Foundations of Classical Electrodynamics
Prof. Samudra Roy
Department of Physics
Indian Institute of Technology - Kharagpur

Lecture – 02
Vector Analysis (Cont.)

Hi students, to the second class of the foundation of classical electrodynamics. So, today we will continue the vector analysis that we started in the first class.

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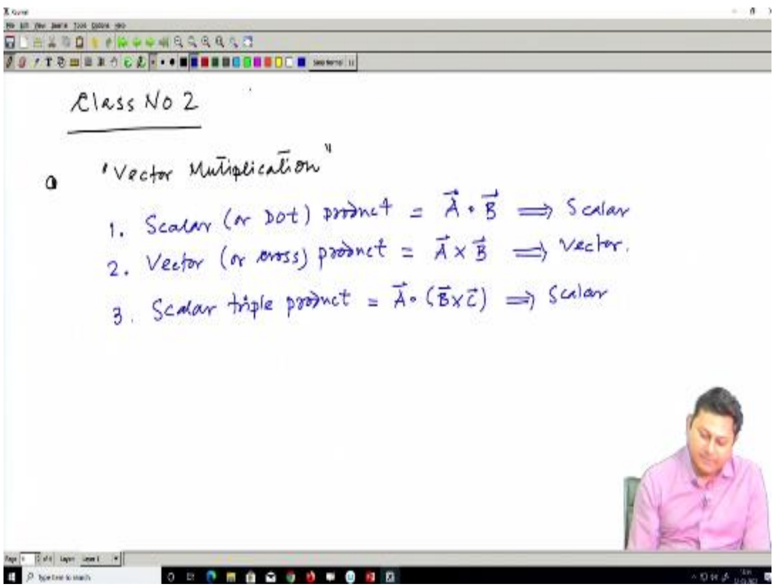

This is class number 2. So, we already know the few important properties of the vector that I believe most of the students are aware of, but still, this is a refreshing kind of stuff we are doing. So, the next important thing regarding the vector is the vector multiplication. So, 3 kind of vector multi 4 kinds of rather vector multiplication we can think of. So, one is scalar or simply dot product, which is if A and B are the 2 different vectors or same vectors,

Then, we define this scalar product as A dot B. Next, vector or cross product, in cross product we write this A cross B. Mind it in the resultant of these things is a scalar. The outcome of this quantity is a vector. What is the third one? It is called scalar triple product, where I have 3 vector, A dot B cross C. Again, these outcomes will be a scalar quantity.

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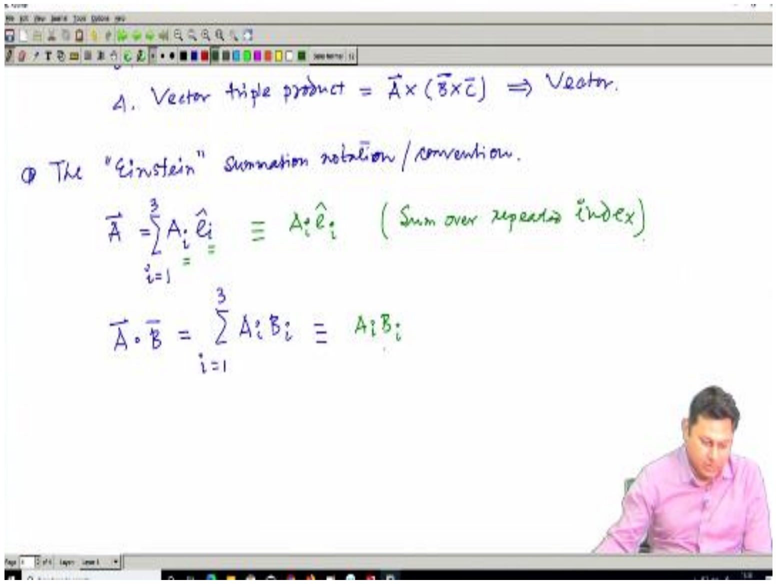

And finally, another kind of a vector multiplication one can think of and that is vector triple product. These are the basic I should say, these are the basic vector operations or vector multiplications. You can think of more complicated one, but if you know these 4 basic vector multiplication procedures you can do whatever you want. This quantity should be a vector quantity. So, these 4 kinds of vector product we will see that in this particular course, in different cases it will appear and then we need to execute these operations in different form.

