Numerical Ship and Offshore Hydrodynamics
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Lecture - 06
Seakeeping - 5

Welcome to the Numerical Ship and Offshore Hydrodynamics, today is the lecture 6 and today we are going to discuss this following concepts.

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We discuss about the force oscillation motion and also discuss that how to get the added mass damping through some experiment and also we are going to discuss something on the heave motion ok.

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So, and also this is the keyword that we are going to use to get this lecture ok. So, let us get back to the some board work.

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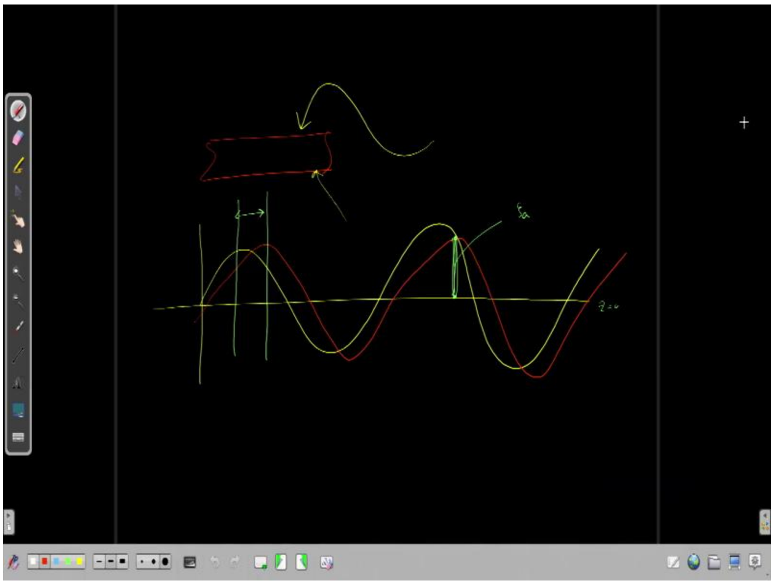

Now, this is where actually we finished in the last lecture and what we are going to discuss is that at which frequency we should get that we could have the maximum response right. In order to in order to find that, so first we need to write let us write the equation of motion and try to find out like how we can get it.

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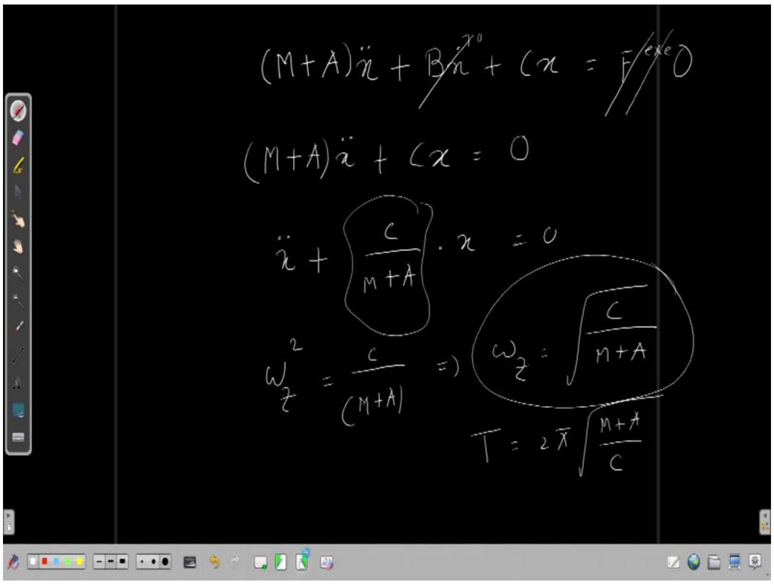

So, we write $(M+A)\ddot{x}+B\dot{x}+Cx=F^{exe}$. So, this is the fundamental equation right.

Now, what let us do one thing let do the you know the free vibration problem. So, we make this right hand side equal to 0 and also we can drop the damping term. So, let us make this goes to 0. So, then what you have then we have a very simple equation $(M+A)\ddot{x}+Cx=0$.

