Foundation of Classical Electrodynamics
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Lecture - 21
Concept of Charge, Charge Density, Coulomb's Law

Hello students to the foundation of classical electrodynamics course. So, today we will be going to start our module 2, which is electrostatic and today we are going to discuss about the concept of charge, charge density and if time permits then Coulomb's law.

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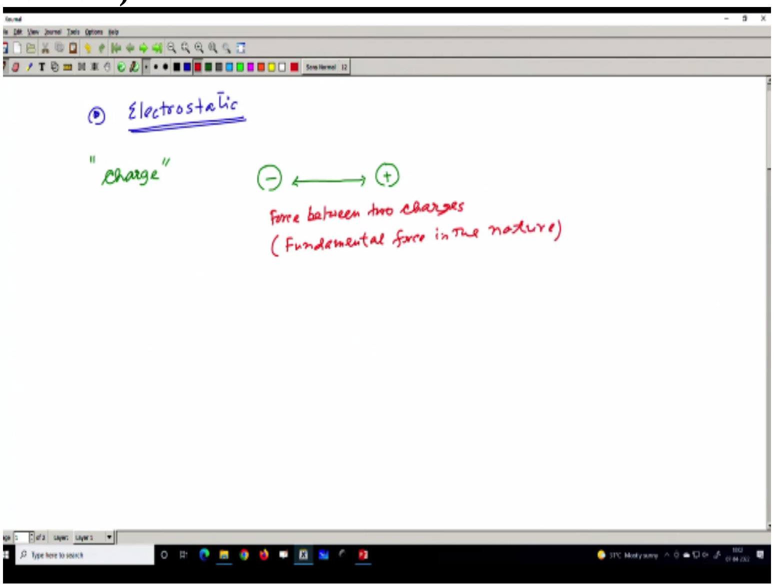

Class number 21 so, our topic today we start electrostatic. So the first thing that we are going to discuss about charge. So, we know that we have 2 kind of charges available in the nature one is negative another is positive and they interact with themselves with certain amount of force like light charge they ripple each other unlike charge they attract each other too. So, force between 2 charges are the force, which is one of the fundamental forces in nature. that means there are other forces also available.

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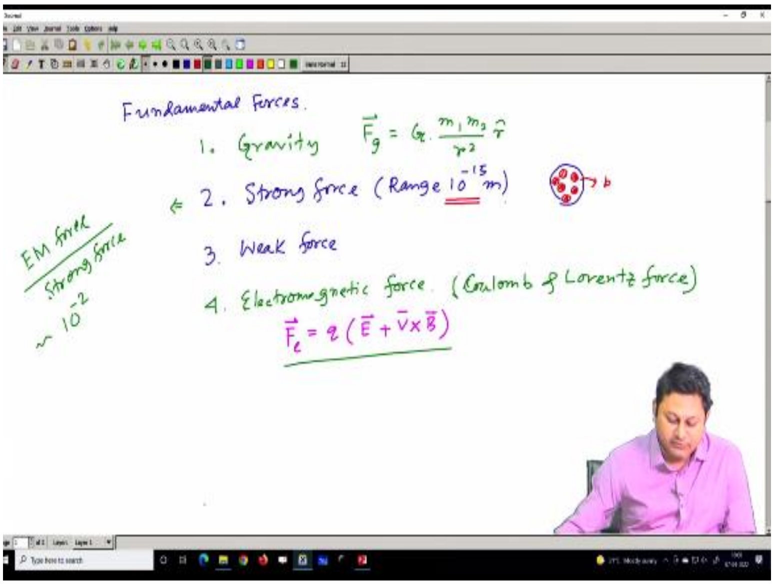

So, if I list them, so, which are the fundamental forces? The fundamental forces are namely the gravitational force were you know that the force is defined by 2 particles m_1 and m_2 these are the mass and height is the distance between them, then this is the way it works. Then the second one is strong force this is a strong nuclear force I am talking about which is a short range force. So, this range is around 10^{-15} meter.

Typical range of the short range forces around this where I mean this is the force where you know that if you have a nucleus and inside the nucleus, these are suppose protons, they are staying in a very tight region even though they are all with positive charges, but they stay there. These are protons and they stay there because of the strong force nuclear force. And basically the binds the baryons and muons. But this is a very short range and this range you can see this around 10^{-15}. So, that is why it is short range force.

Another kind of nuclear force is also available. This is called the weak force. This mediates the neutrino interaction and finally, we have the electromagnetic force. So, these 2 things written in the blue colour are the strong and weak force but these are the short range force on the other hand this gravity and electromagnetic force they are the long range force. So, this electromagnetic force I mean this is a Coulomb essentially this is a Coulomb Lorentz force.

