Numerical Ship and Offshore Hydrodynamics
Prof. Ranadev Datta
Department of Ocean Engg and Naval Architecture
Indian Institute of Technology, Kharagpur

Lecture - 08
Seakeeping - 7

Hello, welcome to Numerical Ship and Offshore Hydrodynamics.

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Today, we are going to discuss about the Froude scaling.

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And, these are the keywords that you are going to use to get this lecture.

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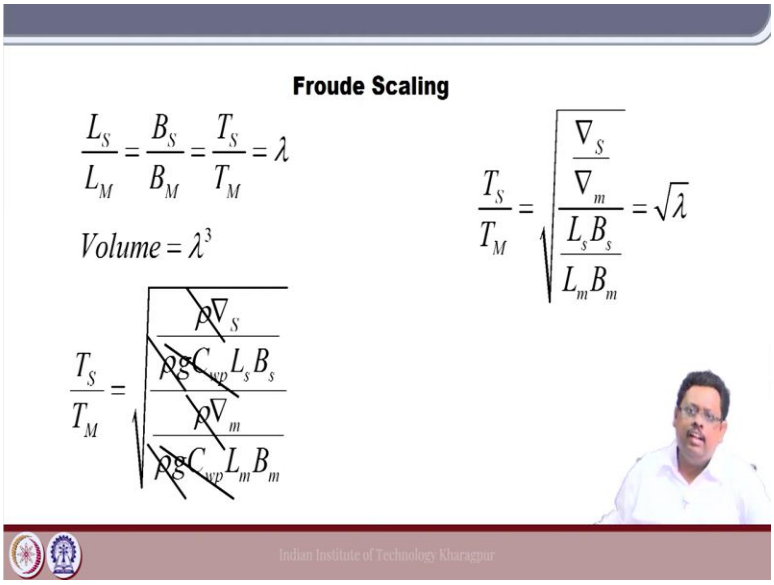

So, let us come back to our topic which is the Froude scaling. Now, when you do the experiment normally you are doing in the model scale. A ship normally conventional ship the length would be the scale of 150 – 200 meter long, right.

So, therefore, going a full scale experiment is really tough. So, then what we do in lab we do the experiment in the model scale and then we try to find out what would be the forces or the parameter that we measure in experimental in model scale what would be that in the full scale and therefore, we have to do some scaling factor. So, we do here the Froude scaling.

Now, if you look at here that we are using that let us say L_S, B_S, T_S be the length breadth and the draft of the original vessel or we can call is a prototype and then we have L_M, B_M, T_M could be the length, breadth and draft of the model scale vessel. And, we maintain that length of the full scale vessel by length of the model scale vessel is equal to length of the breadth of the full scale vessel by beam of the model scale and also the draft also the ratio also is similar and equal to lambda.

Having said this, let us try to find out how we can scale up certain parameters. So, let us start with how we can scale the volume. Now, you can see that volume is nothing but of course, you are doing the dimension analysis the length cube. Now, you can see that length is modeled by lambda. So, definitely the volume should be scaled by lambda cube, right. When you understand that volume is scaled by lambda cube, then we can do many many many parameters also you can scaled.

Let us see that how we can scale the time periods. Now, you see here I am using the formula for time period which is mass divided by the spring constant. Now, here just for sake of clarity we are using for the heave modes. Now, in heave modes this nothing but the mass divided by the restoring force which is ρ g into A_{wp}.

Now, here you can see that I did not write the A_{wp} because most of the time that water plane area is not available. However, which is available to you is the water plane area coefficient. Now, when you design a vessel that is already you know this may be taught in the hydrostatic courses basic hydrostatic courses.

We are going to define so many design parameter. One is called the block coefficient that we discussed before. We did not discuss the water plane area coefficient or (Refer Time: 03:42) area coefficients, this basically when you do the design then a designer can assume that ok that would be my the water plane area coefficient.

And, with the help of that we can get the area using this formula that ρ g C_{wp} equal to l m I mean it is C_{wp} multiply the length into breadth, right. So, from here you can see that very quickly you can get the water plane area if you know the coefficient of the water plane area. Similar to the volume if we know the block coefficient then multiply the block coefficient by L into B into T we will get the volume.

Similarly, if you know the that water plane area coefficient, then we can multiply with the length and beam we can get the water plane. Anyways, now this is how we can get this the water plane and then we can just cancel out the similar terms. So, that we cancel out the rho right, we cancel out the coefficient because this coefficient should be same whether it is a model ship or as a actual ship.