Well, after that I like to introduce something very interesting and that is called the Einstein summation notation this is a notation but very useful notation or summation convention. So, what is says? So, let me go back to the way I define the vector in this basis form. So, A can be written in this way. So, you can see we have a summation sign here. So, whenever we have a summation sign, and if we have indices here, which is repeated for example, here we have i and here also we have ei, so, we can remove this summation sign.

So, I can simply write A_i e_i vector. I just simply remove the summation sign here and it you can do that when you have a repetitive indices here. For example, here i and i both are there, these are repetitive so, that means, it is sum over repeated index. So, if I write this that means, already a sum is there and I am just omitting the sum summation sign because the index are repetitive. I can give you another example and that is A dot B this is essentially A_i, B_i, i 1, 2, 3. A_i, B_i are the components. And this you can simply write in Einstein summation convention it should be simply A_i B_i.

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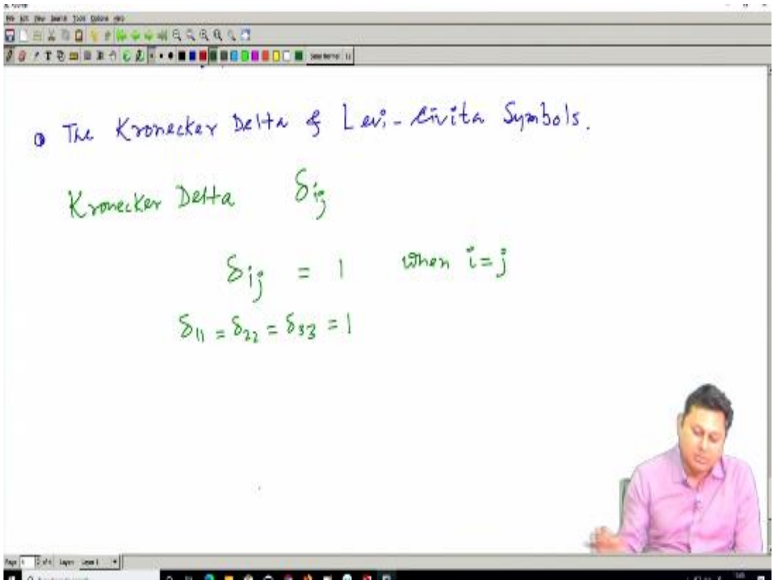

Next again an important thing like define the Kronecker delta and another symbol and that is called the Levi-Civita symbol. So, the Kronecker delta and Levi-Civita symbol very important. So, let us understand what is this Kronecker delta. So, Kronecker delta I already mentioned in the first class that this is my Kronecker delta, δ_{ij} and what is the beauty of this definition or what is the property? δ_{ij} should be equal to 1, when i = j, these 2 indices i and j when they are same then this δ should be 1.

For example, $\delta_{11} = 1$, $\delta_{11} = \delta_{22} = \delta_{33}$ this should be 1. Because i and j are same here and i can run 1, 2, 3; j can run 1, 2, 3.

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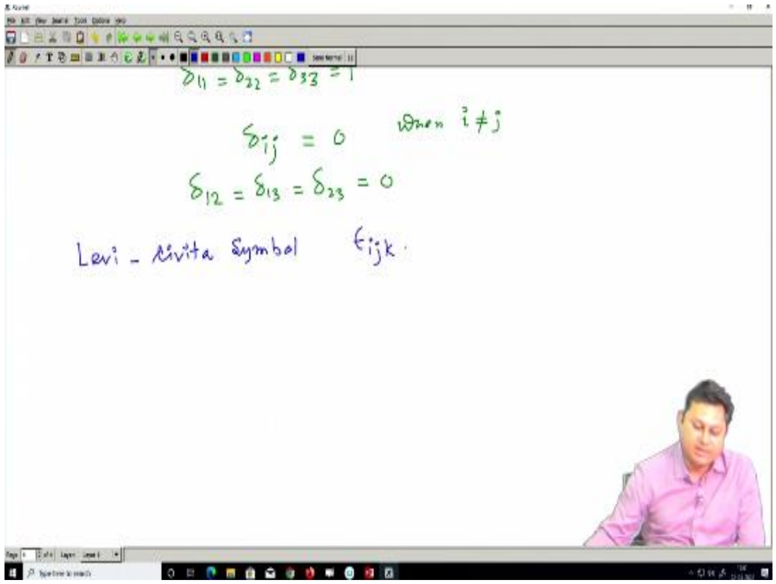