And so therefore, we have here just divide both the side $(M+A)$, so $\ddot{x}+\dfrac{C}{(M+A)}x=0$. And you know this is nothing but your, the natural frequency right? So, then you can define your natural frequency as ω_z.

So, we can we know that this ω_z^2 is nothing but $\dfrac{C}{(M+A)}$. So, from here we can get my $\omega_z=\sqrt{\dfrac{C}{(M+A)}}$. So, you know there is a very common thing. Right? Everybody knows this. Now similar then you can find out my time period nothing but you know $2\pi\sqrt{\dfrac{(M+A)}{C}}$.

Now this is actually very important for us right. So, now, let us go ahead with this the expression right.

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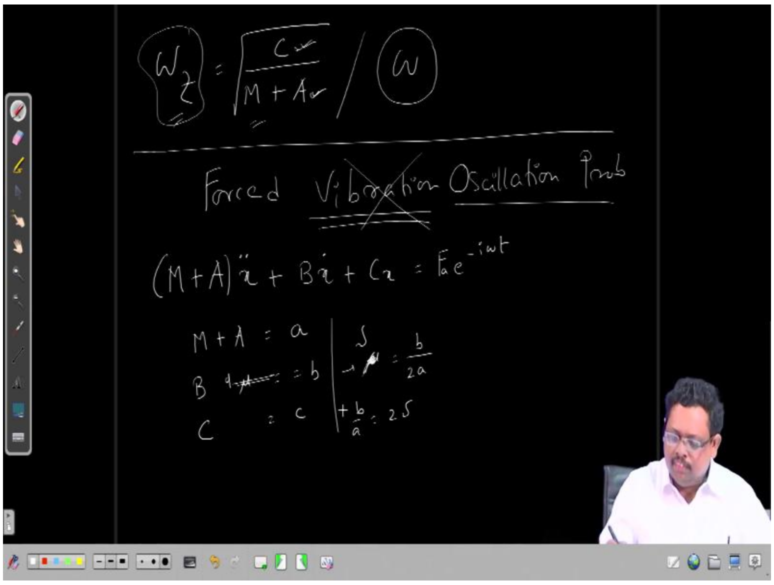

So, we write this ω_z is nothing but the restoring coefficient C divided by mass plus added mass right. Now, you see like now suppose you have a you know frequency of encounter like or that frequency of the oscillation of the forcing function is ω and this is of course, your, the natural frequency ω_z.

So, therefore, if the both are actually you know morely close to each other right? So, at that time you can expect that there is a large response right. So now, you see before actually we started doing the hydrodynamics we can guess so many things right. Because if we if you have the idea about the restoring coefficient C if we have some idea about the added mass A, and then actually mass of course, is we know that the mass of the body.

So, then we can actually approximate the natural frequency of the particular vessel. And therefore, in normal even this is very important for the engineer right. They do not I mean that those are work in that field they do not have time to do some fancy mathematics or do some simulation to find out the responses all the time right.

However, for very simple estimation of the C that added mass A they can actually get some kind of idea about the natural frequency and then you know that helps for the design right ok. Now, this is something about the free vibration. So, now, let us do the forced vibration problem.

Now, you know people when you use the vibration always they assume the high frequency phenomena. So, people do not use it such as, so they can oscillation instead of say right oscillation. However, you can use anything that you like ok both are same. So, now, let me do some modification to do this not modification just rewriting the things it little bit simpler way. So, we are we know that this is my - the classical equation of motion.

So, we are using like a bread and butter for you. So, you have to remember it you can afford to forget this. So now, you just write $F_a e^{-i\omega t}$, this is the forcing function right. And then let us make some kind of simplification not simplification I say that nomenclature. You can call $(M+A) = \text{a}$ and then let us define some term let us say υ equal to you know or ok.

Let us take $\text{B} = b$ ok and then $C = c$ and then you can define $\upsilon = \dfrac{b}{2a}$, not μ, let us say υ. So, then ok so; that means, that $2\upsilon = \dfrac{b}{a}$ ok. Let us assume this and then let us try to apply all this thing in this into this equation right.