So, if I write down the force it is q ($\vec{E} + \vec{V} \times \vec{B}$), the first part is Coulomb force and second part is Lorentz force due to the magnetic field and that is why it is called the Coulomb Lorentz force, so, this is the form of this force. If I want to find a qualitative understanding about the range about you know the magnitude of the strong force and these things, if I considered this strong force to be 1, suppose, the magnitude, then the ratio if I take then the electromagnetic force should be the 100 of the strong force.

So, EM force divided by strong force is of the order of 10^{-2}. So, two order less. On the other hand, if I will calculate that maybe the gravitational force and if we compare the electromagnetic force, so, this value if I make a ratio of the gravitational force divided by electromagnetic force it should be of the order of 10^{-39}. So, gravitational force compared to the electromagnetic force is very weak that we need to understand. Now, at this point maybe we can discuss about the why we call this topic as foundation of classical electrodynamics.

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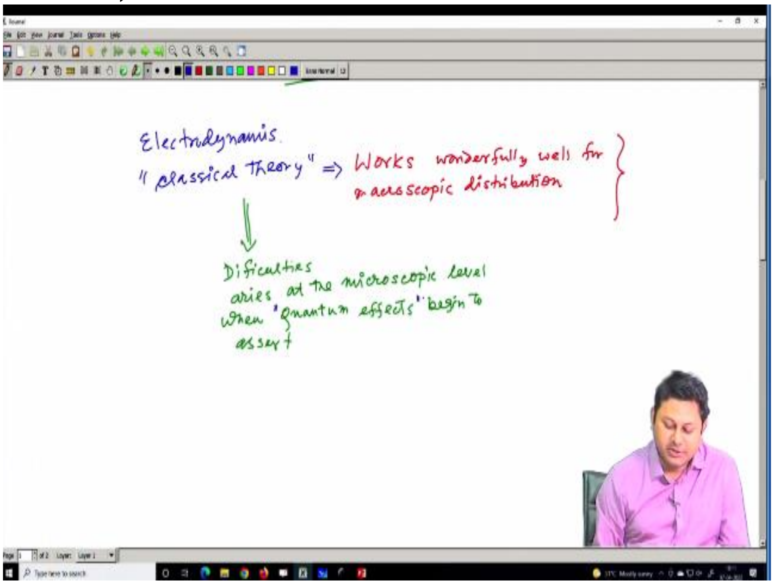

So, in electrodynamics classical theory whatever the theory we are going to discuss in this entire course, this works wonderfully well for macroscopic distribution. What are the macroscopic distributions like capacitor then these kinds of systems like electromagnetic capacitor etcetera where we are not dealing with a microscopic element. So, in this regime the classical theory works nicely, but it became very difficult to work in this classical domain when the scale is microscopic.

So, difficulties arise at the microscopic level when quantum effects begin to assert so that means, when we try to understand these standard electromagnetic theory, which we called the classical theory in microscopic level when the particles are charge particles are microscopic in nature. Then the quantum effects will be going to arise and these classical theory does not work well there. Now, what is in quantum level, what happened in quantum level? If you try to understand very crudely then in quantum theory.

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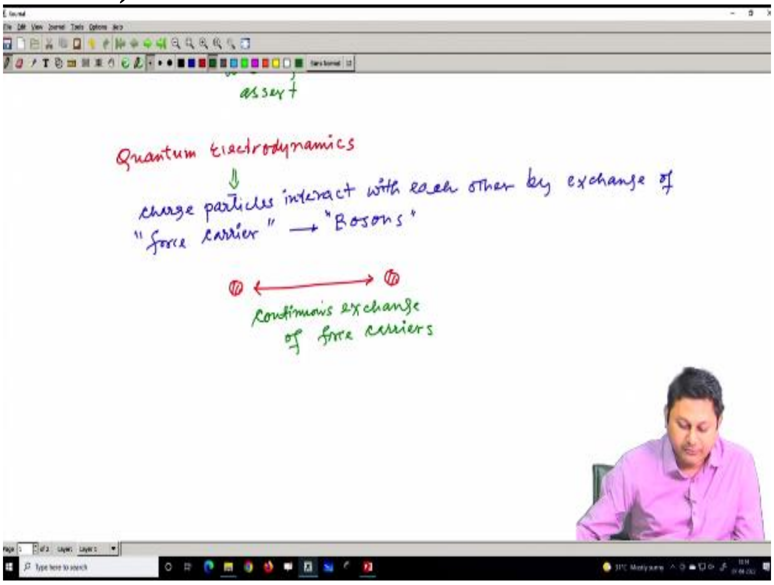

In quantum electrodynamics, what happened so, the charge particle interacts with each other here what happened? The charge particle interacts with each other by say some exchange of force carrier, which is generally called the Bosons. So, that means constantly when 2 particles are there in quantum level these 2 particles they are going to interact with themselves by exchanging you know continuous exchange of force carrier in the form of Boson's.