However, δ_{ij} should be equal to 0 when i is not equal to j. If δ_{12} I consider, if I consider δ_{13}, if I consider δ_{23}, these quantities should be equal to 0 because here i and j are not same. So, this is basically the Kronecker delta by definition. Now, let us understand the Levi-Civita symbols. What is Levi-Civita symbols? Like Kronecker delta we have here something called ε_{ijk}. Previously we have 2 indices one i and j, here we have 3 indices ijk. What are the properties?

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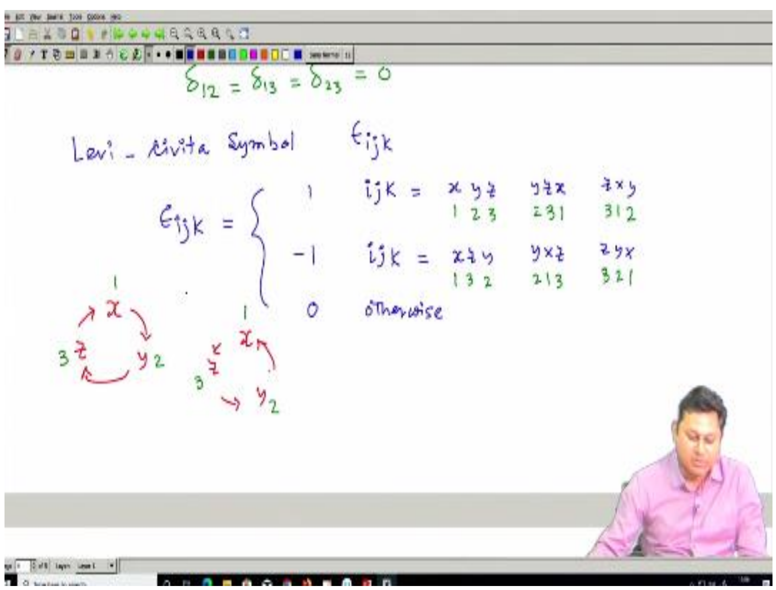

The properties are ε_{ijk}, it should be equal to 1 when ijk, which can take the value like 1, 2, 3 or x, y, z. If they are in certain order, for example, if I consider this as a x y z then when it is x y z or when it is y z x cyclic order or it is z x y in terms of 1 2 3 if I write it should be like when it is 1 2

3, when it is 2 3 1 and when it is 3 1 2, this value is 1. This value should take -1 when ijk are in opposite you know in opposite orientation. What is that? If it is x z y or y x z or z y x, so, if I draw them, things will be a little bit clearer, so, let me do that here.

So, I am having x here, I am having y here, I am having z here and they are forming a cyclic loop. Now, if I write it as 1 2 3. So, 1 2 3, 2 3 1 and 3 1 2 they are in this right-hand cyclic loop and then the all the value of ε_{ijk} should be 1. Now, if opposite happened, so, if opposite happened again I have x and y and z, and this value as usual 1, 2 and 3 and opposite happened means x z y or y x z or z y x, so in this case my here I am making a mistake, my direction should be opposite here because I am going in the opposite direction.

So, it should be in this direction. And what are the values here so, it is 1, 3 and 2 and this is say 2 1 3 and this value is 3, y is 2, 1, so, then ε_{ijk} will take -1. But interesting it will take 0 in all other cases otherwise and what are the otherwise case because there are ijk, so, it can be possible that j and k should be same for example x y y So, if y y is there, then ε_{ijk} should take the values 0. So, there are many combinations.

So, these 2 combinations these 3 combinations will give you 1, these 3 combinations will give you - 1, but all the other combination when we have the repetitive value for example, here xyz all are distinct 1 2 3, 2 1 3 but if you have set 2 1 1, then this quantity will be 0. How many quantities will be there that I want the student to figure out by yourself that how many ε_{ijk} value you can have when i j k can take 3 values, i can take 1, 2, 3; j can take 1, 2, 3; k can take 1, 2, 3 and you can now find out how many ε_{j} ε_{ijk}, how many values are there. So, this is like a home work for you.