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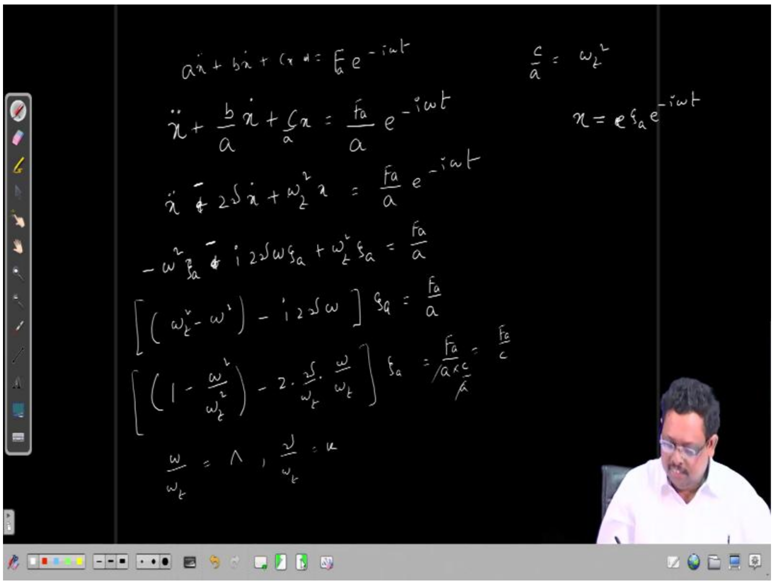

So, if you do that then you have in a very simplified form it becomes $a\ddot{x} + b\dot{x} + Cx = F_a e^{-i\omega t}$ right. If you divide the everything by a.

So, I will get $\ddot{x}+\frac{b}{a}\dot{x}+\frac{C}{a}x=\frac{F_a}{a}e^{-i\omega t}$ right. And now I just simply write it $\frac{b}{a}$, I just- is a $2\upsilon\dot{x}$. Now the c actually you can see that you can see, so I just forget this is $\frac{C}{a}$. Now you know that that $\frac{b}{a}$ is nothing but $\omega_z^{\,2}$ which is the square natural. So, we just we have just found out right.

So, instead of $\frac{C}{a}$ we can write it is $\omega_z^{\,2}x=\frac{F_a}{a}$ right ok. I forgot that $e^{-i\omega t}$ of course, is important. Now again as I said that we can assume my the response also harmonics we can assume $x=e$ to the power or $x=\xi_a e^{-i\omega t}$ and then we just going to substitute over here.

So, let us see what we get we get $-\omega^2\xi_a+i2\upsilon\omega\xi_a+\omega_z^{\,2}\xi_a=\frac{F_a}{a}$ right ok. And then just ok if it is $-i\omega t$ ok, so then definitely this is minus ok. And so therefore, you can find out it is $\omega_z^{\,2}-\omega^2$ ok and $-i2\upsilon\omega\xi_a=\frac{F_a}{a}$ right.

Now I just do little bit more modification. I just divide everything by $\omega_z^{\,2}$ now. So, then what I get is $\left[\left(1-\frac{\omega^2}{\omega_z^{\,2}}\right)-2\frac{\upsilon}{\omega_z}\frac{\omega}{\omega_z}\right]\xi_a=\frac{F_a}{a}$.

Now, remember this $\omega_z^{\,2}=\frac{C}{a}$. So, I just multiply this $\frac{C}{a}$. So, a, a will cancelled out right and then we have $\frac{F_a}{C}$. Now you know why I do all this exercise because I need again you know another nomenclature. So, let us do $\frac{\omega}{\omega_z}$ let us call this you know γ and then you can call that $\frac{\upsilon}{\omega_z}$ you can call something called k right.

Now, what you are going to do is let us substitute everything back to there ok. Let us see what happen.

