Numerical Ship and Offshore Hydrodynamics
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Lecture - 55
Hydroelasticity (Contd.)

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Hello welcome to Numerical Ship and Offshore Hydrodynamics, today is the lecture 55. Now, today we are going to continue that what we have discussed in the last class, in the last class we have discussed some part of the structural analysis FEM; so, today we are going to continue the same. And these are the keyword that you have to use to get this lecture ok; so, let us start.

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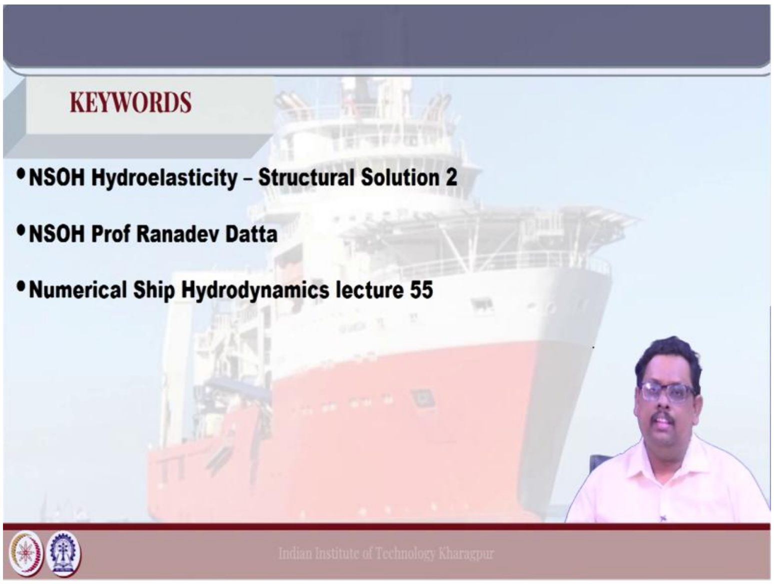

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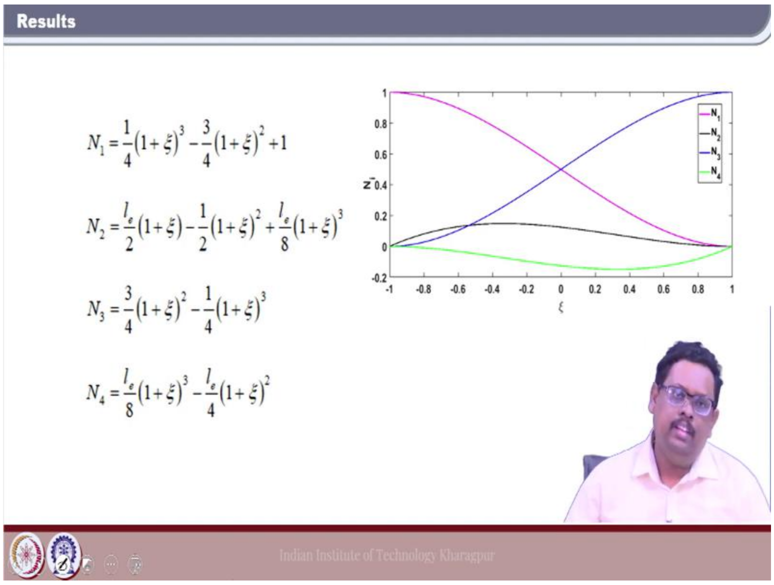

Now, if you remember in the last class we stopped at this point where we actually define how I mean how to get the shape function. Now, here looking at this expression you may be confused because its different from -1 to +1. So, let us not bother about this, this is actually representation of the local coordinate system, I am just going to show you that how cubic polynomials look like and how the shape functions are look like right.

Remember that what we have discussed there is in the global reference frame that is why use x_1 , $\bar{x}$ etcetera. But again, when you do the integration of numerical integration where we are going to see today. So, that time while you need to do that you need to again transform back to this local coordinate system -1 to +1; so, that part let us leave for this moment.