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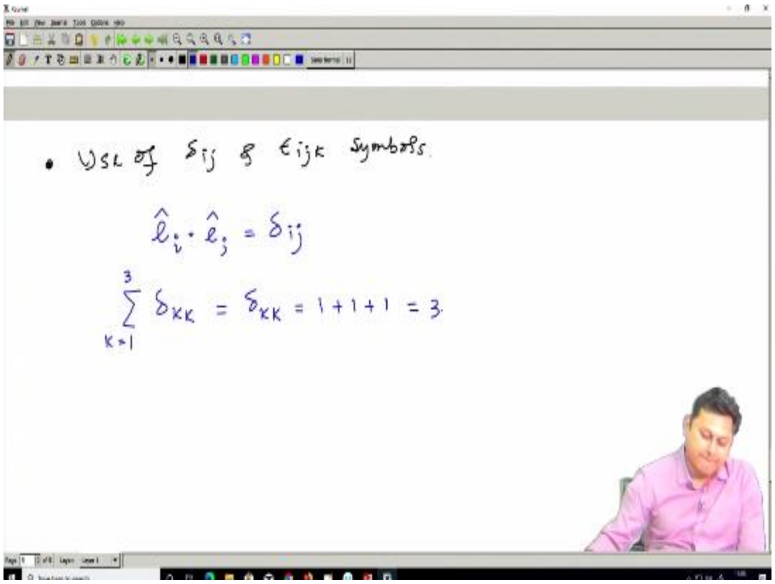

So, after that, we will discuss more about the properties and usefulness of these Levi-Civita symbols and δ functions Kronecker delta functions and δ symbols. So, use of δ_{ij} and ε_{ijk} symbols. What are the uses that we will be going to understand. So, first already it is mentioned that how for orthonormal basis e_i dot e_j we use this δ function Kronecker delta symbols, ε e_i dot e_j is δ_{ij} already mentioned.

Also, another thing for example, δ_{kk} summation, k can take the value 1 2 3, these dummy indices and take 1 2 3. According to Einstein notation, I can remove these summation sign because we have a repetitive index so, simply δ_{kk}. So, what should be the value of this quantity? k can take 1, k can take 2, k can take 3, which should be 1 + 1 + 1. So, it should be 3.

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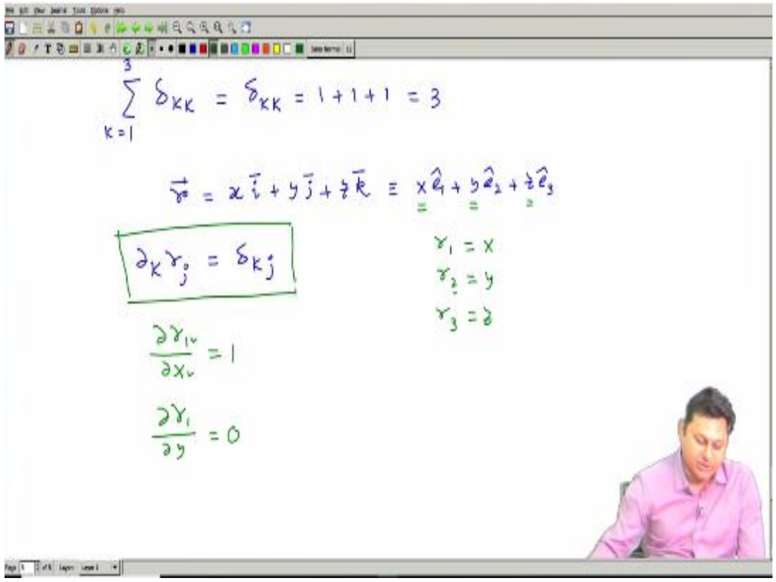

Also, now I define r is a position vector in the last class, this is a position vector and this vector it is like xi + yj + zk. This is the way we normally define, but it can also define like x e_1 y e_2, x e_1 + y e_2 + z e_3, in general way where e_1 e_2 e_3 are the unit vectors the same thing ijk just a notation there is a difference in the notation, but the things are same. Well, now, if I make a derivative for example, any component rj. So, it should be simply δ_{kj}. I already mentioned this is these things.

