Numerical Ship and Offshore Hydrodynamics
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Lecture - 57
Semi Analytic Method (Contd.)

Hello, welcome to the Numerical Ship and Offshore Hydrodynamics. Today is the lecture 57.

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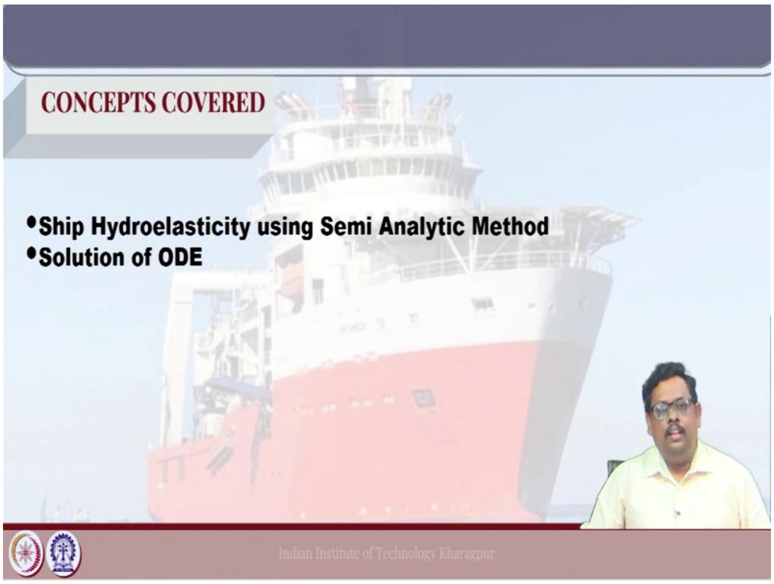

Now, today we are again going to discuss the ship hydroelasticity using semi analytic method. So, today we are going to discuss about the how we can solve the that ODE and find out the structural deflection that amplitude of the structural deflection, ok. And this is the keyword that you have to use to get this lecture ok.

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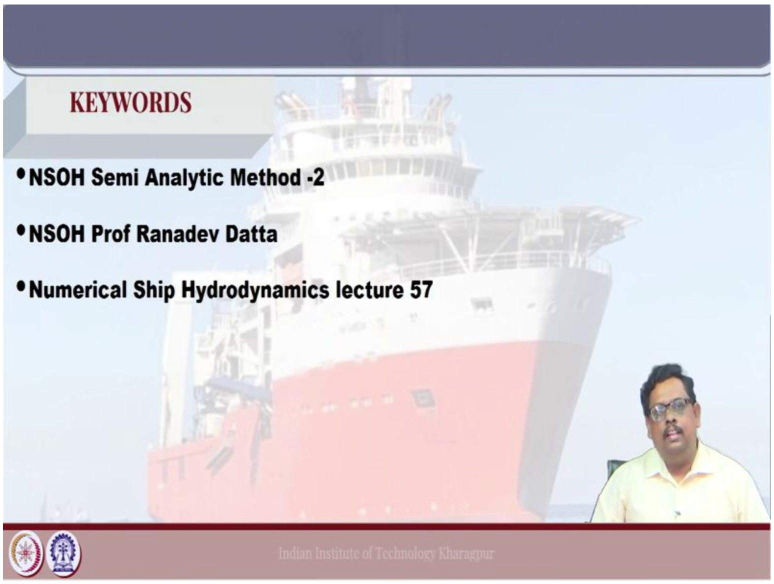

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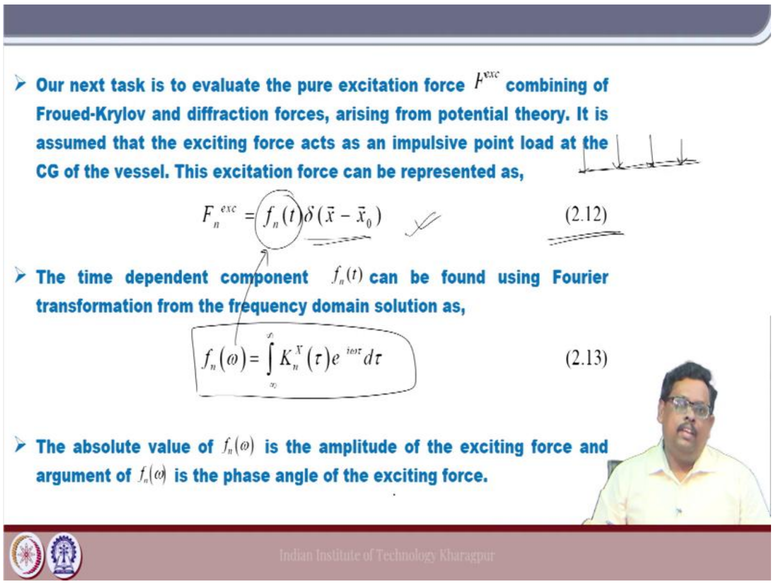

Now, here actually we stopped in the last class. Now, we discussed about how we can get this $f_n(t)$ right, we discussed how we can get this $f_n(t)$ right and, but then we really did not discuss about the second part. Now, this is you as you know that this is kind of Direct delta function now, what is we are assuming that if the exciting force actually it is kind of hammering that each cross section.

Maybe there is not much mathematical base to approximate this, but here the physically what we assume that suppose we have the shape and then wave it is hitting like an impact force in all sections ok. So, that is how actually we thought of right. And then here this is the way I can get frequency domain exciting force.

Of course, that is actually it is required when you write this impulse response functions with method always you record the data from the frequency domain. Now, this $f_n(w)$ is the frequency domain exciting force. So, when you have this, you can get this $f_n(t)$ using the Fourier transformation right. Now, assuming that you have this you have the Fourier transformation you get this $f_n(t)$ and you write this expression $f_n^{exc} = f_n(t)\delta(\bar{x}-\bar{x}_0)$ to write the exciting force.

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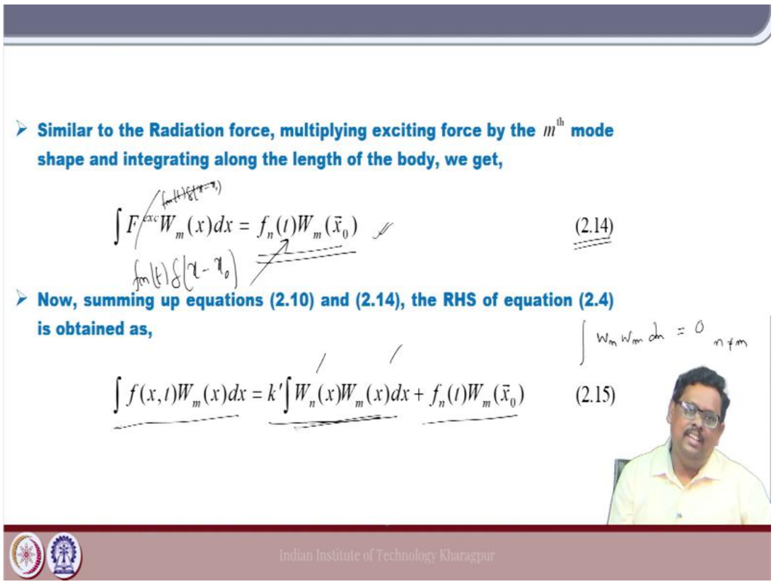

So, then if you integrate along the length l and then this frequency the exciting force is looking like $\int F^{exc}W_m(x)dx = f_n(t)W_m(\bar{x}_0)$. Now, this is the property of the δ function right. So, if you integrate this $F_n^{exc}W_m(x)$ then if you replace this F_n^{exc} as $f_n(t)\delta(x-x_0)dx$ right. So, let me write it little bit bigger way.