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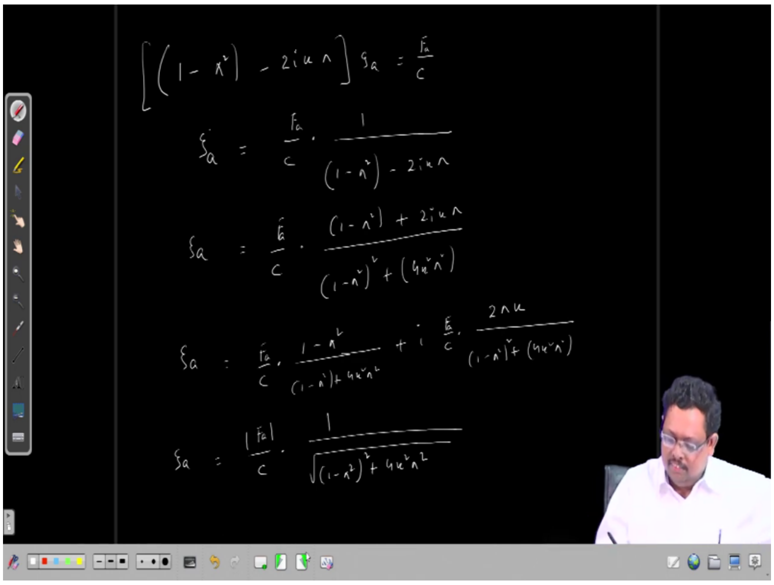

Then we have $[(1-\gamma^2)-2ik\gamma\xi_a]=\dfrac{F_a}{C}$ right?. Now you see is a very nice equation that we are getting.

So, finally, we are finding out my $\xi_a=\dfrac{F_a}{C}.\dfrac{1}{(1-\gamma^2)-2ik\gamma}$. Now how to get the, so now I just do some little bit more manipulation I just write the complex conjugate. So, I just find out $\dfrac{F_a}{C}.\dfrac{(1-\gamma^2)+2ik\gamma}{(1-\gamma^2)^2+4k^2\gamma^2}$ right. That we are going to get.

So, I just break into real and imaginary part. So, it will become $\xi_a=\dfrac{F_a}{C}.\dfrac{(1-\gamma^2)}{(1-\gamma^2)+4k^2\gamma^2}+i\dfrac{F_a}{C}\dfrac{2\gamma k}{(1-\gamma^2)+4k^2\gamma^2}$. fine clear?

Now this part is actually algebra. So, we can find out the modulus and also you can do the $\tan^{-1}\left(\dfrac{b}{a}\right)$ you can get the amplitude as well as the phase right. So, let us find out what is the amplitude. So, then you can get the $\xi_a=\left|\dfrac{F_a}{C}\right|$ and then if you do that it turns out to be 1 divided by, because $\sqrt{a^2+b^2}$.

Now you have the whole square term over here whole square term over here, you can work out right. It turns out to be $\sqrt{(1-\gamma)^2 + 4k^2\gamma^2}$. So, again we can use further you know notation.

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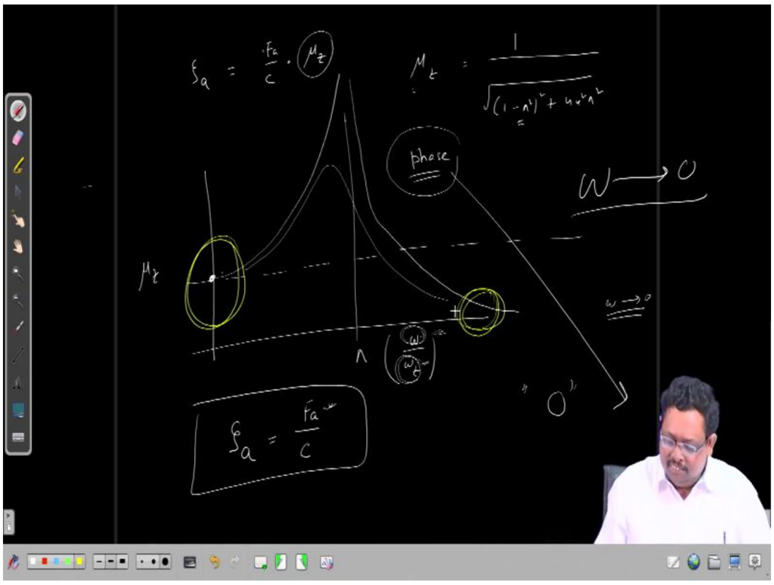

So, again we can use this sum equal to it is $\frac{F_a}{C}$ and you can call this as you know some μ_z, where this $\mu_z = \frac{1}{\sqrt{(1-\gamma^2)^2 + 4k^2\gamma^2}}$.