So, therefore, we have this expression T_S equal to ∇_S by ∇_m that my displacement ratio as well as the length into breadth by full scale divided by length into breadth for the model scale. Now, you know that this displacement is lambda cube as we mentioned over here you can see here and also you can see the other parameter is lambda square.

So, it is λ^3/λ^2. So, therefore, the scaling of time period is $\sqrt{\lambda}$ ok fine. So, now, let us see what the other parameter that we can scale.

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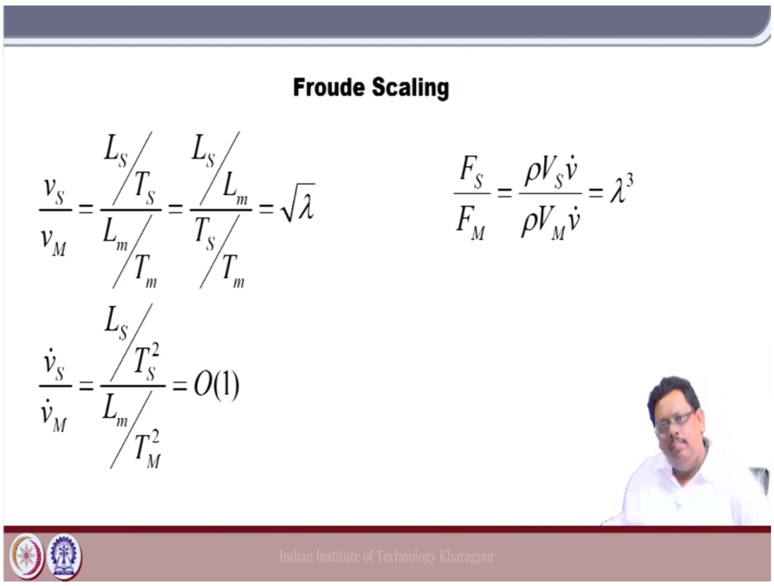

Now, we are doing the velocity. Now, velocity is the L/T, right. Now, I just rearrange it L/T = $\frac{L_S}{L_M}$; that means, the ratio between the actual ship by model ship length divided by the time period of the actual ship by model ship. $\frac{L_S}{L_M} = \lambda$ and $\frac{T_S}{T_M} = \sqrt{\lambda}$. So, I understand that velocity can be scaled by $\sqrt{\lambda}$.

Now, let us see the acceleration then formula for acceleration is $\frac{L}{T^2}$ I mean that is the dimension analysis we do is $\frac{L}{T^2}$. So, again that I can arrange the similar way which is $\frac{L_S / L_m}{T_S^2 / T_m^2}$ and then we can see that it is $\frac{L\lambda}{\sqrt{\lambda^2}}$ which is λ. So, acceleration is order of 1.

Now, once you understand that acceleration it is scaled you cannot it is similar for model or for the ship it is order of 1, then we can really scale the force. So, how we can scale the force it is mass, time you know acceleration. So, I just here they mention that V_S is nothing but the volume. So, volume of the ship we can call this I mean ∇ also here I am using the V.

So, mass and then ρ will cancel out. So, therefore, essentially the ratio of the volume and then multiplied by the ratio of the acceleration. So, from my previous slide I know that ratio of the volume is nothing but λ^3 and then acceleration is order of 1. So, therefore, that I know that my force is the order of λ^3, right.

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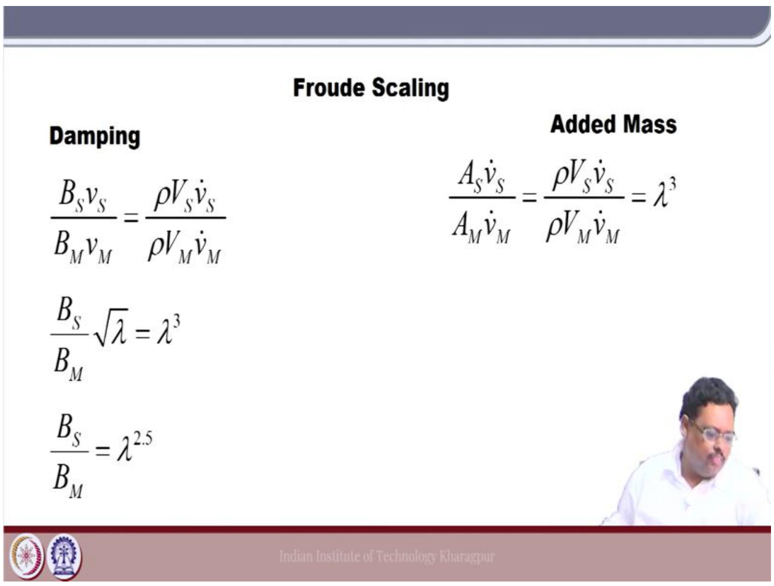