It is if you write it as $f_n(t)\delta(x-x_0)dx$, then definitely you will get this quantity. So, it is the physically it is that each section actually you can as if it is coming as a hammer, I mean as I said there is not much mathematical backup for this particular thinking, but somehow this works ok. Now, you substitute this 2.14 into the original the equation, where you have the radiation force and the diffraction force.

So, if you substitute there. So, this is how actually you can write the total the right-hand side of the equation right that right hand side of the that beam equation. So, now, as you know that we are going to use the property of the orthogonal property of $W_n(x)$ and $W_m(x)$ and where we use that and we ignore the cross-coupling term right.

So, therefore, when $m \neq n$ it is 0, then when $m = n$ we have now if you make it orthonormal, then this becomes equals to one otherwise you have the same values right.

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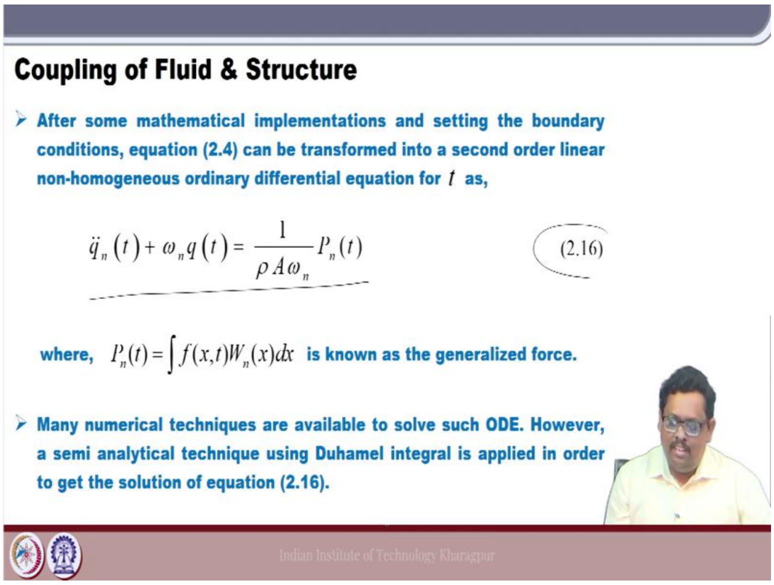

So, this is well known to everybody. So, therefore, if you do this and use this orthogonal property then finally, actually you can arrive at $\ddot{q}_n(t) + \omega_n q(t) = \dfrac{1}{\rho A \omega_n} P_n(t)$, then actually this all the spatial coordinate actually you can remove right. So, when this n not equal to m. So, let us take the let us this is up to you how we are writing this $W_n(x)$ and $W_m(x)$ if you write it in orthogonal way.

So, then this when $\int W_n W_m dx = 0$, when $n \neq m$. And then it is equals to some value k or 1 it depending on whether it is you are using orthonormal mode shape or orthogonal mode shape anyways. Now, this is the part of the structural and I really know I know that you all know this very well and we do not have much time to discuss all these things very elementary right.

So, we can skip it and but if you use this property definitely you will get this equation $\ddot{q}_n(t)+\omega_n q(t)=\frac{1}{\rho A\omega_n}P_n(t)$. Now, here the idea is how I solve this differential equation $\ddot{q}_n(t)+\omega_n q(t)=\frac{1}{\rho A\omega_n}P_n(t)$. Now, numerically you know there are lot of numerical methods are available like you can use the Euler method then Newmark Beta method and then so many methods I mean Adams Moulton method, Predictor Corrector method.

So, many methods are there right, Runge Kutta method, but here we did not use any of these methods, where you are using some of some kind of semi analytic technique using the Duhamel integral. Now, what is again Duhamel integral we are not discussing here you can just browse and you can get the get the knowledge about the Duhamel integral it is very well known. Now, how we use here that is the question.

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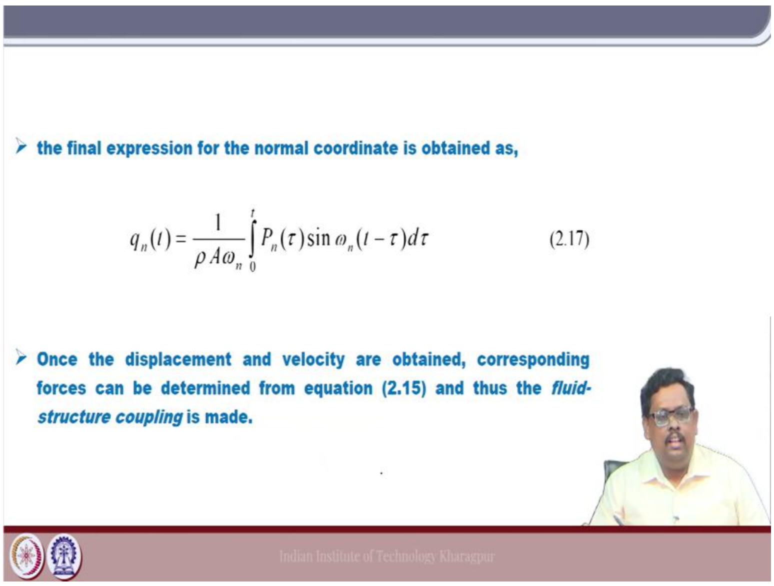

Now, here if you apply this Duhamel integral, then your displacement actually looks like in this fashion. Now, as I said we first of all we have not much time to discuss how applying Duhamel integral we are getting it. So, we can ignore at this point it is really not that necessary, necessary is try to find out this $q_n(t)$ that the value for the displacement right. Anyway, or how to solve this equation $q_n(t)=\frac{1}{\rho A\omega_n}\int_0^t P_n(\tau)\sin\omega_n(t-\tau)d\tau$.