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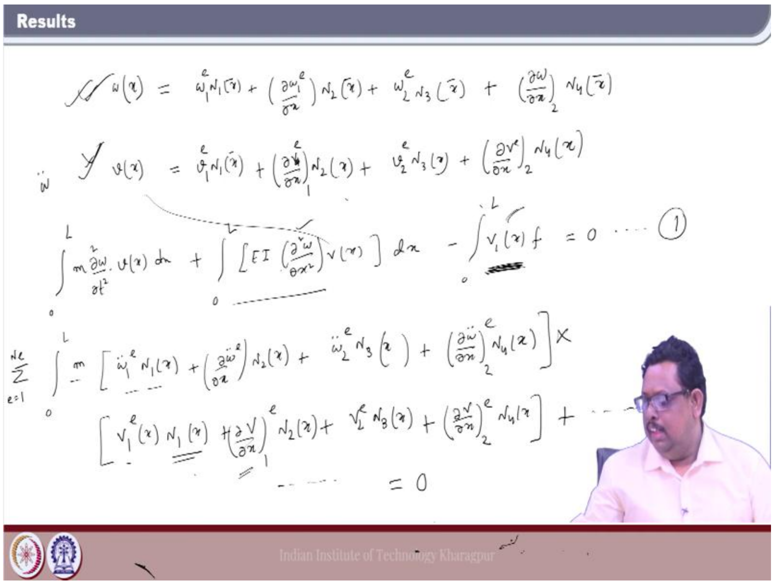

So, if you remember in the last class, we have defined the shape function $N_1, N_2, N_3,$ and N_4. And then, we write our global displacement or wx this equals to in terms of a shape function right. So, in terms of a $w_1 N_1(x)$ and then plus of course, for the element e, let us not forget the element t. And then it is $\left(\frac{\partial \mathrm{w}^e}{\partial x}\right) N_2 x$ right plus again, now it is for the second element $w_2^e N_3(\mathrm{x})$.

And then plus which is the $\left(\frac{\partial w}{\partial x}\right)_2 N_4(\bar{x})$; so, this is how actually we are we wrote the displacement. The similar way that the test function also we can write that v x that is the test function that we have used in the last class. So, that also you can write as some v 1 into of course, element $\mathrm{eN(x)} + \left(\frac{\partial \mathrm{v}_1}{\partial x}\right)$ the slope for the element e del will be 1 for the element e. Again, it is $N_2(\mathrm{x})$, then it is $v_2 N_3(x)$; of course, for the element $t + \left(\frac{\partial v}{\partial x}\right)_2^e N_4(\mathrm{x})$; now, this is my that that how I approximate.

Now, if you want like you can use also the $\bar{x}$ etcetera because these are the approximate solution ok; here, you can use the $\bar{x}$ also I am not I am leaving all these things right. Now, the next job is substituting this w and v in the original that weak form solution. Now, what is the weak form solution if you remember it is 0 to L into m into which is $\left(\frac{\partial^2 w}{\partial t^2}\right)$ right.

And we multiply this with my shape function $v(x)\,\mathrm{dx}$, this one and then $+\int_0^L\left[EI\left(\frac{\partial^2 w}{\partial x^2}\right)v(x)\right]dx-\int_0^L v_1(x)f=0$. Now, this is actually my formulation and you can just if you want to mark a mark equation let us say 1 something like this. Now, what you need to do is, you need to replace here you need to replace all the, I mean the w and v ok.

So, now if you substitute this wx and vx in this equation 1 then what you get. So, you get

$\sum_{e=1}^{Ne}\int_0^l m\left[\ddot{w}_1{}^e N_1(x)+\left(\frac{\partial\ddot{w}^e}{\partial x}\right)N_2(x)\right]+\ddot{w}_2 N_3(x)+\left(\frac{\partial\ddot{w}}{\partial x}\right)_2^e N_4(x)$. Now, here I am just I am not going to only for mass matrix I am actually selecting I do not split the other transits not necessary in either similarly you can break these two. But, now let us see that here you need to multiply by the test function V also.

So, then it is $\left[v_1^e(x)N_1(\mathrm{x})+\left(\frac{\partial V}{\partial x}\right)_1^e N_2(x)+V_2^e N_3(x)+\left(\frac{\partial V}{\partial x}\right)_2^e N_4(x)\right]$ so, this is how I write. Now, what I do over here, now I take this common ok; of course. Because, I need to do this integration in for all element right and then plus if you want you can write the other term then make equal to then you make this equal to 0.

I am not writing the other terms ok I am just writing this the mass term, similarly you can substitute here the same thing this equation you can substitute over here and also this equation substitute over here and then you can do the things right ok. So, having said that I am having this expression ok; now, I am writing this expression I am taking that v 1 x common ok.