Now, once I know the force is also lambda cube so, let us try to find out how we can scale the damping and the added mass, right. Now, you see the added mass also a force and damping also a force. So, definitely my right hand side should be lambda cube, right. So, let us see that what is there in the left hand side.

Now, I know that right hand side is of course, lambda cube and left hand side also added mass also the added mass force is added mass multiplied by the acceleration. So, therefore, it is the ratio between the added mass of the prototype divided and the and model multiplied by the ratio of the acceleration equal to lambda cube. Now, as you know that that $\frac{\dot{v}_S}{\dot{v}_M} = 1$. So, therefore, I understand that added mass also can be scaled by λ^3.

Now, let us see how we can scale the damping, ok. Now, in damping in the right hand side of course, it is lambda cube because it is force and in the left hand side it is damping coefficient multiplied by the velocity that represents a force. So, now if you do the again the scaled analysis this $\frac{V_S}{V_M} = \sqrt{\lambda}$ that we have found out right that velocity the ratio of velocity just you know we found know it is a root lambda.

So, therefore, that $\frac{B_S}{B_M} \times \sqrt{\lambda} = \lambda^3$ which is the dimension of the force and from here I understand that damping actually we can scaled by $\lambda^{2.5}$. Now, this is how we can do the scaling for the different parameters that actually we can measure through our through experiment in lab, ok.

So, how we can contact the experiment that to get the damping and added mass we have already discussed it before. So, in connection to that once we have the damping coefficient in scaled model or the added mass in the scaled model. And, sometimes the natural frequency also you can get through the scaled model and then we can use this scaling factor and we can guess that what would be the added mass in the real ship, what would be the damping coefficient for the real ship and what is the natural time period for the real ship, ok.

So, now, let us see one let us solve one problem.

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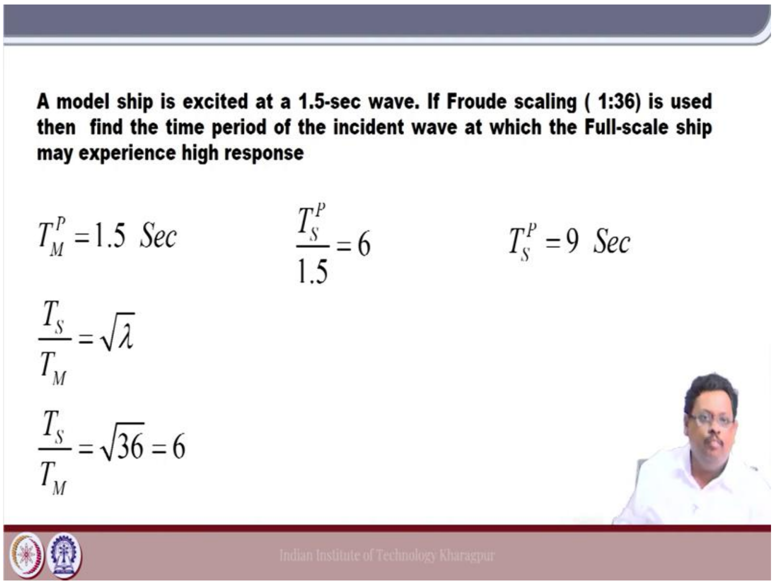

Now, it is written that a model ship is excited at 1.5 second wave. Now, if you maintain the Froude scaling $1:36$, so that means, that that lambda scaling of lambda is $1:36$ ok. So, then if it is used; that means, it means that that length of the ship by length of the model equal to 36, right that is what the meaning of 1 is to 36 scaling. So, the question is asked that what the time period of the incident wave at which the full scale ship maybe experience high response.