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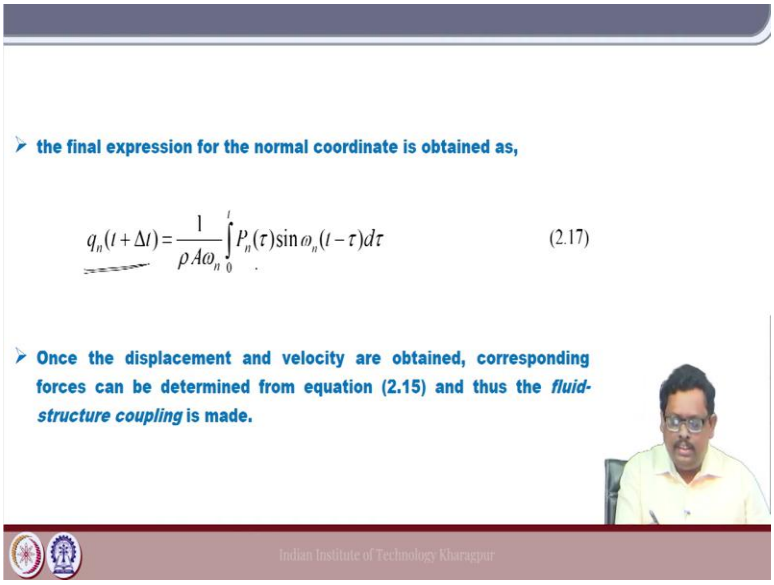

Now, here this $q_n(t)=\frac{1}{\rho A\omega_n}\int_0^t P_n(\tau)\sin\omega_n(t-\tau)d\tau$ we actually solve in semi analytic method. Now, how we solve this semi analytic method, let us see this. Now, here if you solve this actually that we are getting the velocity in the next time step based on the force. Now, if you look at here the force is this $P_n(t)$ is nothing but the generalized force which is the combination of the radiation force diffraction force and sometimes it is the combination of the radiation diffraction as well as the hydrostatics is depending on how we are how you are actually writing the external force ok.

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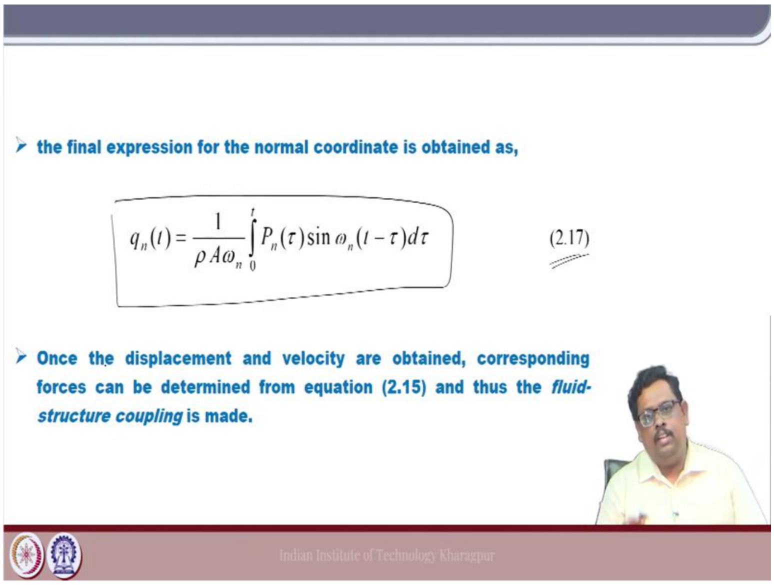

Now, in this particular case we are only taking the exciting force and the radiation force ok.

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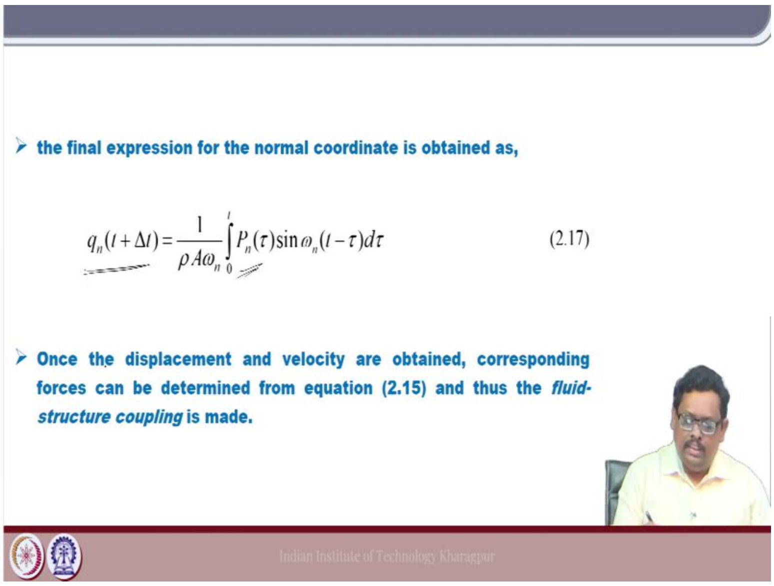

So, now this $P_n(\tau)$ is nothing but the total generalized force right and then ω_n is the frequency and then A is the sectional area and ρ is the density right density of the structure ok.

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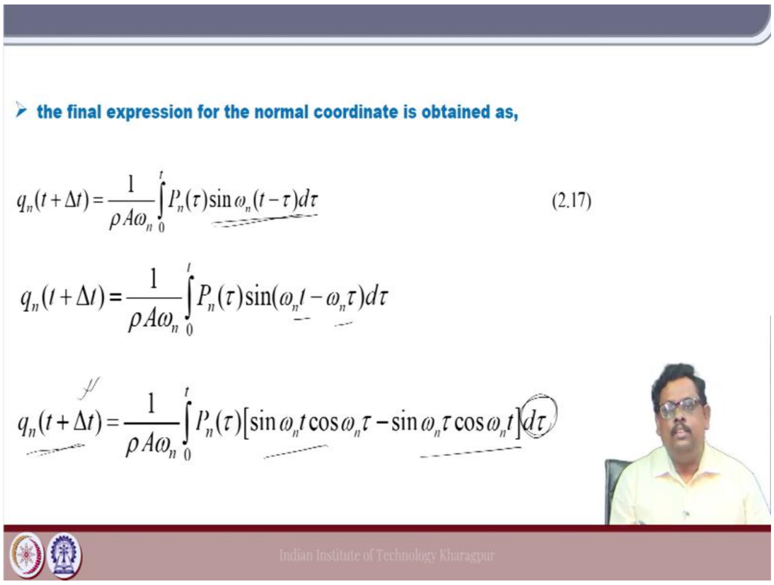

So, now how I solve this $q_n(t+\Delta t)=\frac{1}{\rho A\omega_n}\int_0^t P_n(\tau)\sin\omega_n(t-\tau)d\tau$ we are writing this $\sin\omega_n\tau$ is $(\omega_n t-\omega_n\tau)$ and then I just write into the sin and cos term right. So, now here you can see in 2.7 this $q_n(t+\Delta t)$ actually having two components. Now, here if you see that this integration is about the τ, this integration is not about the t.