And now, component wise this r_j is the different component. Here these different component means, I am talking about this x, y and z. So, that means, r_1 is nothing but x, r_2 is nothing but y and r_3 is nothing but z. Now, if I look carefully what should be the value of this derivative. So, and δ k 1 2 3 means, this k_1 means it is x, k_2 means y, k_3 means z. That means, I am making a derivative, partial derivative with respect to x y z by taking the value of these dummy indices k 1 2 3. So, 1 2 3 is nothing that it corresponds to x y z here.

Now, if you look carefully that how these things is defined that $\frac{\partial r_1}{\partial x}$ that should be 1. So, here 1 and x that is it is k is 1 and j is 1, δ_{11} so, it should be 1. What about the other values? k r1 and delta y, that is this y is 2 here because this corresponds to 2 so, that means, in this formula I have δ and k here is 2 and j here is 1, so, δ_{21}, so, that that means it should be 0 and so on. You can please check this and you will find that this relationship how this relationship is holds for position vectors.

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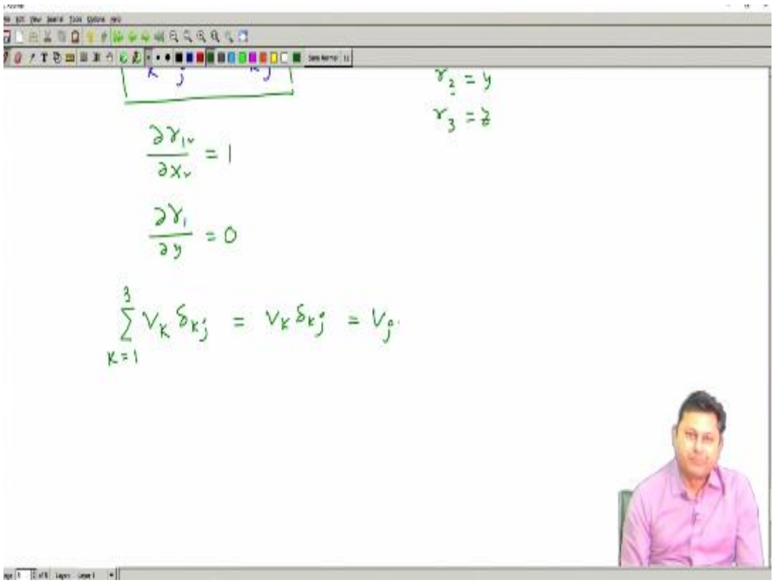

Also summation of say v_k δ_{kj} so, I multiply v_k so k is a variable here it can take k 1 2 3 and also j can take the value 1 2 3, but here k is repetitive. So, I can use the Einstein notation I can have you know I can have v_k and δ_k and j, j is a given value, k is changing 1 2 3. So, these values if you look carefully, when k = j then only this is a meaningful, apart from that, this δ value is always 0. So, that means, it will give me the j th component of v. This is the way this δ one can use and one can operate.

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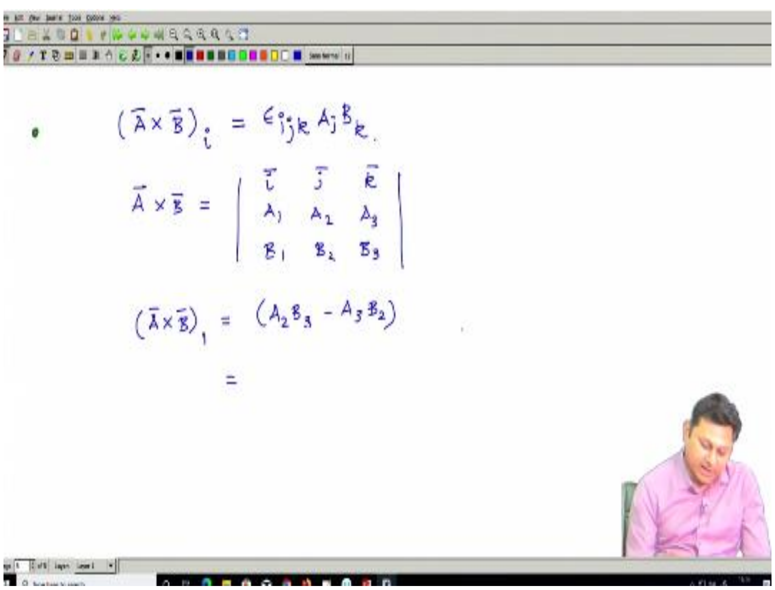