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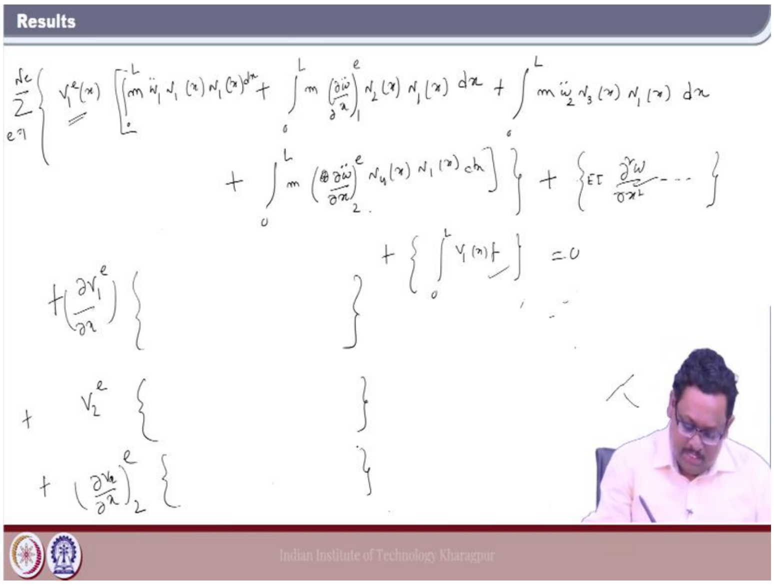

So, if I do that now my now next job is basically, I am taking this $v_1^e(x)$ I mean I am taking this common and I let me write the other stuff ok. So, if I take it common here $v_1(x)$ this multiply with this element right. So, I just write over here; so, V 1 x equal to $\int_0^L m\ddot{w}_1 v_1(x)\,\mathrm{N}_1(x)dx$ right.

Now, we take the next term of course, I should not forget that 0 to L into d x it is always there right. So, and then again, I just write 0 to L; now, I take the next term which is $\int_0^L m\left(\frac{\partial \ddot{w}}{\partial x}\right)_1^e N_2(x)N_1(x)dx$ now similarly you can write the other two term also right.

So, here let me write again the third term which is 0 to L and remember of course, it has like e equal to 1 to Ne of course, it is there right and then it is under this bracket. So, now, again if I take $v_1(x)$ common next term will be $\int_0^L m\ddot{w}_2\,\mathrm{N}_3(x)\,\mathrm{N}_1(x)dx$.

And then the fourth term remains which is nothing but $\int_0^L m\left(\frac{\partial \ddot{w}}{\partial x}\right)_2^e N_4(x)N_1(x)dx$ right. So, then the x; so, I close this and then I mean I should not close this for this because we have another term. Again, the similar term can be written for this $EI\frac{\partial^2 w}{\partial x^2}$ into.

So, for that also you need to do that and then also you need to do that for another term which is nothing but integration 0 to N here I guess; so, another term $\left\{\int_0^L v_1(x) f\right\}$ like this equal to this. So, you have to do all this term also right, I am focusing on only this and not also the focusing fully I am writing this for $v_1(x)$. Now, similarly basically I can do the same for $\frac{\partial v_1^e(x)}{\partial x}$ also, if I take common and then again, I can have the similar term right, is it not.

And then plus I can have this v_2^e; again, I have the similar term and then I have this $\frac{\partial v_2}{\partial x}$ right for the element or you can say 2 over here e and again you can have some similar term right. It is very easy it just you have to be patient and you have to write the term; so, I just take common for v_1.

Similarly, you can take common for this the second term which is nothing but del V and del x right. And then you try to figure out what other term left and you can do all these things. Now, idea is after having this all exercise what you are going to have as follows.

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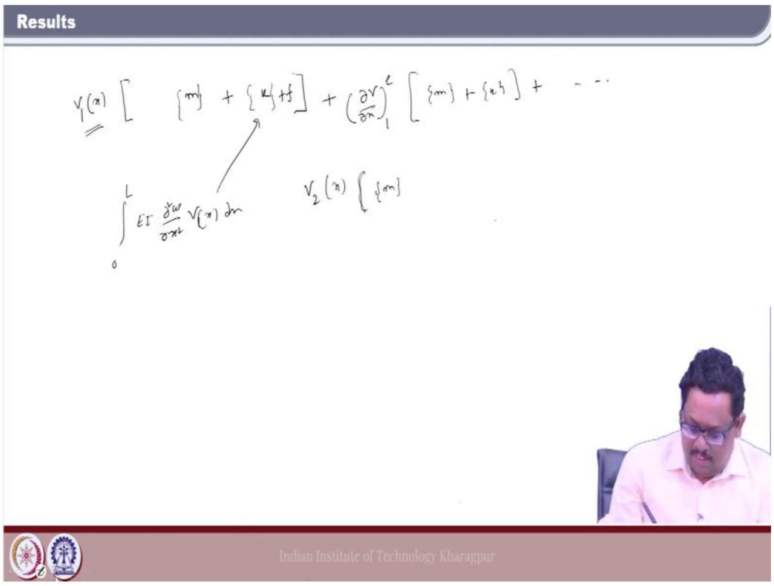

Now, after you do all these things then actually what you are going to have is, now for in case of a mass matrix you have some term with you know $v_1(x)$ V 1 x and if you take a common you have term some terms that is the part of the matrix m right. And then you have some term, which is again coming from the stiffener term that the second term.