Now, it has all name like you know you can find out there is a different name for all these things like it is called the tuning factor, amplification factor all these things, but we are not that into. We just try to find out the phenomena some phenomena right.

And of course, you can just you can work out how to get the phase ok. Now what is my interest over here why I derive this I just trying to plot the one thing, which is the μ_z, which is essentially the response divided by the tuning factor γ, which is essentially that $\frac{\omega}{\omega_z}$ right.

Now here in this location if you put this γ tending to 1 then you can see here it goes up to ∞. Now if you put $\gamma = 0$ right then actually you can see that that $\mu_z = 1$. So, μ_z can be you know you can say that μ_z basically the response with respect to this. Now you can see that at this equal to 1; that means, when this forcing, I mean frequency of the forcing function equals to the natural time period it goes up shoots up to ∞.

And again very quickly it comes down and it goes to 0 because when the tending to infinity this goes to 1 right. So, γ means this when the ω tending to ∞ it goes to 1. So, you can see that how important it is to find out the natural frequency right.

So, we can now you can assume the no damping of course like if you have used some damping it might come down of course. So, but our idea to show all these things just to show you that at when this natural frequency equals to the forcing the frequency of the forcing function there is a huge response you can expect. Even if you put some damping definitely you can ignore this point.

And also you can see that when ω tending to 0. when ω tending to 0 ok just write this bigger so, that you can get it when ω tending to 0 you can see that response goes to 1. Now let us see what happened that time so; that means, the $\mu_z = 1$. So, in that case you can see my response ξ_a is equal to basically the ratio between the hydrostatics and the amplitude of the exciting force.

Now, you see its very important thing like you know you see that is what happening actually when you are you know going in the beach and you are just finding out the that your response is you are moving up with the wave and moving down with the wave. And if you calculate the phase also you can find at that particular moment when this ω tending to 0, at this particular moment this phase also goes to 0.

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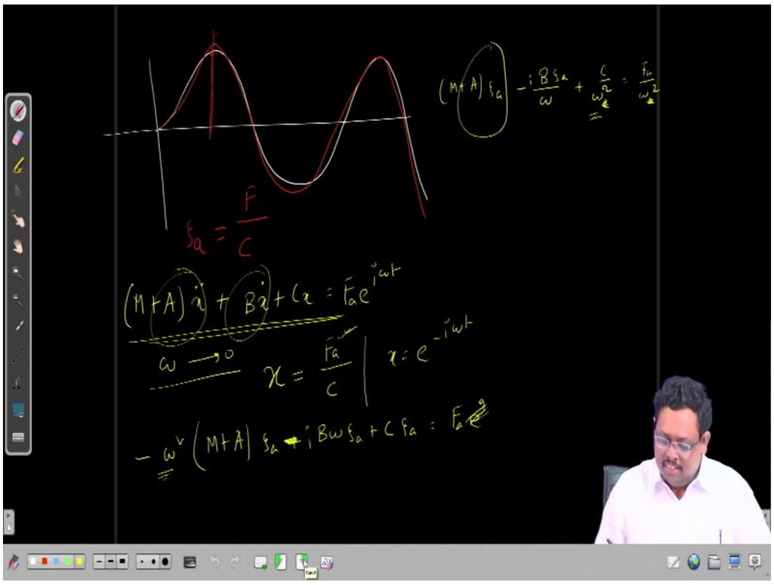

It means that when you are in top ok. And then this is exactly what is happening in the low frequency region right ok so; that means, there is no phase first of all and second you are moving with the wave. Now if you see that what is happening you see that that response ξ_a, is equals to that amplitude of the exciting force divided by the linear restoring component.