Now, try to understand the operation mechanism of the Levi-Civita symbol. So, we have the cross product, A cross B and if I want to find out this i th component of this inter vector quantity and simply it is ε_{ijk}, then A_j B_k. This is the way they ε operated, the Levi-civita operator operates. So, you can find out, A cross B what we are having A cross B, i j k A_1 A_2 A_3 B_1 B_2 B_3, these are the x y z component, I am just writing A_1 A_2 A_3, what do we have, if I want to find out the A cross B.

I find the 1 that means the x component, if I want to find out the x component, whatever you get, you will get A_2 B_3 - A_3 B_2, this is the value we are getting. Now, if I write in terms of these symbols, you can see that here j and k are repetitive that means the summation sign is already there. So, I am just removing the summation sign. So, j and k are repetitive means you are changing these j and k to 1 2 3 and 1 2 3.

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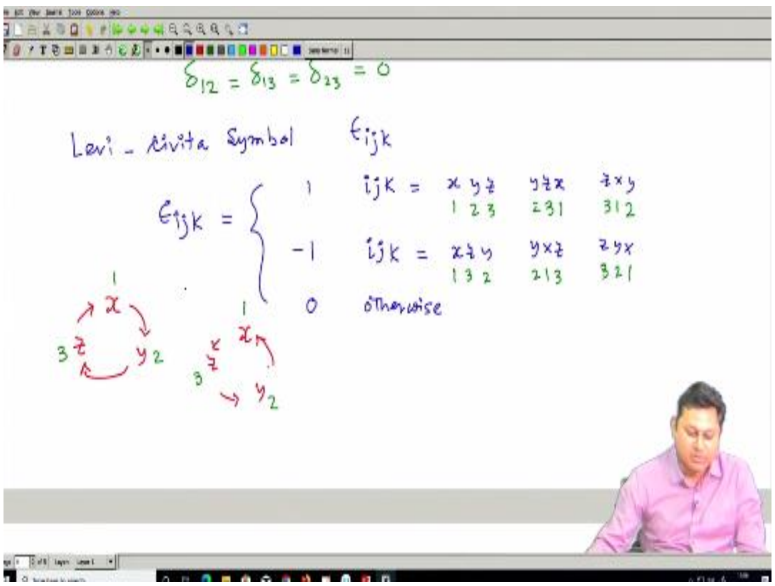

But ε has this property that not all the values is 1, not all the values is 0, there are some values these are 1, there is some values, which are -1 and 0.

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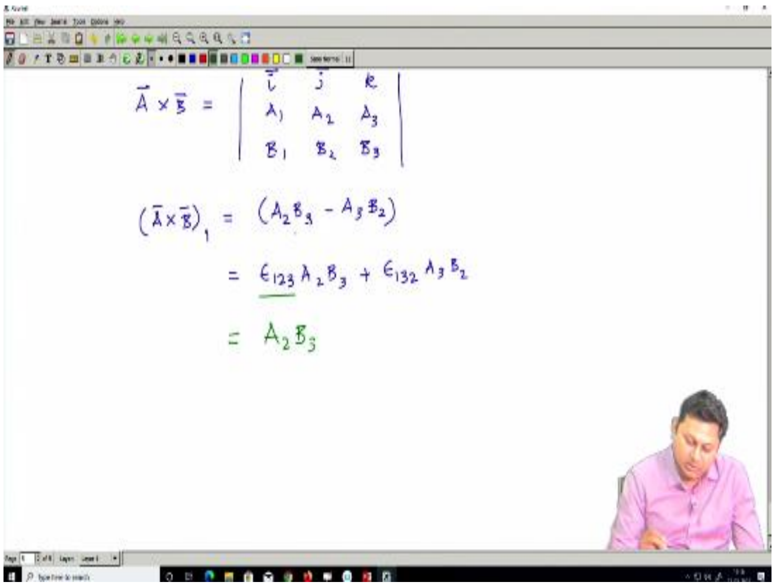

Now, if I put all these values together, you will find only nonzero value are ε_{123} and ijk all are distinct then only we have done nonzero ε value. And another term you will ε_{132} and you have A_3 and B_2, why is that, because according to the notation is it ε_{ijk}, j can take 1 2 3, k can take 1 2 3. But if j and k same then if ε is 0, so, all these values should be 0 when j and k are same.