So, let us call as a KI am just writing in this way ok; similarly, if you take common like $\frac{\partial v}{\partial x}$ and it is 1 e. And you do the same thing right as I said you substitute everything in the second integral like that integral is nothing but in $\int_0^L EI \frac{\partial^2 w}{\partial x^2} v(x) dx$. So, in this here if you split again w and the V that with the test function and again you take everything common then you have some component with if you take a common v N (x) you have some component definitely here.

Similarly, in case of here also you can have some component this m right and then some component again having with the K and so on; so, again you have taken $v_2(x)$ common and then you have like this m. In fact, you have some term from the external matrix f also right. Because, here also sorry here in this equation also if you substitute this $v_1(x)$ with this definition definitely some component you are having with this f matrix also right.

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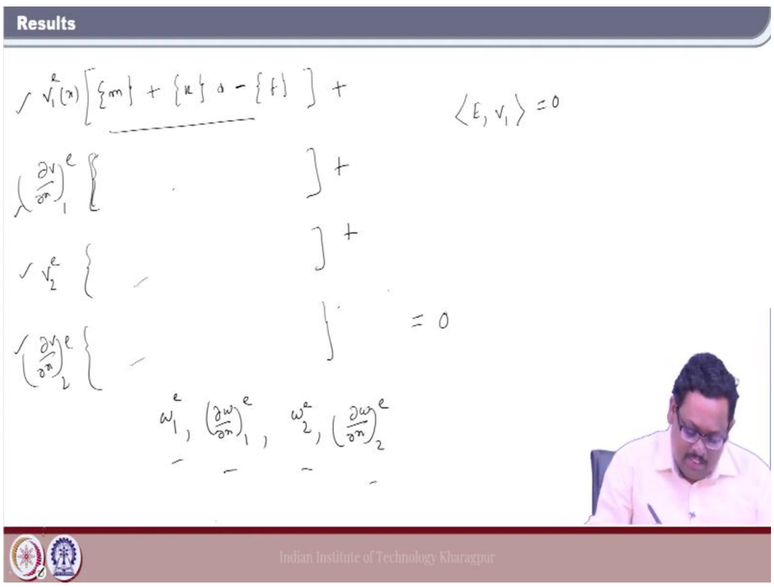

So, frankly speaking ok, let me write a refresh it again; so, if you take this common $v_1(x)$ then you can have some component we write as m some component I am writing as K and minus some component as in having a f something like this you can have right. And similarly, if you take $\left(\frac{\partial v}{\partial x}\right)_1^e$, for element e again you have similar these 4 components right. Then plus if you take this v_e^2 again you can have this component, and if you take $\left(\frac{\partial v}{\partial x}\right)_2^e$ again you are having this component that is equal to 0.

Now, my now initially if you remember I said that orthogonal property under this orthogonality, I call that this error function and this test function v_1, it the norm equal to 0; so, that is the idea. Now, if these are independent to each other; now if this has to be 0, then all this term associated with this should be equal to 0. So, if I do that then actually you can have 4 equations, you have one equation for this one equation, for this you have one equation, for this and you have one equation for this right.

So, this is the idea now you see I have a 4 unknowns, my 4 unknowns are what my 4 unknowns are w_1^e, $\left(\frac{\partial w}{\partial x}\right)_1^e$, for each element, then I have w_2^e, and then I have $\left(\frac{\partial w}{\partial x}\right)_2^e$ for each. So, I have these 4 unknowns, and now I have individually all these 4 should be equal to 0 right is it not; so, for each element you are having this one right is it not. So, this is the idea if you do that if you do that then actually finally, what you are getting is finally, you are getting a you know a matrix, you have some four matrixes right, I mean if I write the write everything in matrix forms then how do I write it.