Now, we discussed in seakeeping that in the natural time period only natural time period that ship tend to experience the high response, right. So, therefore, here it is given that that natural time period we can guess because it is not explicitly mentioned here that moderation is excited at 1.5 second wave it means that that natural $T_M^P = 1.5 Sec$, right.

So, if it is so, then we try to find out what is the natural time period for the real ship and scaling also is given 1.36 second, right. So, the model scale the $T_M^P = 1.5 Sec$. So, we are using the Froude scaling. So, $\frac{T_S}{T_M} = \sqrt{\lambda}$. So, here the λ =36. So, therefore, that $\sqrt{\lambda} = 6$, right.

And then we understand that t I mean that time period of the $\frac{T_S^P}{1.5} = 6$ and therefore, we can find out that our in real ship $T_S^P = 9 Sec$ wave can cause a huge damage. I mean I do not say I mean it is a high response. So, I cannot say whether it is a damage or not at that particular anyways.

So, this is how actually we can solve some problem using the Froude scaling. Sometimes in fact, it is possible to share the both of the data like we can say that what is the natural time period for the model scale, right and also we can say the scaling factor and also we can say that what is the let us say C_{wp} coefficient and one can ask that what would be the added mass also.

So, let us see let us go back and just see that what I said just here. So, here you can see that sometimes it might be given that what is the model scale the scaling factor is given to you and also it is given to you is what is the value for the model time period this also gives to you and also the C_{wp} will be given to you.

So, if the all the data is given to you some even one can ask what would be the natural time period for the you know what is the added mass for the ship. So, in that case you have to use the this standard formula and use the Froude scaling and you can solve the problem, ok.

Anyway, so, let us move on today we are also going to discuss some more problems. So, let us take the next one.

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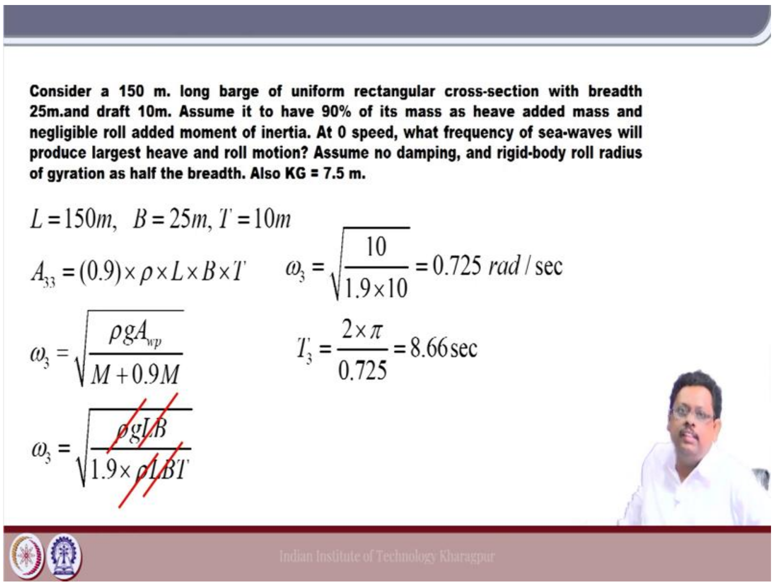

This, ok. So, it says that consider a 150 meter long barge I mean uniform rectangular cross-section. So, that means, that each cross sections it is uniform and it is 25 meter and then the draft is 10 meter and assume that we have a 90 percent of the mass can be considered as a added mass, right.

So, it says that at 0 speed what the free at what frequency of sea waves will produce the largest heave and roll motion? And, also it is given that we you know there is no damping, fine and then that the rigid body roll radius of gyration is equals to the half the breadth. So, it is half of 25 which is 12.5 and also kg is given ok. So, now how do I solve this problem?

So, let us first write that what is the formula let us solve the first problem and try to figure out what is the formula for you know the natural time period for or frequency for heave mode. Now, this data are given to you. So, $\mathrm{L} = 150m, \mathrm{B} = 25\,\mathrm{m}, \mathrm{T} = 10\,\mathrm{m}$ and also that it is given that it is 90 percent of the mass should be your divided mass.

So, it is $\mathrm{A}_{33} = (0.9)\times\rho\times L\times B\times T$, right ok and now this is the formula for your frequency ω_3. It is $\omega_3 = \sqrt{\frac{\rho g A_{wp}}{M + 0.9M}}$. So, I replace everything $\sqrt{\frac{\rho g L B}{1.9\times\rho L B T}}$. So, I know that that this expression will come, right.