What is the meaning of that? Let me write the equation again you can understand it. So, you can, the equation is $(M+A)\ddot{x}+B\dot{x}+Cx=F_a e^{i\omega t}$. Now so as omega tending to 0 what is assuming that that all those the radiation component A the damping component B absolutely has no role in fact.

So, that time the response is very simple it is simply the ratio between the restoring coefficient, I mean the amplitude of the exciting force divided by the restoring coefficient right. Now, this is a very important thing, because you know for this particular situation you really do not need to calculate all these tedious, that that all these software's you do not need to run all this software. And simply from the physical point of view we can simply say ok.

So, this is the low frequency region very low, so we can assume that added mass damping that should goes to 0. So, therefore, only the dominating component is exciting force amplitude and the restoring term. Similarly, if you go to the other part of this side when actually that the omega tending to infinity that very high frequency region.

So, that time as you see the response is very small right. And then actually from here you can see that in this equation when omega because if you write this if you write this in terms of that $x = e^{-i\omega t}$, if you apply this right what you get is $-\omega^2 (M+A)\xi_a - iB\omega\xi_a + C\xi_a = F_a$.

Now, if you divide whole thing by ω^2. So, then what you have what is going to happen is that your $(M+A)$ is free of ω, but then it is $(M+A)\xi_a - \frac{iB\xi_a}{\omega} + \frac{C}{\omega^2} = \frac{F_a}{\omega^2}$. Sorry, it is not 2 it is ω^2 sorry yeah.

So, then what is happening in that situation you can see that in high frequency region your added mass become very important not the restoration. So, you can see that in that is the beauty of this like you can guess from this from the low frequency region, that your restoring force become important and in the high frequency region your added was becoming very important.

So, that is in vibration added mass become very important for floating structures, but intermediate region both added mass damping restoration all are very important right. And actually we are going to discuss those range only we are not discussing when ω tending to 0, we are not discussing when ω tending ∞, we are going to discuss when ω neither in low frequency region neither in high frequency region intermediate region, where all this radiation component and all other component is very important ok.

Fair enough. Now let us now go into that how experimentally actually we can calculate the added mass damping ok.

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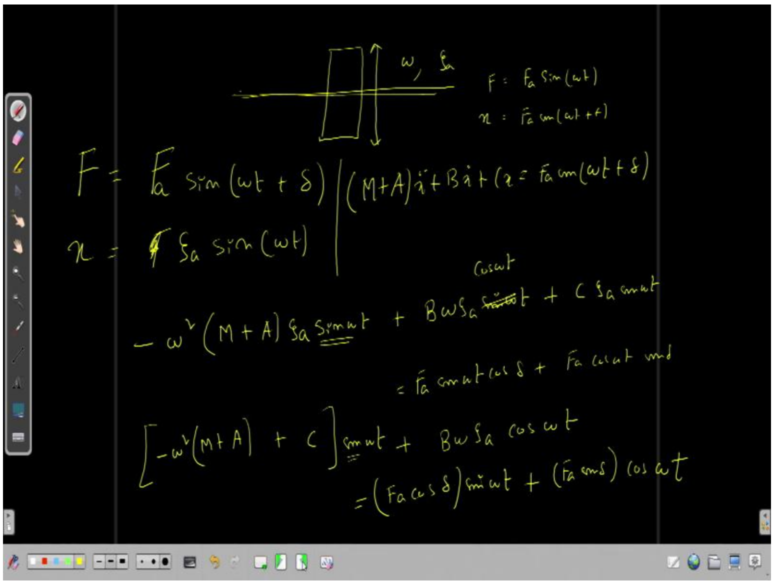

So, it is again a very simple thing that suppose you have a. So, what is the radiation force or is nothing but in calm water you are oscillating a body in some frequency right. And then some pressure field generated around this body if you integrate the pressure field you will get the radiation was fine.

So then, but you have to oscillate here with some frequency omega and of course, with some frequency ξ_a. Now let us do that with respect to as I said like not using the complex number rather let us use it a you know the cos sin function ok. So, now, let us assume that your the forcing function now you know now, so see you can do both like you can assume that your forcing function $F = F_a \sin \omega t$.

