function and then, we can assume this x equal to some amplitude multiplied by minus i omega t term.

So, now I substitute this in the left hand side and once you substitute this in the left hand side, what we get? We get we can check it is $-\omega^2(M+A)\xi_a - i\omega B\xi_a + C\xi_a = F_a$ because $i\omega$ terms will cancel out from the both side right.

Now, you see here the solution is little bit, you can see from at this expression the solution is little bit straight forward because you can take that $\xi = F_a$ and then, divided by the whole thing which is $-\omega^2(M+A)$ $-i\omega B$ and then plus C right.

Now of course, now you see it is in a complex domain. So, of course, this should be is equal to some complex number. So, I just write $p+iq$, where p and q both are real because when you solve this, definitely you are going to get a complex number right.

Now, if you get a complex number, then definitely this I take a mode which is get the amplitude of my response right.

So, which is the $\sqrt{p^2+q^2}$ and definitely, I am going to see the phase right and we define this as ε and this is nothing but the $\tan^{-1}(q/p)$. Now, you see once we solve the whole thing, we get two things; one is the amplitude and then, the second thing what you get is the phase right.

Now, suppose if you use it you know now you can say that where is that phase actually are just if I use this complex domain, the phase directly comes in comes here right. But you know remember if you not very much you know very much what I said that comfortable with this with the complex thing, you can assume this to be a harmonic function in sin or cos function also.

But remember in that case you have to you know consider the phase also. So, in case of a you know in a if you do not just you want to deal with only the real part of this or the sin function cos function explicitly; that means, you want to say that F^{exe} exiting force equal to some F_a into sin or $\cos(\omega t)$ ok.

Let us take $\cos(\omega t)$. So, then, you have to add explicitly the phase delta and again, $F^{exe} = F_a \cos(\omega t + \delta)$. You repeat the same process and again, you come up with the same thing ok. Because now, you see like in the right hand side, you have the $\cos(\omega t)$ term and $\sin(\omega t)$ term with the phase epsilon, ε and here also you can get $\sin(\omega t)$ term and $\cos(\omega t)$ term and therefore, you can just compare the sin term and cos term will get the same thing that we are getting using the complex number.

Why are we using the complex number? In complex number, we do not have to explicitly find out the phase. It is comes automatically; but in case of a, if you deal with the real number, you have to consider this delta, δ. So, just give a small exercise you try to do this and then, you just say and check that if you are again coming to the same problem or not ok. Now, what we are going to actually get from this exercise ok? Now, you see what we are getting here, we are solving this problem.

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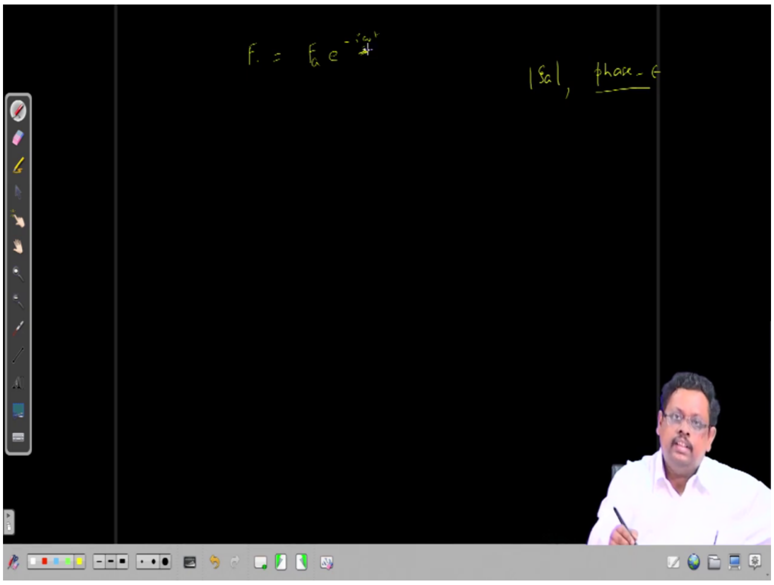

So, we are assuming my forcing function, $F = F_a e^{-i\omega t}$, F is equal to we have some amplitude with some frequency ω_t right? Now, once we have this frequency ω_t and then, we are getting some amplitude, we call this as a ξ_a and not only that we are getting some phase ε, right? So, that means, if you; that means, what I am going to say is that means, if you have a forcing function with respect to a frequency ω, so. So, what we are actually getting with all this exercise ok.