So, A_{wp} I replaced by LxB here and also that mass actually I use this ρLBT. And, now, I can cancel the similar term. So, ρ is cancelled out, L is cancelled out, B is cancelled out. So, finally, I got that it is $\sqrt{\frac{g}{1.9\times T}}$. So, just I am taking approximately g=10 $\mathrm{m/s^2}$ and $\mathrm{T} = 10\,\mathrm{m}$.

So, then I can get $\omega_3 = \sqrt{\frac{10}{1.9\times 10}} = 0.725 rad/\sec$. We really do not understand much I mean we cannot visualize the frequency we it is much more clearer to me at least for me to know that what is the time period. Now, if I just time period if you use the formula it is $\frac{2\times\pi}{\omega_3} = \frac{2\times\pi}{0.725} = 8.66\sec$ which is almost 9 seconds.

So, what meaning? It means that a 9 second waves if heat that shape then it can have the high oscillation in heave mode, ok. Now, so, actually this problem gives a some nice practical feeling that I am going to discuss after this the problem. Now, let us solve the next problem which is here you can see that added moment of inertia for roll is ignored ok and radius of gyration is given.

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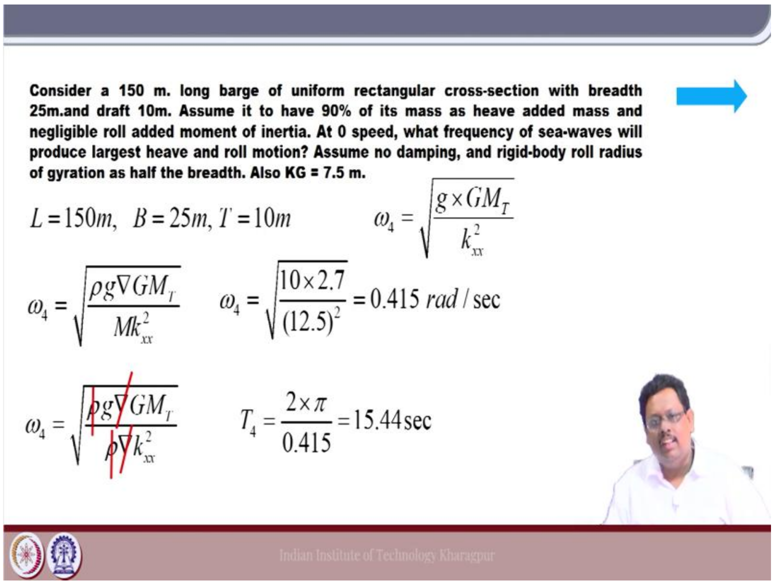

So, this is the basic parameters $L = 150m, B = 25m, T = 10m$ and this is the formula for the natural frequency $\omega_4 = \sqrt{\frac{\rho g \nabla GM_T}{Mk_{xx}^2}}$ in the roll.So, again I substitute M by the $\rho \times \nabla$; that means, the volume or ρ into ∇ and then if I cancel the similar terms here also the ρ and ∇ will cancelled out. So, finally, I have this ω x g x GM_T x k_{xx}^2.

Now, this the radius of gyration is given there it is half of the breadth, so, it is 12.5. However, I have to calculate what is the value of GM_T right, ok yes ok.

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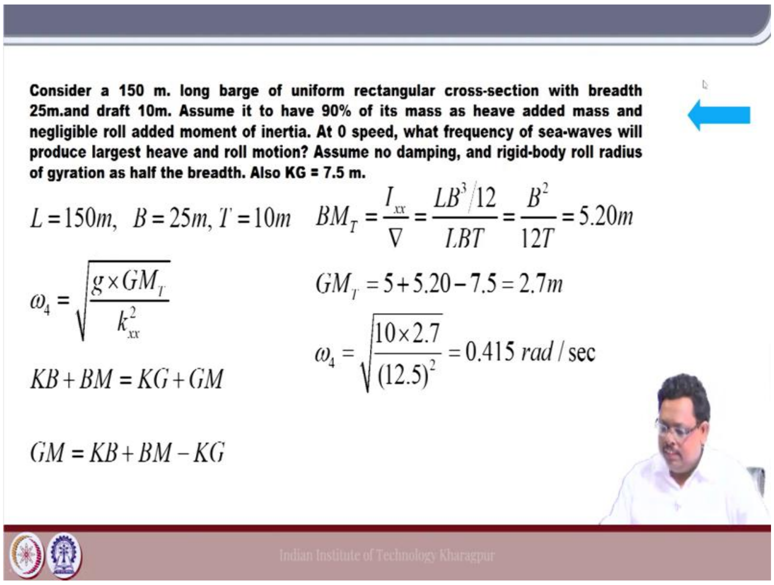