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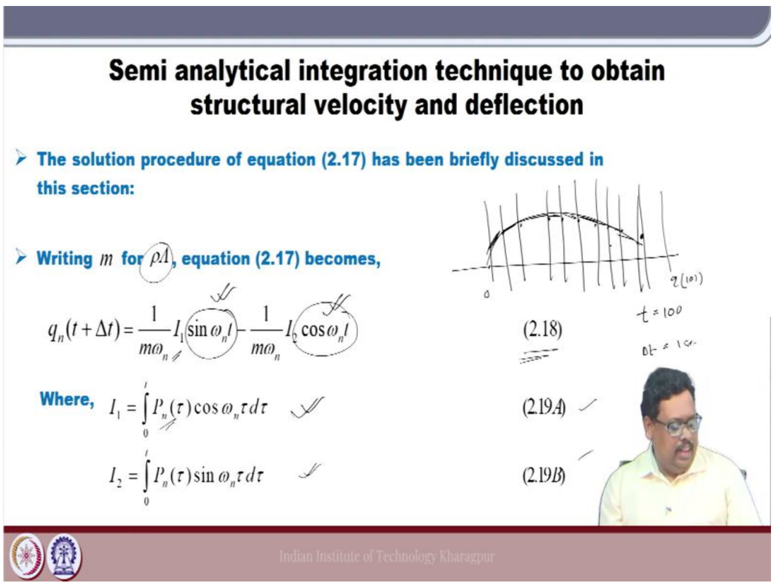

So, therefore, actually this equation $q_n(t+\Delta t) = \frac{1}{m\omega_n} I_1 \sin\omega_n t - \frac{1}{m\omega_n} I_2 \cos\omega_n t$ and also it is not that difficult to find out right. Because, you see here nothing like you just write this sin a cos b minus sin b cos a and then you can see here it is the $\cos\omega_n\tau$ term is there, which is actually involve the integration, but this $\sin\omega_n(t)$ is independent of the τ.

And similarly in this expression also you can see this should be involved into the integration, but this is actually outside the integration term. So, therefore, and also you can if you write the $\rho A = m$. So, therefore, q this equation. Now, our major job how I could do this integration $I_1 = \int_0^t P_n(\tau)\cos\omega_n\tau d\tau$ and $I_2 = \int_0^t P_n(\tau)\sin\omega_n\tau d\tau$ Right.

Now, here also now see that what is $P_n(\tau)$? $P_n(\tau)$ is nothing but the force term that you are getting from 0 to the previous time step. Now, if I draw this a graph and let us say now, I am trying to calculate let us say t is equal to I mean some 100 second let us say ok and let us take your delta t equal to let us say 1 second. So, I have the data from 0 to 100 and then I try to approximate about the q at you know 100 to 101 second that is actually I am going to do.

So, but I have the data; I have the data till this much right. Now, therefore, I can draw this graph right and therefore, actually this integration I can again think of you know always as I said that the best way to deal with the assume everything to be linear or straight line. So, then I just split the whole thing in small, small, small, small, small the time interval right.

And then I can assume that it is a straight line the force term is a, behave like a straight line in between the time interval.

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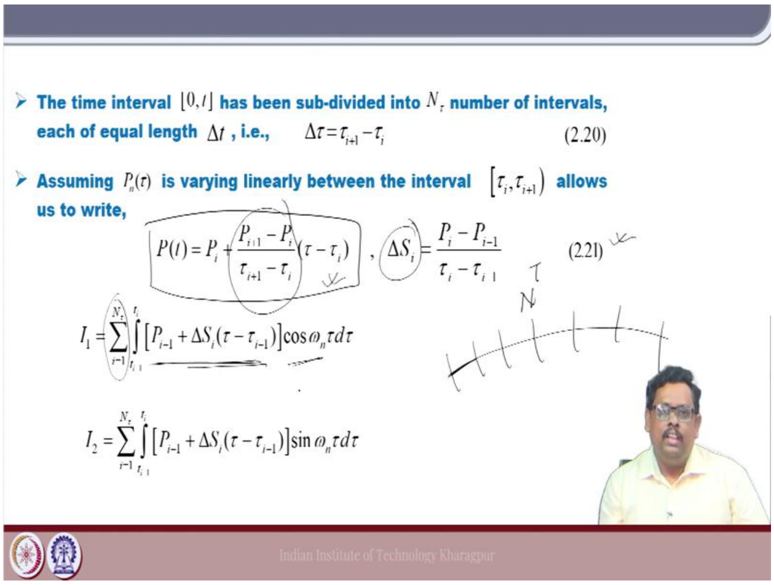

So, the moment I do that. So, the moment I do that I can write the general force as a in a in this fashion right, whether this you can see the slope and I can make it as a ΔS_i right. Now, you see like the idea is that things are fairly complicated. However, we are using some kind of approximation some kind of you know what you say that ignoring some crucial I mean ignoring some non-crucial thing and then so, that I can approximate the solution is little bit simpler way.

So, that I can get some values or some realistic values, right. So, here what I did we assume that now what is the assumptions? The assumptions are that again over the time interval it is the force is not abruptly changing with time. Now, if force does not abruptly change with time so, therefore, definitely this curve I can approximate as a sum of the straight line. So, therefore, at any interval I can write this equation $P(t) = P_i + \frac{P_{i+1} - P_i}{\tau_{i+1} - \tau_i}(\tau - \tau_i)$ right.

Now, you see that remaining part is now how to do this right because we have done it before also right. So, here what I do is suppose I assume there is a N^τ number of segments. So, therefore, I can sum of each segment right and then I substitute this $P(t)$ for each segment I multiply with the cos omega n tau and then I can do the analytic solution right.

So, you know this is that is what we do for in case of a when you discuss the rigid body solution for IRFs for IRF based method. That time also we use the same technology here also we are using the same technology.

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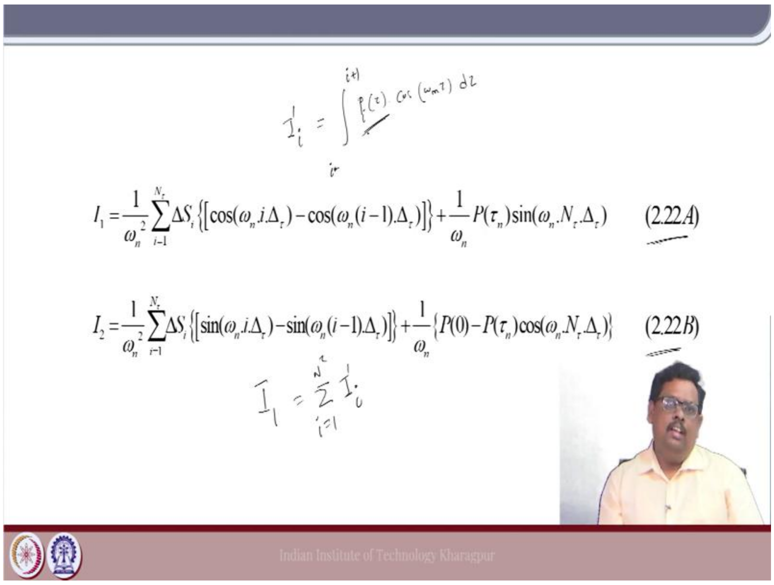

Now, once we use it then actually this is the equation $I_1 = \frac{1}{\omega^2}\sum_{i-1}^{N_\tau}\Delta S_i\left\{\left[\cos\left(\omega_n.i.\Delta_\tau\right) - \cos\left(\omega_n(i-1).\Delta_\tau\right]\right\} + \frac{1}{\omega_n}P(\tau_n)\sin(\omega_n.N_\tau.\Delta_\tau)$ and equation $I_2 = \frac{1}{\omega^2}\sum_{i-1}^{N_\tau}\Delta S_i\left\{\left[\sin\left(\omega_n.i.\Delta_\tau\right) - \sin\left(\omega_n(i-1).\Delta_\tau\right]\right\} + \frac{1}{\omega_n}\{P(0) - P(\tau_n)\cos(\omega_n.N_\tau.\Delta_\tau)\}$ you are going to get this expression. See now see, why I am not doing it here right now simply because this has been done for many times right before also in initial each and everything that actually we have discussed and we know that how to do this.