And then you can assume your the displacement $x = F_a \sin(\omega t + \varepsilon)$. But we do opposite let us because that is more easier for me because the ultimate the end result is same. So, we can assume my the forcing function $F = F_a \sin(\omega t + \delta)$ whatever nothing. And then you can use that that $x = \xi_a \sin(\omega t)$ ok.

Now, what I am going to do is I am going to substitute this into the equation of motion ok when I do that let us see what are you going to get. So now, remember like it is like your every time you need to remember this equation $(M + A)\ddot{x} + B\dot{x} + Cx = F_a \sin(\omega \mathrm{t} + \delta)$, now in this case it is $F_a \sin(\omega \mathrm{t} + \delta)$; I am going to do it for a real number right.

So, I just substitute over here. So, it is become $-\omega^2(\mathrm{M}+\mathrm{A})\xi_a \sin\omega t + B\omega\xi_a \cos\omega t + C\sin\omega t = F_a \sin\omega t\cos\delta + F_a \cos\omega t\sin\delta$

Now I am just you know accumulating the sine term and cos term right. So, $-[\omega^2(\mathrm{M}+\mathrm{A}) + C]\sin\omega t + B\omega\xi_a \cos\omega t = F_a \sin\omega t\cos\delta + F_a \cos\omega t\sin\delta$.

Now next as I said it is very easy, so just compare the sin omega t term.

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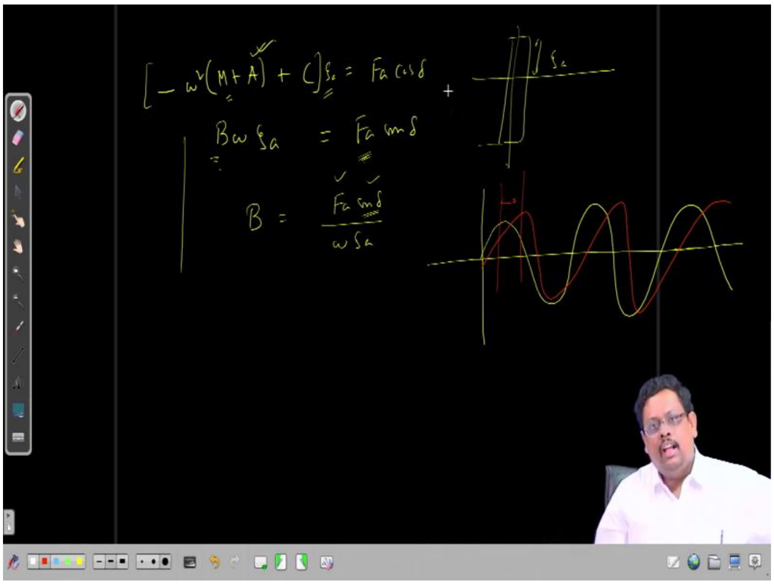

So, will get $[-\omega^2(\mathrm{M}+\mathrm{A}) + C]\xi_a = F_a\cos\delta$ and if you look the sin cos term. So, $B\omega\xi_a = F_a\sin\delta$. Now you see from here you can write the expression for a because all other term is known to you know the mass M you know the ξ is the harmonic the forcing function the displacement of the body, like you know you are displacing it with ξ_a right.

So, this is known to you and then you know this force is known to you because that is what you are going to you are read it from the machine you are putting some pressure sensor to get the pressure and forces. So, this is known to you right and from here also you can get the damping right. So, $B = \dfrac{F_a\sin\delta}{\omega\xi_a}$ right.

So, it is straight forward, so to get the B. Now from here actually this all this known to you and then only thing is need to do that what is the delta, now delta also very easy now if you plot it the signal getting from your machine, this is the signal of your forcing function and if this is the signal of your another that the resultant. So, then from here you can get the information about the delta right. So, here from these two equations you can obtain what is your added mass and what is the damping ok. So, today we are going to stop here.

Thank you very much. So, we are going to see in the next class.