So, let us see now here we are going to use this formula up to finding out the GM_T. Now, we know that the $KB + BM = KG + GM$, right. Now, this is already discussed in our initial days lecture, right and I think you all know this formula very well. So, I just physically I can demonstrate here yeah. So, this is the pain here. So, this is the last point we call the kill, right and then you can take then G sum over here and B may be some over here and m maybe this.

So, I just say this $KG + GM$ should be equal to B sum over here. So, $KB + BM$ right and then from there I can get $GM = KB + BM - KG$. Now, here I will tell you that since it is applicable in case of a roll or in case of a pitch also, I did not mention explicitly this GM_T or GM_L. In both the cases $GM = KB + BM - KG$.

Now, in case of a roll that $BM_T = \frac{I_{XX}}{\nabla}$ ok. I_{XX} is nothing but $LB^3/12$ ok and volume is LBT. So, this is very well known, right. It is $LB^3/12$, it is this is the for a rectangle this is the second moment of area everybody know that so, nothing new over here.

So, I can get this equal to B square by 2 LT because if you cancel out the similar terms, it will become this $\frac{B^2}{12T} = 5.20m$. So, in case of a GM_T I am using that I mean BM_T I use 5.20. And, now since the draft is $T = 10\,m$, so, since the draft is 10 so, therefore, that buoyancy should be the middle point so I can take this as 5 and KG is given is 7.5.

So, therefore, I can get the value GM_T equal to KB is 5, BM is 5.20 and KG is 7.5. So, the value is 2.5, right and then we have that $\omega_4 = \sqrt{\frac{g \times GM_T}{k_{xx}^2}}$ just use this formula I can take $\omega_4 = \sqrt{\frac{10 \times 2.7}{(12.5)^2}} = 0.415 rad/\sec$ this comes out to be 0.415 radian per second.

So, here I get that value is $0.415 rad/\sec$ and let us take that what is the value here the time period $\mathrm{T}_4 = \frac{2 \times \pi}{0.415} = 15.44 \sec$ because it is you are much more comfortable with this time period. Now, you can see here it excite at 15.5 second which is very normal in ocean going vessel. That is what we discussed during our lectures about the natural time period of heave mode roll mode and pitch mode I discuss that in case of a roll it is much bigger compared to the not I mean in 5 second is huge actually I will tell you.

Normal in ocean going in ocean normal it is mostly this 10 to 20 second waves that is actually operating this the sea. So, therefore, this roll is very much into that domain. However, heave is 10, so, still it is better. Now, here let us try to this is not there in this problem, let us try to find out the pitch natural frequency, ok. Now, no data is given to you for pitch, but still we can approximate.

So, that is how I say that it is always of course, we are going to do everything numerically what where we do from the first principle and we solve this problem from the first principle I accept. But, however, it is always very important to get some engineering idea about the things so that at least you can do some very quick estimation and you can find out what is the natural frequency or search some basic things like this. Let us see here we do not compute anything which is using some kind of approximation.

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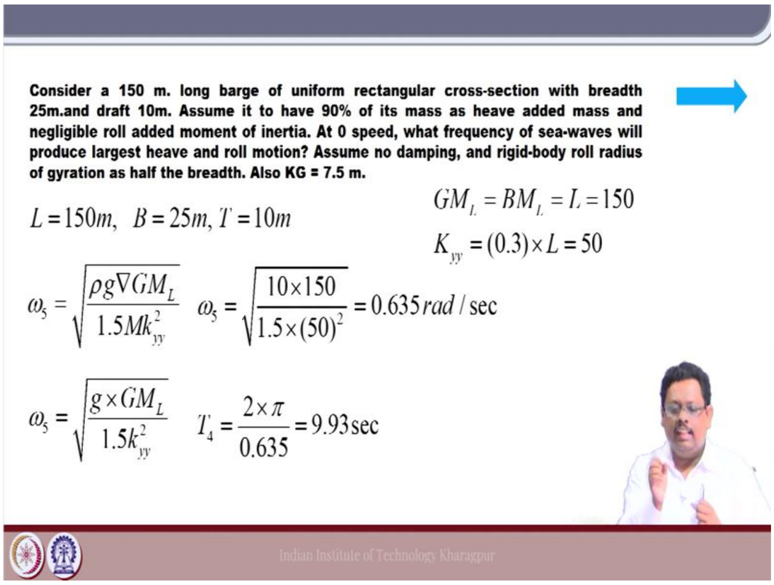