So, you should be able to get this expression $I_1 = \frac{1}{\omega^2}\sum_{i-1}^{N_\tau}\Delta S_i\left\{\left[\cos\left(\omega_n.i.\Delta_\tau\right) - \cos\left(\omega_n(i-1).\Delta_\tau\right]\right\} + \frac{1}{\omega_n}P(\tau_n)\sin(\omega_n.N_\tau.\Delta_\tau)$ and $I_2 = \frac{1}{\omega^2}\sum_{i-1}^{N_\tau}\Delta S_i\left\{\left[\sin\left(\omega_n.i.\Delta_\tau\right) - \sin\left(\omega_n(i-1).\Delta_\tau\right]\right\} + \frac{1}{\omega_n}\{P(0) - P(\tau_n)\cos(\omega_n.N_\tau.\Delta_\tau)\}$ if you follow the previous classes. Now; however, one way is to that Brute force so, as I said like you can take this $\int I_1 = \sum_{i=1}^{N^\tau} I_i^{'}$ right and this $I_i^{'} = \int_i^{i+1} P_i(\tau)\cos(\omega_n \tau) d\tau$. So, this is one way it is called the Brute force way to do that right. And then each time you define this as a straight line and then you know that how to split up into the constant term and the linear term right and then how we can find out the expression for I_i. So, we know this part.

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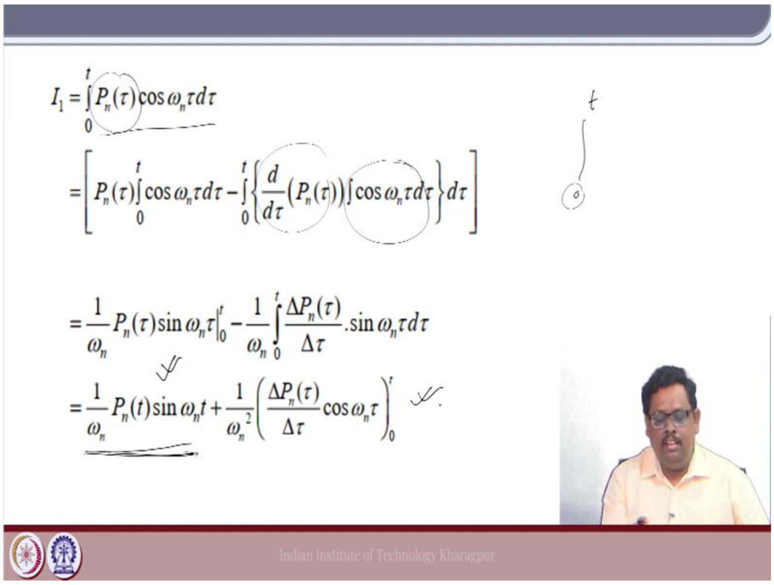

Now, the same thing actually we can do that little bit more sophisticated way also it is we will suppose here what we do is this we do this is the integration by parts now and then we are taking this $P_n(t)$ as a the first function and this is as a second function. So, as a rule it is $P_n(t)$ and then we integrate this one and then minus again 0 to 2, if we differentiate the first term and then integrate the second term that is what we do.

Now, if you do this now we can see that here you can get this differentiation $\frac{\Delta P_n(t)}{\Delta t}$ which is nothing but my ΔS_i and multiply $\cos\omega \cos\omega_n\tau$ is $\sin\omega_n\tau$. So, $\frac{1}{\omega_n}$ is coming here. Now, here also I am writing 0 to tau now if you see here when I do that for 0 to t. So, since sin 0 equal to 0. So, we are only having the final term.

So; that means, we are integrating from 0 to t if I do that since the sin 0 equal to 0. So, we are only having the final term right and then if we and again if you do this integration right. So, now, here also you have to do this integration so, again this omega n square is coming and then this much you have to do 0 to t. So, this is the fundamental idea of the integration by parts right.

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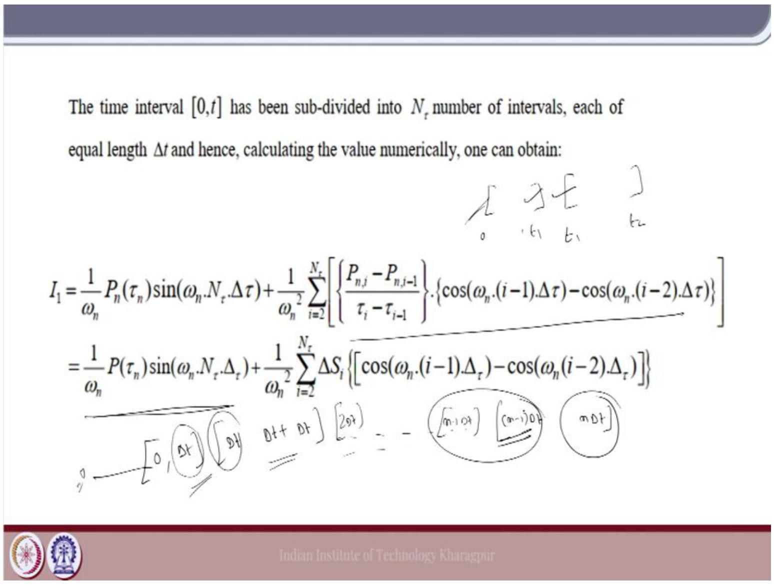

Now, the fun is that. Now, if you now discretize it in N_t number of segments. Now, if you discretize the N_τ number of segments now this is actually you can get from the second term, because you have to do this integration into each segment. However; you can see the first term is basically the last term. Why? Because when you do this integration from 0 to t, now when you do this integration from 0 to t then sin 0 is always equal to 0. So, always we are getting the final term.

Now, that can be verified very easily right if you make a segment let us say 0 to t_1 and then again another segment t_1 to t_2. So, if you do that and we can see that because this part is 0 and then this part and this part is cancelled out so; that means, what I try to say is that suppose, you have $[0,\Delta t]$ and then you have $[\Delta t, 2\Delta t]$ and in so on.