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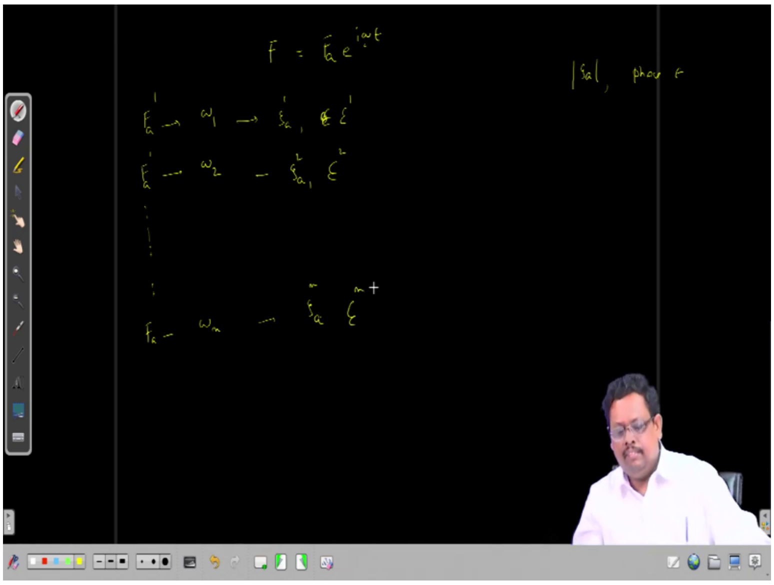

Now, what actually do let us see we are using a forcing function F with a amplitude F_a and then, you know it is a harmonic function with some frequency ω right. So, this is my forcing function. So, you can assume this forcing function. Right now, you assume this forcing function as your waves right; still we are not do any study on the waves and all. So, at this moment assume as a harmonic forcing function right with a frequency ω.

Now, if you do that we expect that there is a response right and what is the frequency of the response? Definitely is ω and then, we have this response have a amplitude ξ_a and also it has a phase ε. So, that means, let us say we get ω_1, a forcing function is let us say F_a^1 with ω_1 and then because of that we get some amplitude ξ_a^1 and also the some phase if ε^1. So, that is we are getting.

Now, similarly, we can let us take the amplitude always same F_a^1; but this ω is some another frequency ω_2. Definitely, we are going to get a different response; even that response should be different that amplitude definitely going to be different and therefore, the phase also could be different. So, we keep doing it for n number of frequency. So, making it ω_n and then, we have this ξ_a^n and then, let us say it is ε^n. Now, we let us plot it. Now, if we plot this what we get?

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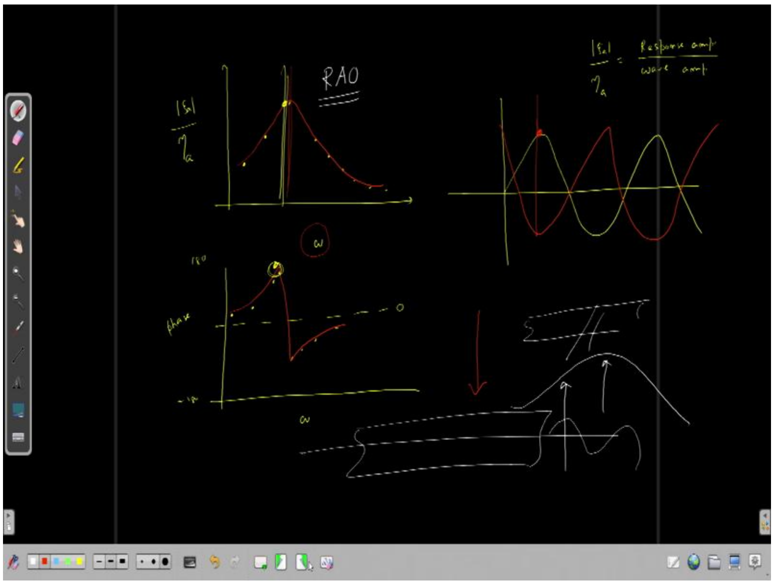

So, let us plot this graph. Now, in this x axis, we have this frequency omega, where the ω_1, ω_2, ω_3, ω_n all are there and here, we are plotting the amplitude and sometimes what we are going to do is; not sometimes, most of the time, all the time let us say we divide this by the wave amplitude.

So, we are going in the y axis basically the response amplitude divided by the wave amplitude. So, we can say that it is response amplitude divided by the wave amplitude ok and then, we plot the point. So, now, let us say for ω_1, this is the response; for ω_2, this is the response; from ω_3, this is the response; then ω_4, this is; ω_5, this.

So, we are going from higher to lower, right? and then, actually we can get a nice plot like this and this we call as RAO. This is the response amplitude operator or sometimes, we can call this as a transfer function also. Because it is the frequency domain representation of the amplitude; but this is not all everything because you know we have to plot the phase also similar way ok.