So, this is the given to you $L = 150m, B = 25m, T = 10m$. Now, what I approximate first that at and pitch added moment of inertia maybe the 50 percent of the you know that moment of inertia pitch moment of inertia. So, added moment of inertia I take is a 50 percent of this. So, therefore, in the denominator I am taking $1.5Mk_{yy}^2$, ok.

Now, we need to approximate so many things over here, right because again you know that volume will cancelled out and we and then we have the $\sqrt{\frac{g \times GM_L}{1.5k_{yy}^2}}$. Now, nothing is known to me, right. GM_L I do not know, K_{yy} I do not know, right then how do I understand that what would be the possible natural frequency for this particular vessel.

Now, here comes the idea of the approximation. So, I just approximate my $GM_L = BM_L$. Now, simply you can take here that instead of you know $LB^3/12$ it should be $L^3B/12$. So, you leave with earlier it is $\frac{B^2}{12T}$. Now, we have $\frac{L^2}{12T}$ that you can do. No problem. Otherwise what insert I did not do that also.

What I did is basically I am taking simply BM_L is the order of length 150. See here the whole point is the approximate. We are not computing anything. Similarly, I am doing with the $K_{yy} = (0.3) \times L$. So, I just take the one third of the length. Now, with this approximation, if I try to calculate the natural frequency ω_5 it comes out 0.6.

You know if you just do that it comes for 10 seconds. Now, you see like it is very much impractical you know in realistic answer I get that normally that heave and pitch time period are very close by. So, it is also comes out 10 second and that and then heave comes around the 9 seconds. So, you know you can do the same conventional process that I did for the roll because KG is given to you can simply find out what is your GM_L directly, right.

And, also you can take that you know K_{yy} is one third of L that also you can take and you do the analysis and you can find out that what would be the it is T_4 is given should be T_5 ok, anyway. So, what is the natural now here what the inter you know interesting observation I want to make as follows. Here it is 10 second for pitch; it is almost 10 second for heave. Now, you see suppose a 10 second wave is hitting the ship.

Now, then the ship will excite both in heave and pitch mode and why that is what I said it is very important to know the information about the phase. Now, you see in both situation heave and pitch excited by the same wave and then both tend to get to the maximum amplitude. Now, if both coming at the same time it means that it is heaving maximum at that time pitching also maximum. Now, you can see that how much of bending moment will come in this particular case.

So, that is why this phase is also very important. Now, if I see that both are not in a 10 second wave, but both have a phase like let us say it has this let us say some 10 second or sorry 0.1 second phase lag. So, therefore, when heave got maximum pitch may not be get maximum at the time, may be pitch may be the minimum at that particular time, right.

So, that is why this problem gives a very nice idea about you know when that is why this coupled equation of motion analysis is also very important for us right, ok. So, with this I have one more problem. So, let us very quickly see this problem and I will you know I will leave that most to you to do this in home, ok.

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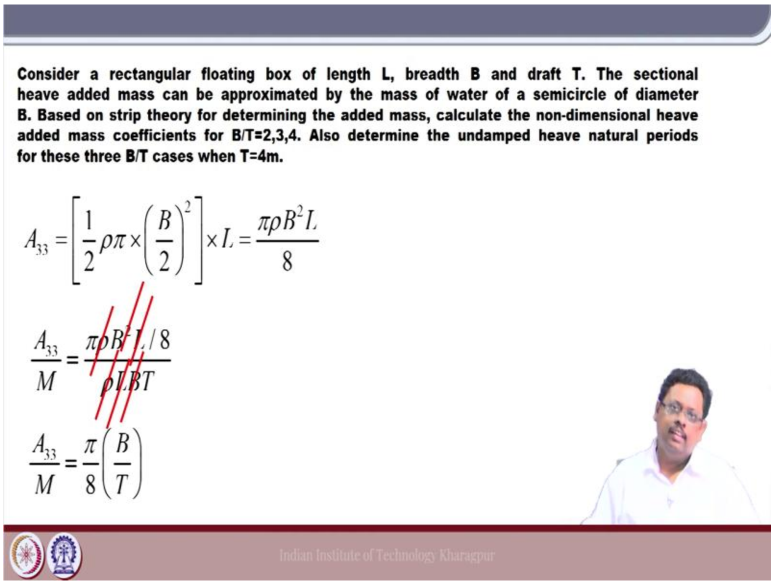