And then it is $[(n-1)\Delta t, n\Delta t]$. You have this number of segments, now since this part equal to 0, now the second integral because this is coming in the second integral and in the next time step if it is a first integral. So, these two will cancelled out. Similarly, $2\Delta t$ here and $2\Delta t$ here will cancelled out. So, all these things will cancel out.

So, here also $(n-1)\Delta t$ will cancel out with the next $(n-1)\Delta t$ and only we are leaving with this $n\Delta t$ right. So, this way also we can arrive the same equation ok. So, now, we understand how to find out the value of I_1 similar way you can find the value of I_2 also.

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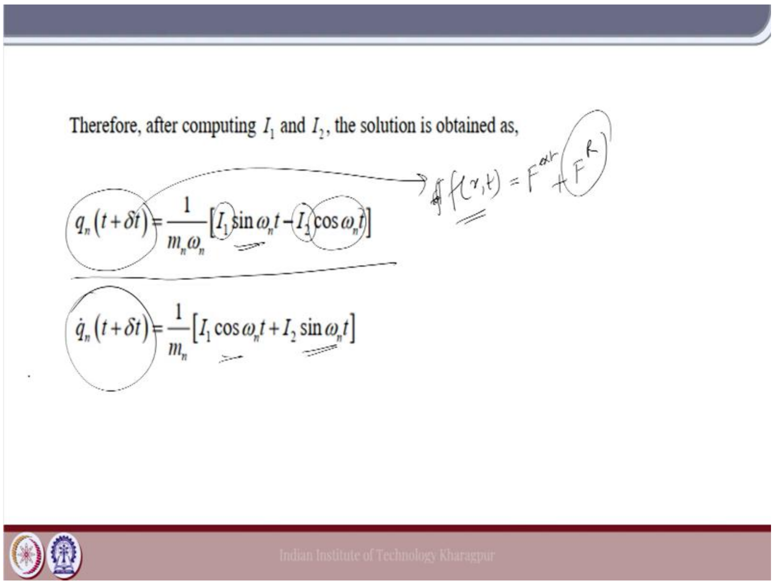

So, then once we know that value of I_1 and we know the value of I_2. So, we can get the total displacement in this equation $q_n(t+\delta t) = \frac{1}{m_n}\left[I_1 \cos\omega_n t + I_2 \sin\omega_n t\right]$. Now, if you differentiate with respect to t and remember I am differentiating with respect to t. So, therefore, I_1 and I_2 you know it does not affect.

So, therefore, it is only sufficient to differentiate this $\sin\omega_n t$ which is nothing but the $\cos\omega_n t$ and if you differentiate the $\cos\omega_n t$ you will get the value for the $\sin\omega_n t$ right and so, this is how I can get the solution for q_n and solution for $\dot{q}_n$. Now, once we know the solution for q_n I know the solution for $\dot{q}_n$. So, I know the entire solution at time step $(t+\Delta t)$ right.

So, now in the next level we use this $q_n(t)$ again to figure out my that $f(x.t)$, the total external force which is combination of this plus combination of the radiation force. So, when you calculate the combination of the radiation force. So, that time again you have to use the value for $\dot{q}_n(t)$ right.

So, now, this is how actually you can do the fluid structure coupling and you can get the solution for the deflection and then you can calculate the shear force the bending moment everything using the semi analytic method.

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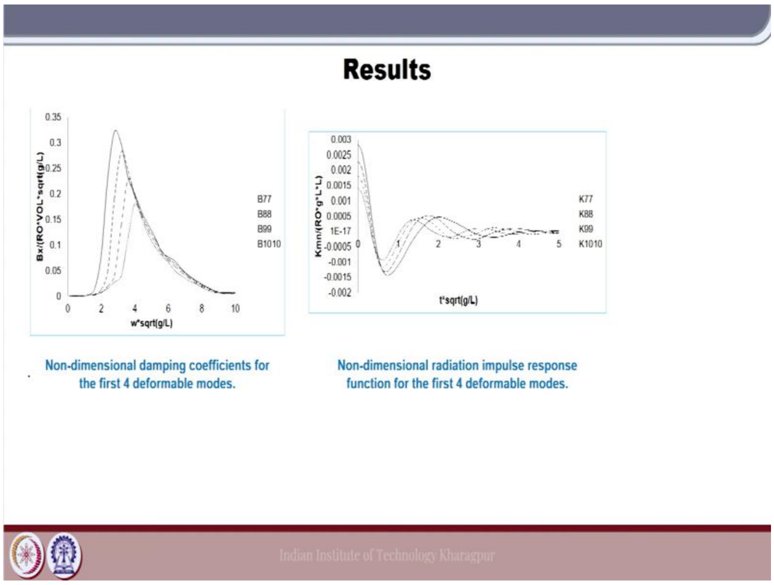

Now, let us see that some results on this semi analytic method and let us see that how good this result is with comparison to the finite element bound element coupling method ok.

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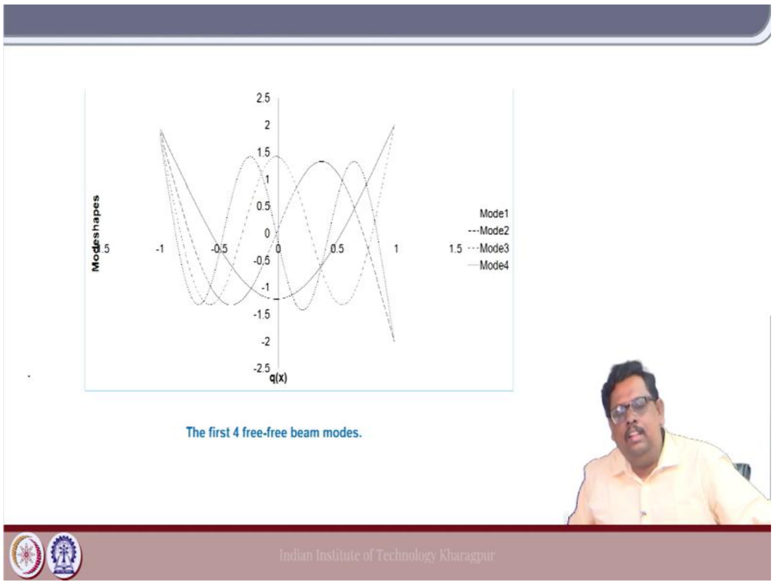

Now, if you see that this is the results for the different modes that we are getting it using the modal superposition method.