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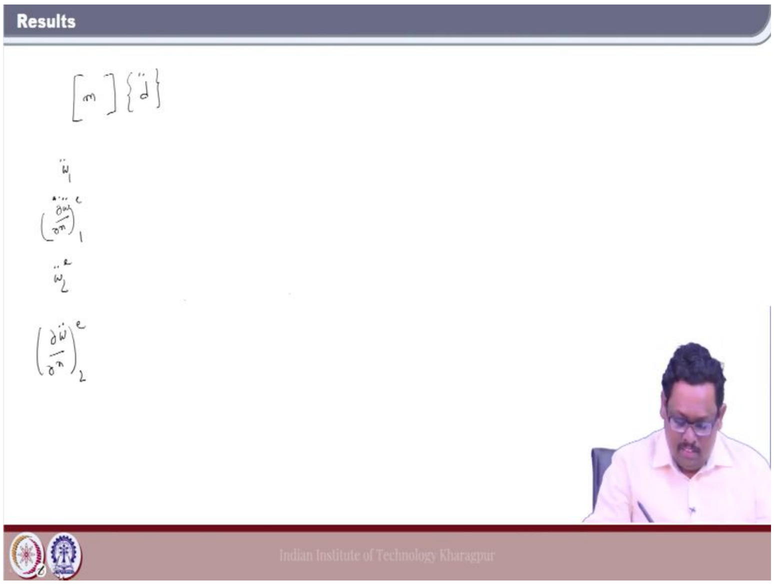

So, if you want to write in a matrix form then definitely you have a matrix m and then it should be it is, now I just write $\ddot{d}$. Because, you see it is del it is $\ddot{d}$ means it is a column matrix, it is $\ddot{w}_1$ $\left(\frac{\partial \ddot{w}}{\partial x}\right)_1^e$, then $\ddot{w}_2$ and then $\left(\frac{\partial \ddot{w}}{\partial x}\right)_2^e$. So, I have these 4 elements right; so, therefore let me rub this thing first ok.

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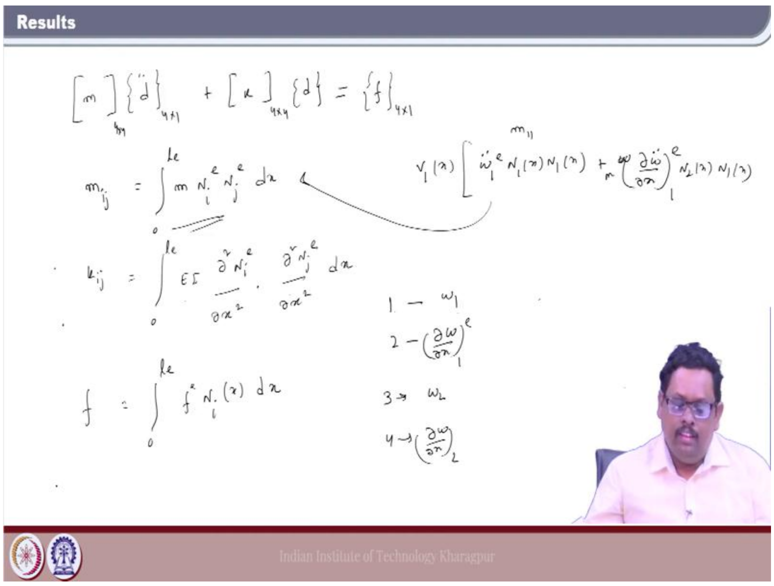

So, therefore, this is actually your 4x4 matrix and this becomes 4x1 matrix and then you have again as I said is a K matrix if you do this other term, it is again it is 4x4 matrix and it is multiplied by the distribution matrix d. And then minus you have this or you can take equal you can take the other side of this which is the f matrix and which is again this 4x1.

So, this is we can call is a element matrix; so, it is a localized, now this is one. Now, here this I you can also confirm that this let me first write it here this element $m_{ij} = \int_0^{le} m N_i^e N_j^e dx$. Now, again this $k_{ij} = \int_0^{le} EI \frac{\partial^2 N_i^e}{\partial x^2} \cdot \frac{\partial^2 N_j^e}{\partial x^2} dx$ ok.

And also, you can write this $f = \int_0^{le} f^e N_i(x) dx$. Now, you see I just I can see that this is same; so, you can just verify the first one here you see, I multiply $v_1(x)$ which is now if I multiply this with this so, what you get ok. Let me write in the right-hand side, I just rub it; so, I just write only one you can understand I just take a common $v_1(x)$ and let me write that what term I can have.