So, this is the problem we have the rectangular again we have rectangular barge with the length is L, B and T and it is given that the sectional added mass is given to you which is the which is the area of the semicircle whose diameter is equals to the beam, right. So, now, then you have to find out the added mass coefficient with respect to B/T ratio. So, this is the problem.

Now, how to do that? Now, here is given the sectional added mass is nothing but you know that that area of a semicircle whose a beam is equals to the I mean whose radius diameter is equals to the beam. So, therefore, the area is nothing but you know $\pi \times \left(\frac{B}{2}\right)^2$ for the circle and then if you multiply by another $\frac{1}{2}$, then it is the area for the semi circle now if you multiply by the row. So, it is a it is a mass.

And, then if you multiplied by the length L then it becomes a you know the volume. So, it is that dimension equal to the dimension of the mass. So, I understand A_{33} is equal to $\frac{\pi\rho B^2 L}{8}$. Now, what is the dimensionless quantity? So, I just divided the added mass by mass. So, mass term is LBT and this is the added mass I just compute here and if you cancel out the similar term it becomes $\frac{\pi}{8}\left(\frac{B}{T}\right)$. So, this is the first part of the question.

Now, if you replace $\left(\frac{B}{T}\right)$ by 2 3 4, then you can find out what is the ratio. Next question is the what is the natural time period for the undamped system.

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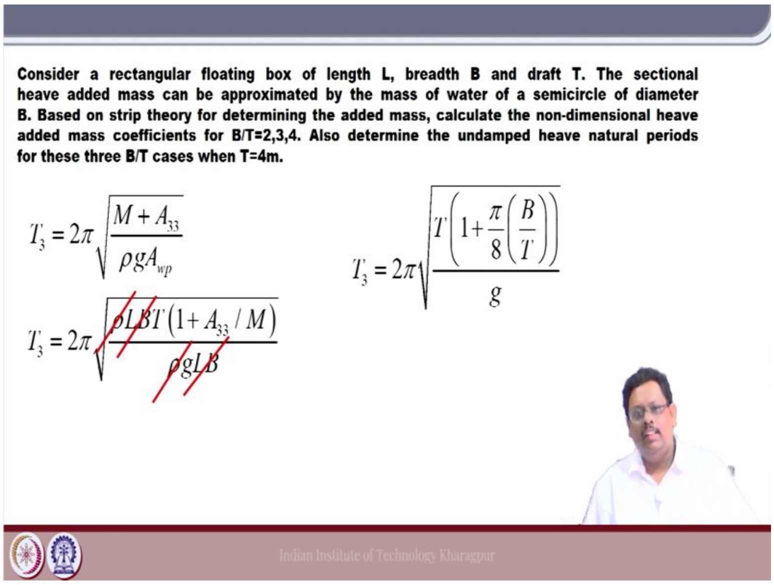

And, now this is the formula for the undamped system, $T_3 = 2\pi\sqrt{\frac{M + A_{33}}{\rho g A_{wp}}}$ right. So, now, what I what you can do over here you know I know the ratio $\frac{A_{33}}{M}$. So, I can divide the whole thing by mass and I replace the mass by ρLBT and then $A_{wp} = LB$ and again you can cancel out the similar term and finally, you have this term

$$T_3 = 2\pi\sqrt{\frac{\rho LBT(1+A_{33}/M)}{\rho gLB}}$$

$$T_3 = 2\pi\sqrt{\frac{T\left(1+\frac{\pi}{8}\left(\frac{B}{T}\right)\right)}{g}}$$

Now, you can put $\left(\frac{B}{T}\right)$ ratio 2, 3, 4 and you can take that the T = 4 and then you can find out what is the natural frequency for this particular vessel with different $\left(\frac{B}{T}\right)$ ratio ok, fine. So, just work out this and find out those numerical values ok. So, with this let us stop today.

Thank you.