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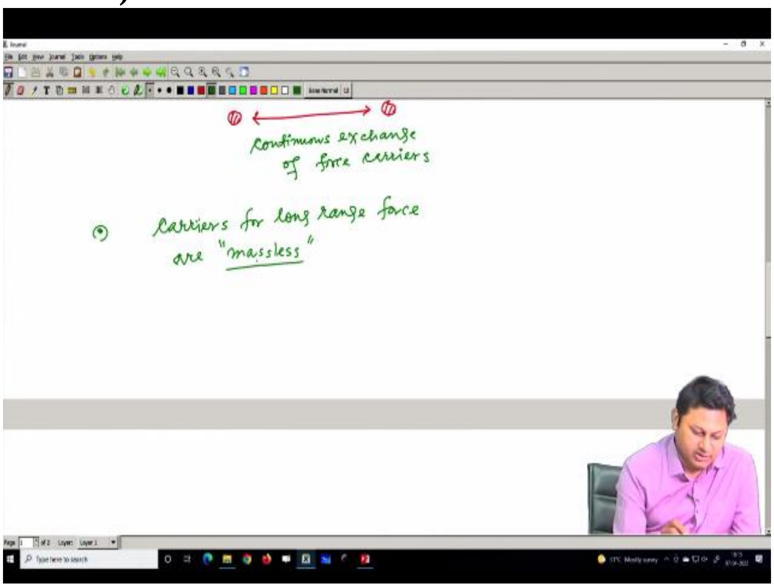

So, now for another thing we need to do so, this is one and another thing is if I try to understand this is a very crude statement we are making just to try to understand that what happens in quantum level so, this carriers whatever the carriers I mentioned for long range force are massless and this is eventually this massless things are nothing but massless Bosons are nothing but photon and what is long range force? Long range force I already mentioned here the gravity and electromagnetic force these are the long range force.

So, you can see that for electromagnetic force when we have the electromagnetic force then in quantum level the 2 charge they are interacting with each other by exchanging particle and this particle should be massless in nature and we call it photon, later we will maybe going to discuss this in detail.

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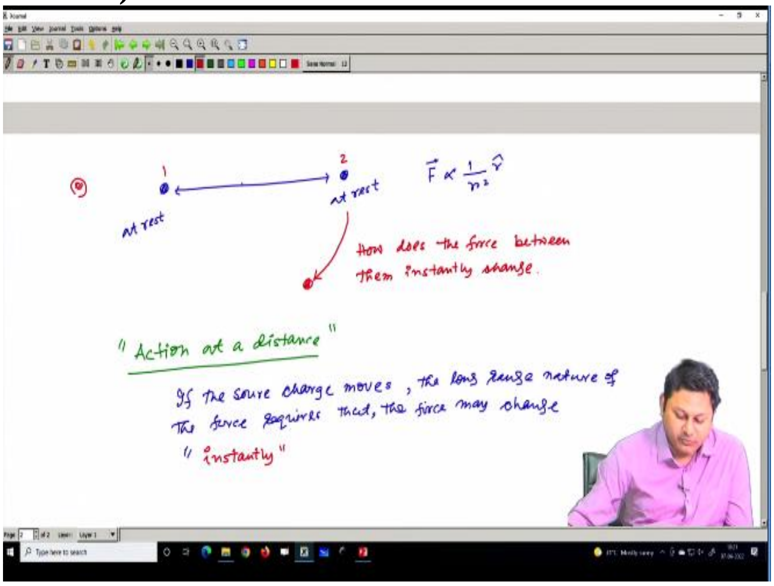

So, you can see that this is one aspect the next very important aspects that we need to understand at this point is something called action. So, these 2 charge particles are there suppose and the inverse square law whatever the force we are having here which is proportional to $\frac{1}{r^2}$ this works very nicely unless these 2 charge particles are static at rest, but if one charge particle moves from here to here then the question is how does the force between if this moves.

The question is how does the force between them whatever the force they have instantly change. So, the meaning is suppose this particle charged particles sitting here and a very long distance another particle 2 is sitting. Now, they have the interaction with this force the inverse square law so, they know each other, they are interacting each other with that force now, suddenly one particle is moving from this point to this point. So, what happened that how instantly the force will change here?

Because, how this particle will know that this particle 2 is changing from this point to this point, because we know that it will take some time the information to go from this point to this point, this is point the basic understanding of the special theory of relativity, where it is considered that no information can move faster than light. So, obviously, they are very long distance. So, there should be some time it should take to go the information go from this particle to this particle and the information is it is now moving in another point.

So, that is here we consider that they are changing instantly and in physics it is called action at a distance so that means as soon as the particle change from this point to this point, this particle will be going to know instantly that there is a change and accordingly the force will be going to change there. So, this action at a distance is saying that if I say if the source charge moves, the long range nature of the force requires that the force may change instantly.