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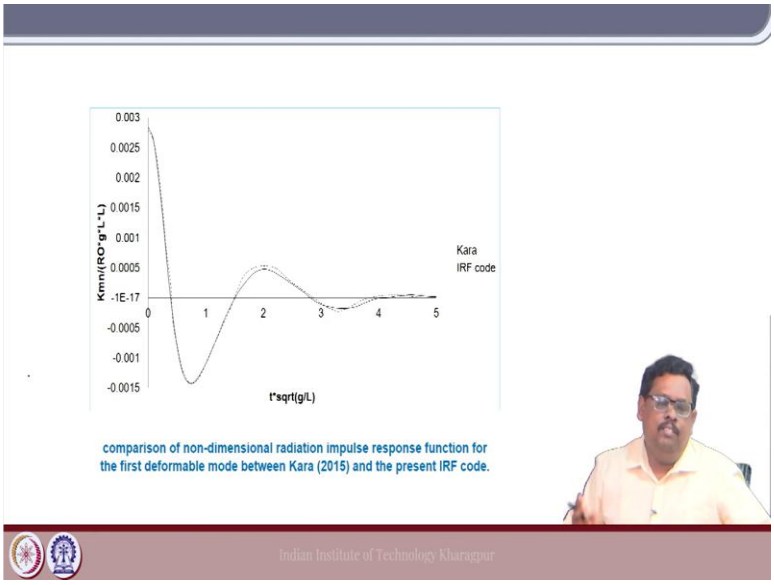

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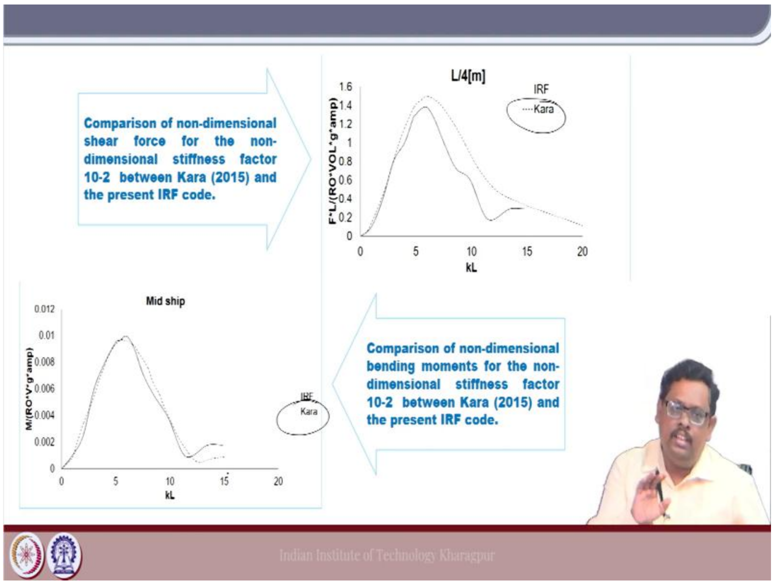

And then you see that here we are comparing the impulse response function which is matching very nicely. So, now this is the result of the non-dimensional shear force and bend and I think the second one is the bending moment and we can see that this it is absolutely matching very well with the public results and this Kara actually using the that it he uses the time domain solution.

However, the structural solution is similar he uses the structural solution using the modal superposition that is how he solve the structural problem; however, he uses the time domain panel method to calculate the radiation force diffraction force etcetera ok, but now you can see that even if we see we are telling that this is a simplistic method or maybe semi analytic method, but the results are matching very well.

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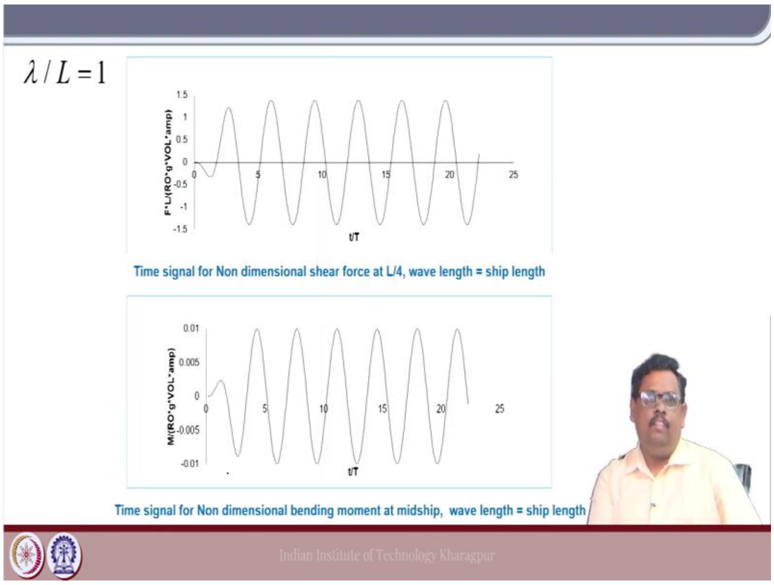

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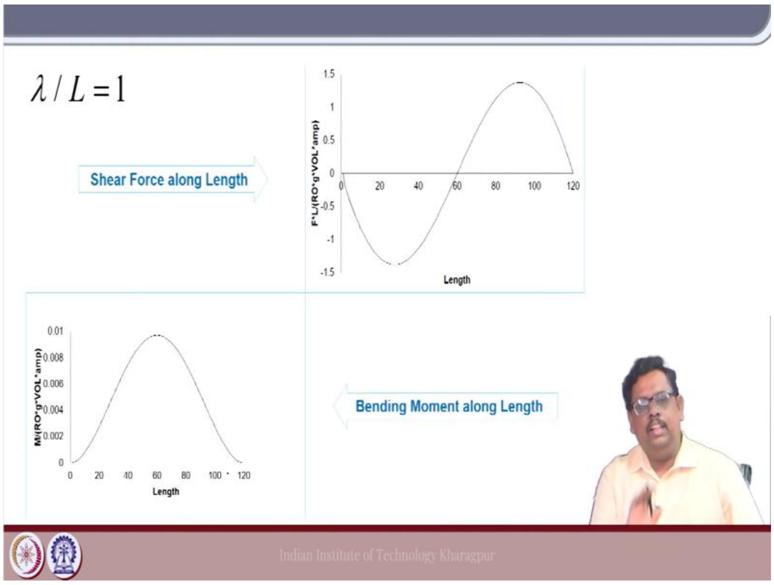

And also, you can see that the results are very stable and also you can see that is how we can get the stress shear force and the bending moment right this is the shear force and bending moment dynamic when actually I can have the maximum bending moment at the mid ship. So, at that time we are calculating the shear force and when the wavelength is equals to the ship length ok.