Apart from that which distinct value it should take, i is 1 here. So, the distinct value 1 j and k are taken 1, j is 2, k is 3 and when j is 3, k is 2, so that 2 value I take when j is 2, k is 3 and when j is 3 k is 2. So, these 2 values combinations we take and we will get these results. Now, if you look carefully, ε_{123} this quantity is 1, + 1. So, I should have simply A_2 B_3 whatever ε_{132}. If you go back here and check that ε_{123}, ε_{132} that is this one 132 first one these values -1.

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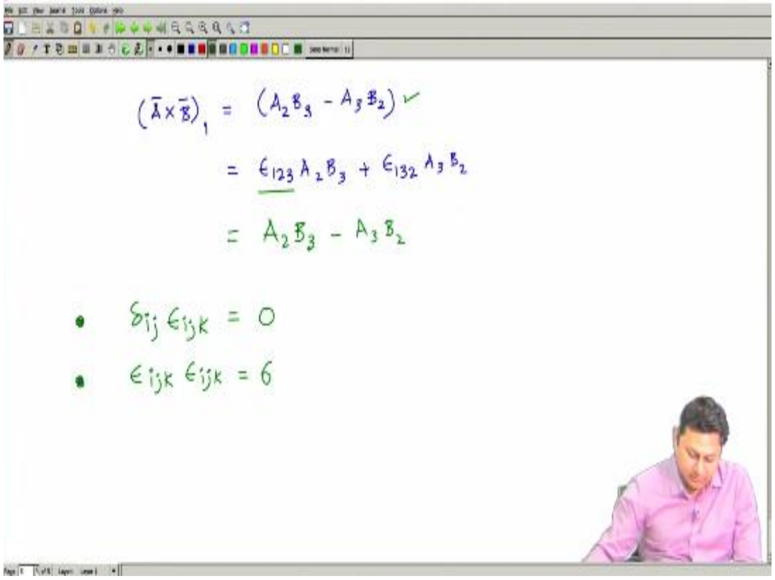

So, that -1 if I put so, it should be simply minus of A_3B_2 which we already had here. So, this is the way we can define the cross product in terms of this Levi-Civita symbol, ε_{123} very, very powerful and very, very useful. Now, few more things. So, δ_{ij} ε_{ijk}, what value we will get now? Now, you can see this is a very interesting kind of problem. So, you can see that i and j and k here. So, when δ is nonzero then i and j should be same, but if i and j same then Ɛ is always 0.

So, that means, this value should always give you 0. Another problem what value of ε_{ijk} and ε_{ijk} multiplied by when ijk here. Again, you are going to see that there are different combinations for which Ɛ will be 0, because when j and k are same, i is changing 1 2 3, j is changing 1 2 3, k is changing 1 2 3 only the meaningful value you will get that when they are distinct. And in that case, you will find this value if I add because there is summation over that because these are the repetitive index. So, you will see that these values should be equal to 6.

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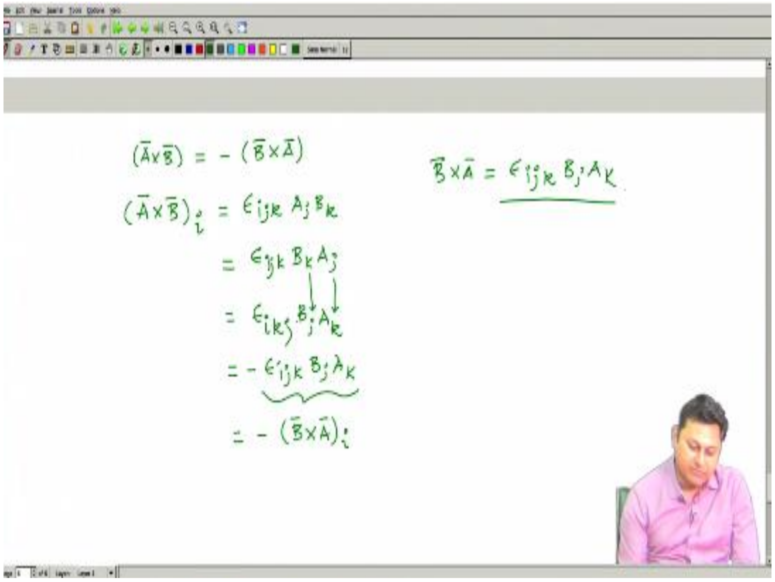