So, first one is $\ddot{w}_1^e N_1(x) N_1(x)$ and you have $N_1(x)$; now, you see you are having this term; so, this is your m 1 1 term right is it not. Now, similarly the second term would be $\left(\frac{\partial \ddot{w}}{\partial x}\right)_1^e N_2(x) N_1(x)$ right and of course, here the multiplied them. Now, you see N_{12}, now your 1 refers for w_1, your 2 refers for $\left(\frac{\partial w}{\partial x}\right)_1^e$, your 3 I mean your 3 refers for w_2, and your 4 refer for $\left(\frac{\partial w}{\partial x}\right)_2^e$. Now, you see you can just break it up you can get this is the compost though you can get this expression right. Now, once you have this local matrix; now, you need to convert this local matrix to the global matrix ok; so, now, how we can do that.

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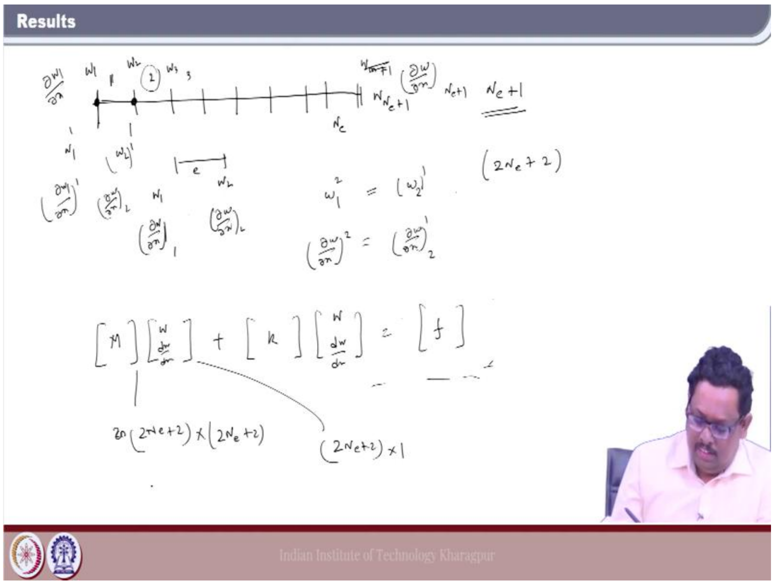

Now, here what you need to do is as follows; suppose, I have now this is the beam. Now, what I am going to do is I am break it up with the N number of element right; now, I break it up the, a in a way. So, I take any e take element e and then I define here it is w_1 , $\left(\frac{\partial w}{\partial x}\right)_1$, w_2 and $\left(\frac{\partial w}{\partial x}\right)_2$. So, here you can see now let us start with the first element, second element, third element; now, if I have the N e number of sections; so, definitely I have $N_e + 1$ number of nodes right.

So, here now what I need to do is, I have N_1, I mean that element 1; here, I have w_1 and then $\left(\frac{\partial w_1}{\partial x}\right)_1$. And here also I have w_2 and $\left(\frac{\partial w}{\partial x}\right)_1^2$ right; so, here I have the contribution from the element 1. Now, when I write for this second the second element; so, here my w_1 and $\left(\frac{\partial w}{\partial x}\right)$ then I have this w_2 and $\left(\frac{\partial w}{\partial x}\right)_1$ is the same node right; so, therefore, I need to have a mechanism to make it globalized right. So, how do I deal with what I do is I make each node as w_1, w_2, w_3 and here; of course, it is W_{N+1} or $N_e + 1$ whatever ok, just make it W_{N_e+1} ok.

Now each node I have 2 degrees of freedom; so, therefore, here also I have $\frac{\partial w_1}{\partial x}$ and similarly I have here at the end node I have $\frac{\partial w}{\partial x}$ which is $N_e + 1$. So, therefore, the total number of the nodal element I have or nodes I have it is $2N_e + 2$ right, $N_e + 1$ for this w and again $N_e + 1$ for the $\frac{\partial w}{\partial x}$.

Now, I need to write the whole thing in the form of there is now there is no element will be there. So, I need to write a global mass matrix M, which is multiply by this element which is w and; of course, $\frac{\partial w}{\partial x}$. Similarly; I just going to write this stiffness matrix K again I have this matrix ∂w and $\frac{\partial w}{\partial x}$ then I have a matrix the right-hand side which is the force matrix.

So, then what is the dimension of this matrix it should be $2N_e + 2$ Xv 2 right and this 1 is nothing but $2N_e + 2$ X1 and so on right. So, now, my job is how do, I write this element matrix; now, for that actually we use some kind of a connectivity matrix right. Now, what is this connectivity matrix let me try to explain.