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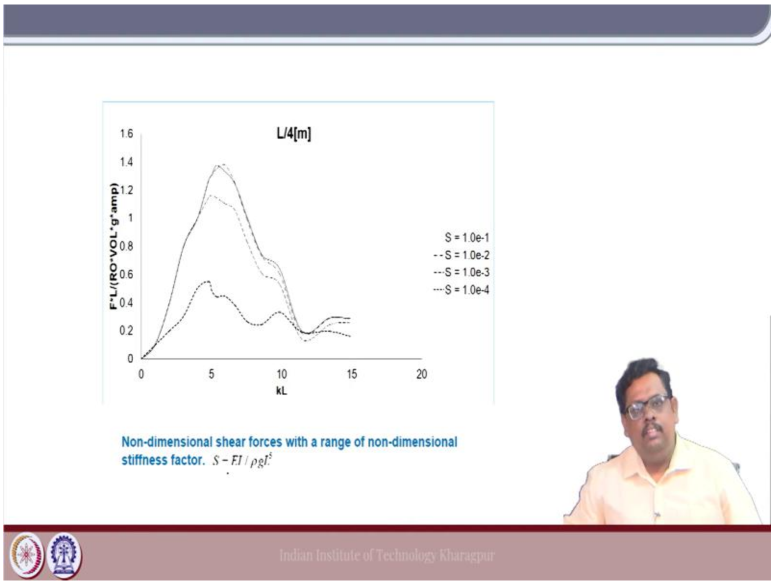

So, now and also, we are just showing that how the flexibility actually plays a significant role if we change the flexibility, you can see that when it is more flexible than we can have the less shear force and bending moment of course, it is possible when it is which it is more flexible definitely the it absorbs the more energy. So, therefore, the deflection or the bending it should be less right.

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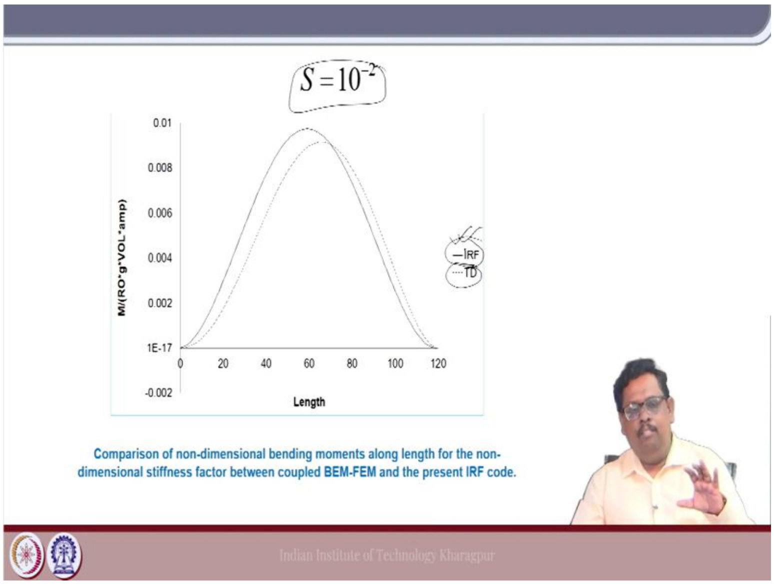

Now, here that is a interesting result that this is the IRF; IRF stands for this Semi Analytic method and then the TD it is stands for the time domain method that we discussed in the previous class and you can see that TD is much sophisticated method compared to this one, but; however, when you see the result $S = 10^{-2}$ it implies more or less the structure is made of steel is a dimensionless quantity.

Now, idea is that we can see that though we are saying this IRF is bit simplistic method, but quality wise the results are as good as the sophisticated. So, called sophisticated panel method I mean that FEM-BEM based panel method right. So, that is the idea like. So, this is the that major advantage of this particular method.

Only thing that we required is the beforehand we have to have the added mass and damping of the frequency domain data code the frequency of added mass frequency of damping frequency of exciting force for flexible structures this is the only thing that we required. So, we can say that is the only disadvantage of this particular code.

But if it is available to you and if it is given in some literature. So, definitely this is a very powerful tool to capture the flexibility of a large floating structure ok.

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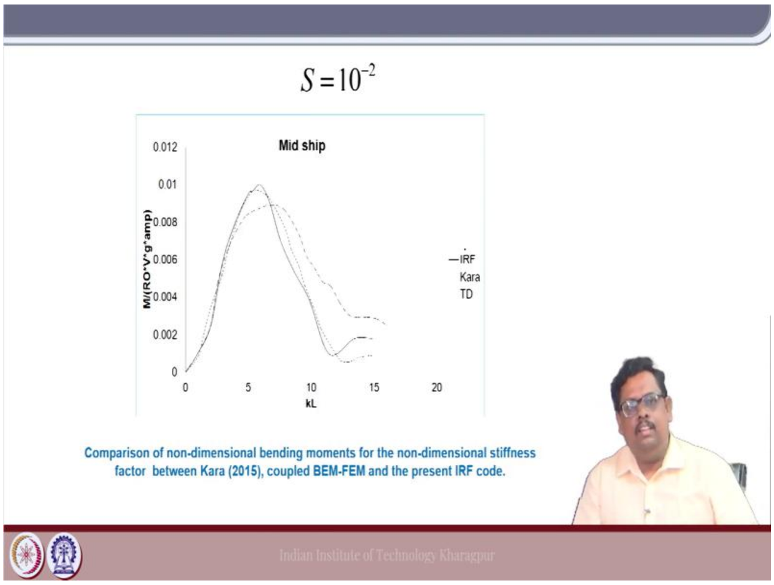

Again, you can see that all the results are matches quite well. So, definitely we can see that these results are really good using the semi analytic method.

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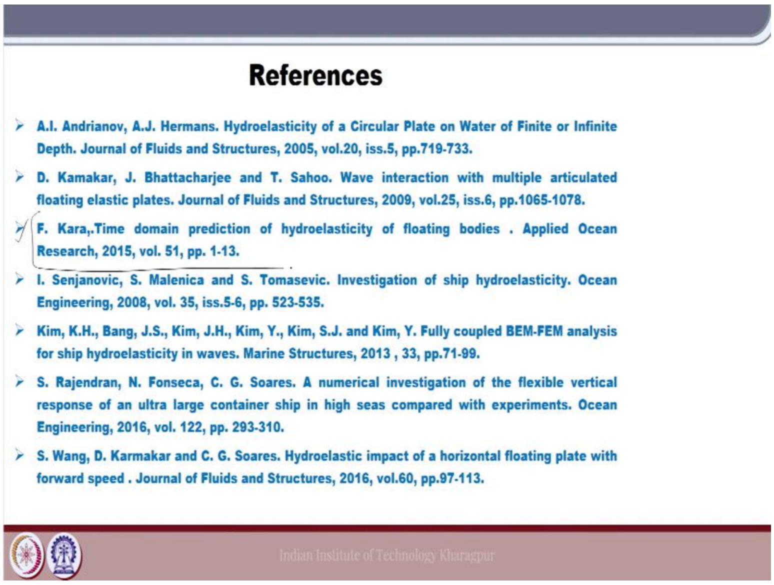

Now, this is the references that we like to use that. And then I would recommend you to see this paper and, in this paper, actually you can see that all these frequency domain data are available right and also this also this paper you can get lot of results. So, these two actually all are very important; however, these two actually it is very useful for our particularly this course ok.

So, with this we stop this semi analytic method and from the next class we are going to start that discuss some of the basic thing about the slamming and then green water and then how we can use this slamming green water, I mean the impact of slamming on green water into this semi analytic code as well as the time domain method code those things we are going to discuss from the next class.

Thank you.