Well, also with these notations, you can find something. For example, A cross B and we know that it is minus of B cross A. So, now, we will be going to prove this using this Levi-Civita symbol. So, A cross B, I need to put the vector sign here i th component it is simply ε_{ijk} A_j B_k that is the way one can have. Now I can manipulate this right-hand side. Let us write you know ε_{ijk} B_k A_j. Now, I change this k to I can write it in this way. I just can change this k to this form usual form j and k, j to k.

If I do then I should write Ɛ, I replace this j, i is there, I replace this j to k and k to j because these are the dummy. I can have B_j and A_k. Mind it I replace j to k here and k to j here. k is replaced by j, j is replaced by k because these are the dummy I can always do that interchanging. Now, I have ijk I now write it in a proper way. So, if I write ε_{ijk} I need to put a negative sign because I am changing this order. So, B_j A_k so, now, these things are now following whatever the cross-product symbol is there.

So, this quantity is minus of B cross A with i th component. Because if I write B cross A in Ɛ symbol it should be ε_{ijk} B_j A_k. So, in order to have this form, I just manipulate it and find that A cross B i th component is simply B cross A i th component with a negative sign and that we know.

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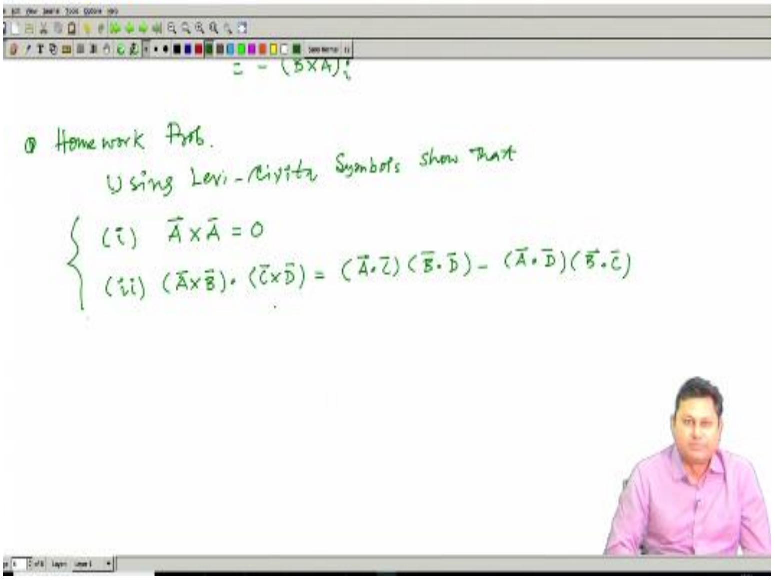

Now I am giving a two homework problem and then I will complete this class. So, these are the homework problem. So, the problem is using symbol show that so, you need to use Levi-Civita symbol to show one these 2 vector identity A cross A = 0 and second this big identity, A cross B dot C cross D is equal to you need to prove it A dot C and then B dot D - A dot D and B dot C. So, these two problems I like to give you student as a homework.

So, I believe you can do this problem by using the symbols please go through the class very carefully this whatever the steps are done all the steps is there. And I like to you know, gave you these two simple problems to check whether you can use this Levi-Civita symbol and δ symbols Kronecker delta symbol properly to find out this once you know, once you understand these symbols, the important thing is that once you understand these symbols, you do not need to remember these complicated vector identities.

You can whatever the vector this kind of problems is given to you by using these Levi-Civita symbols. You can find it out by yourself. You do not need to remember anything this big, you know, expressions. So, with this note, I would like to conclude here in today's class because I do not have that much of time to continue in the next class. Again, we try to understand the concept of Levi-Civita symbols and try to solve few other vector identities like the triple product identity and others identity. So, thank you for your kind attention. So, let us meet in the next class and do more about this vector analysis. Thank you.