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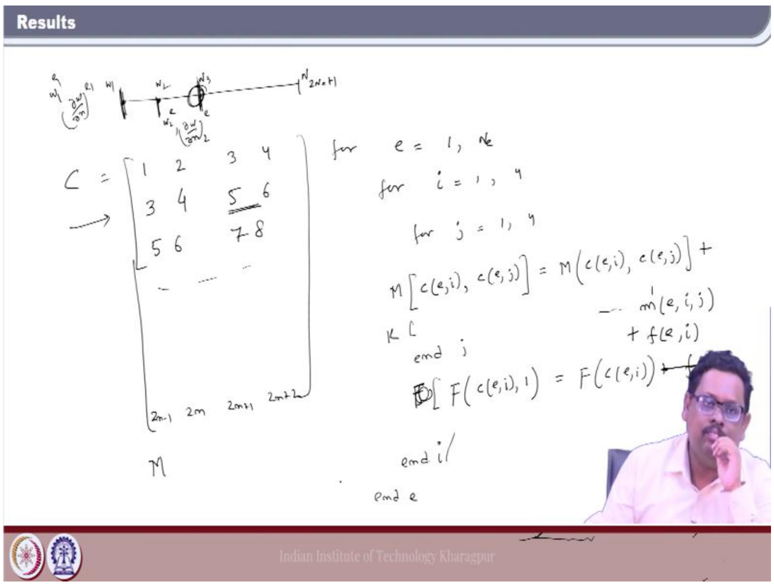

Now, thing is that here this you have this w_1 then w_2; now, these are the global; so, I am not writing e and it is W_{2N_e+1} something like this. Now, I write this matrix connectivity matrix as this is 1 2 3 4. Now, again now you see here this 1 2 corresponding to my element stiffness matrix e 1 and $\left(\frac{\partial w}{\partial x}\right)^{e1}$ right.

And 3 4 corresponding to w_2 and then $\frac{\partial w}{\partial x}$ for the I mean for that same element 1; so, now, I break it up into 4. Now, then effect of this also will be there; so, when I write this second matrix it should be 3 4 5 6. So, this 5 6 again for the third element it; so, when I write for the second element, I am taking the contribution from the first element also.

Similarly, when I write for the third element; so, I need to take this contribution of the second element. So, I am writing here 5 6 7 8 and so on so this is called the connectivity matrix and finally, it should be 2n-1, 2n, 2n+1, 2n+2. So, this would be my connective in matrix; so, I use this connectivity matrix to get this the global transformation the global matrix to the local matrix the global matrix.

Now, here I am just writing a just a pseudo code for you and then it is your job to you substitute this pseudo code in the matrix and you check that whether you are getting this effect or not ok. So, now, this code goes like this way; now, just take e equal to 1 to N e; so, this is the total number of elements.

Now, you have this element twice you have 1 2 4 and then ok I just take 4 this is a MATLAB 4 and then you have j also equal to 1 to 4 and then you are writing I am just writing for the mass matrix and the stiffener matrix and also the force matrix also the same.

Now, I can write here $M[\mathrm{c(e,i)}, \mathrm{c(e,j)}] = \mathrm{M}[\mathrm{c(e,i)}, \mathrm{c(e,j)}] + \overset{1}{m}(e,i,j)$ ok. So, then you can simply your end the j, and then actually because here similarly you can write K of K also the similar the same thing.

So, I am not repeating, for the K also you can write the same thing and again now your global force matrix F ok just $\mathrm{F}[\mathrm{c(e,i)},1] = F[\mathrm{c(e,i)}] + f(e,i)$. Now, it is your job ok; so, this force matrix it is not ok visible, I just write is in just top; so, that you can see that f of e, i ok; now, you can see it right ok.

So, if you use this pseudo code and once you use this pseudo code actually you can get this local matrix into the global matrix ok. So, it is your job to put these values I do not have much time to show you that, but you can take this as a homework and you can check that actually it is happening ok.