Now, what we are going to do is in that case, in the left hand side, again we have the ω and then in this side, we have the angle; the phase, right?. So, here let us take the phase is 0 and then, it is let us say minus 80 and then it is a plus 80; 180 and then, we again try to plot the points; here, here, here, here, here, here, here, here, kind of this and then, we just add it. Now, you see, this is the only thing that we want; all this calculation of the added mass, the damping, everything that final aim to get this two graph with different frequency what is the my response and what is the phase.

Now, this two gives you know very important information about the ship. What - It will tell you that at which frequency, the ship might have the peak response right. So, in this case, let us say here and let us say that moment of time, what is the phase? Now, here let us say it is here. So, we can see that at this peak point, now let us try to you know physically get the meaning what is actually happening. Here when the response is maximum, we can see that there is a 100 degree phase lag.

So, what is happening it means that if I try to plot this graph and this phase is with respect to what? With respect to my forcing function, right?. Now, or the η. Now, here now, this is the my forcing function or some this and let us take the response is 180 degree phase. So, this should be the kind of thing.

Now, you assume that when you know it has exactly the opposite phase, so it means that when this wave is trying to go up, the ship is you know it is I mean ok that is the most realistic thing. When what I try to say is when let us say this is the mean level, now when this wave when this wave is going to going up, this fellow the ship is try to go down.

Now, you see that what is going to happen because of this. Now, if you think physically, this what is going to happen is you can have a slamming; that means, that bottom can hit the surface and sometimes, you can have the green water also. What is the meaning of the green water?

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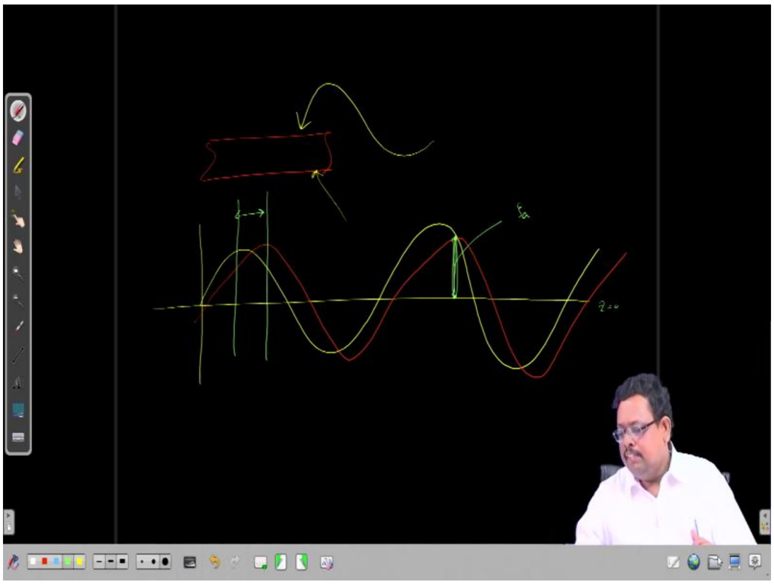

Because when it comes down and then, waves is piled up and then, it can also hit the surface; I mean sorry hit the main deck. So, therefore, two both the things are possible; it can hit here, it can hit here and then, you can have a severe thing. So, that is why this is very important to understand not only the amplitude of the motion, but also the phase.

Now, you know it is for frequency domain, it is very easy; how you can calculate the phase because you take the amplitude, you can get the amplitude, you get $\tan^{-1}(B/A)$ or $p_a q/p$ you will get the phase. But in case of a time domain, how we are going to fix this? Now, in time domain what at the finally, what you are getting?

You get a signal; a time domain signal. Now, if you assume I mean if you assume or not, if the forcing pass is harmonic, of course this would be the graph for the forcing function right and then if you calculate the motion, then definitely you can see that motion has this is the signal for the motion. Now, here, clearly you can see there is a difference;

difference between the peak right?. We are definitely going to discuss later on like how to calculate in time then, but he just given an idea. Now, this will give you the phase information and of course, the distance from z equal to 0. Now, I mean this is I mean that $z = 0$ whatever you can see.

Like the distance from this, this gives you the amplitude ξ_a of the response amplitude. Now, this is how we can find out from the time domain signal. We get the time signal, we get the signal of the forcing function and then, we can find out the difference from the difference of the peak time and also, from 0 to this, we can get the value for the amplitude.

So, you see like even if we use the time domain if we use the frequency domain, definitely all the time you can able to draw the RAO as well as the phase. So, today, we are going to stop here ok and then, from the next class, we are still continuing with the same discussion and we will try to find out at you know at which frequency the- this response could be maximum; how to gauge that frequency right which is nothing but the natural time period ok.

So, till then, thank you very much.