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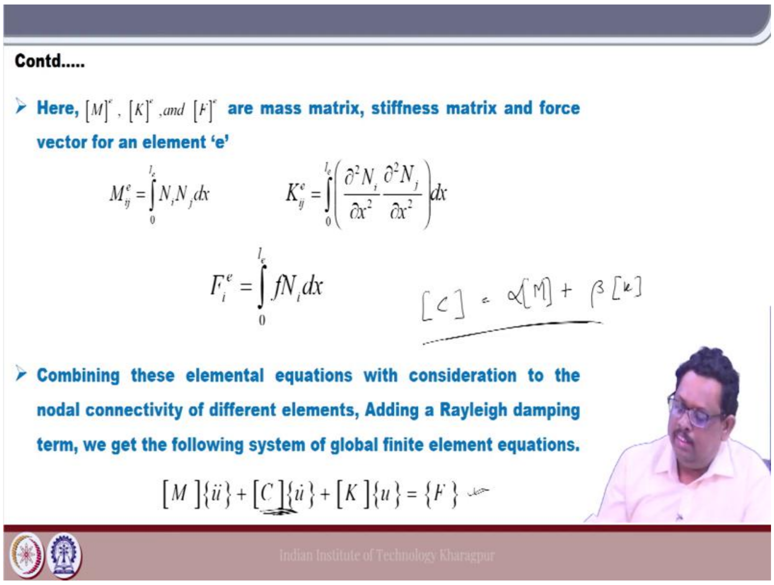

So, once you have this then actually you can get this global matrix over here. Now, once you have this global matrix now you say differentiate ordinary differential equation right. Now, once we have this ordinary differential equation, I already take lot of classes how to get the velocity and displacement from this ordinary differential equation. We are using Euler Scheme, you know apart from that actually you can use Newmark beta method, you can use the Runge kutta method, you know how to solve this equation right.

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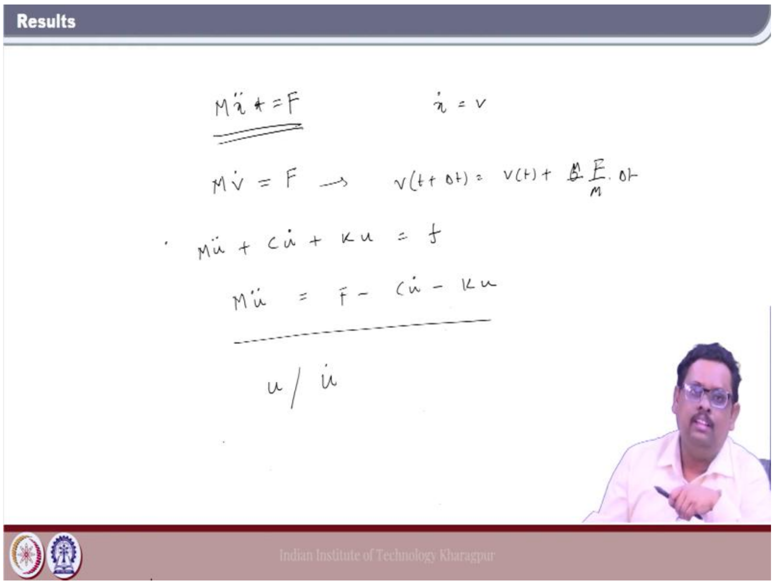

It is very simple as you know that I am just showing you that if you remember we are having this $m\ddot{x} = F$, we solved it quite often like then you take it is 2 degree I mean equation of degree 2 you have to convert for degree 1. So, we take $\dot{x} = v$; so, then we can convert into $m\dot{v} = F$. And then we are using the Euler scheme right we are using Euler scheme we are getting $v(t+\Delta t) = \text{v(t)} + \frac{F}{m} \cdot \Delta t$ and so on.

Similarly, here also you can have $m\ddot{x} + c\dot{x} + kx = F$. Now, here you can see that where this C term is coming, now this damping term actually external you need to add and then lot of literature are available for this you can use anything any damping scheme ok. So, normally we are taking the, this damping matrix the ratio between the mass matrix and the stiffness matrix.

So, we can take this $[C] = \alpha(\text{m}) + \beta[\text{k}]$. So, in this form actually also you can take a damping; so, you can check it out like that how to add the external damping ok. Anyways in fact, inside this you know you just check the Rayleigh damping is the most popular one actually that is what I explain little bit; so, you can check it out.

So, similarly here also instead of that we can write as now here we have $m\ddot{u} + c\dot{u} + ku = f$. Similarly, also here you can simply way you can write this equal to $m\ddot{u} = f - c\dot{u} + ku$ and then you can find out the solution for I mean you can get this placement u as well as the velocity $\dot{u}$ right.

So, this is all about the structural solution of this hydro elasticity problem ok. So, today we are going to stop and maybe from the next class we are going to discuss some other way of solving the same problem. Now, here we are using the finite element, and then in the in next class maybe we can see the same problem, but the solution technique not finite element it may be the modal superposition technique ok. So, with this we stop over here.

Thank you.