So, this is an instantaneous change of this force and that assumption is called actions at a distance, but as I mentioned that the special theory of relativity does not consider this if you consider the special theory of relativity this concept is no longer valid. So, what we get from this brief discussion is the classical electrodynamics?

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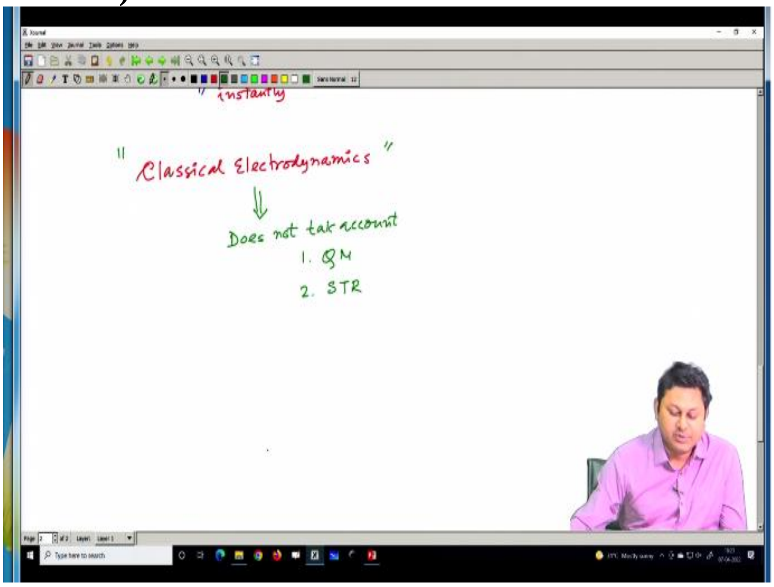

This thing does not take account 1 quantum mechanics and 2 special theory of relativity, so, this is not classical electrodynamics the subject we will be going to discuss here in this level it does not take account the quantum mechanics and special theory of relativity at all. So, all the charge particle there stands still static and the static charge gives rise to electric field, static electric field and when it is moving with the constant velocity that gives rise to you know this magnetic field.

And when they accelerate they will radiate electromagnetic radiation will be there and at this stage, we will consider this action at a distance and purely the classical picture we are going to discuss where the quantum mechanical aspect that means, in the microscopic level what happens or what happens if the charge is moving relatively these things will not be going to take account. So, that is why this course is in classical level also, based on that too, we like to mention one thing.

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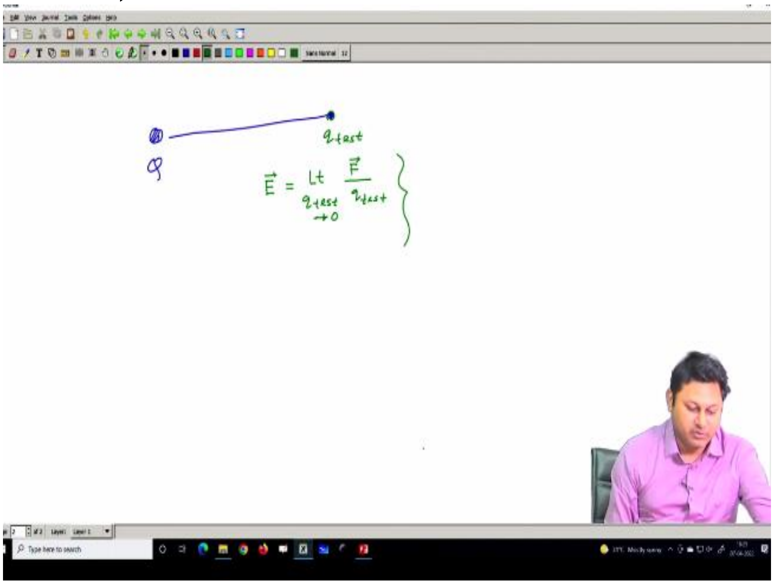

So, suppose we have one charge here say be Q and I am putting a taste charge very far away. So, this is say q_{test}. Now, obviously, by putting this test charge here, it should have some field with itself. So, there should be some change in the force between them. So, we will consider that this is infinitesimally small. So, when we calculate the electric field should be calculated under the limit that this test charges is vanishingly small. And whatever the force we are having, this is force divided by that test charge.

So, for unit amount of test charge, what will be the force we are having that we consider as electric field here and with the limit distance to 0 and this is the way we define the electric field now on that I just have a very tiny amount of charge here, which is having vanishingly small amount and then whatever the electric field we have and mathematically this is the way we define to get rid of the interaction of these charges, which is present here, with this very brief overall discussion about what happened for classical picture.

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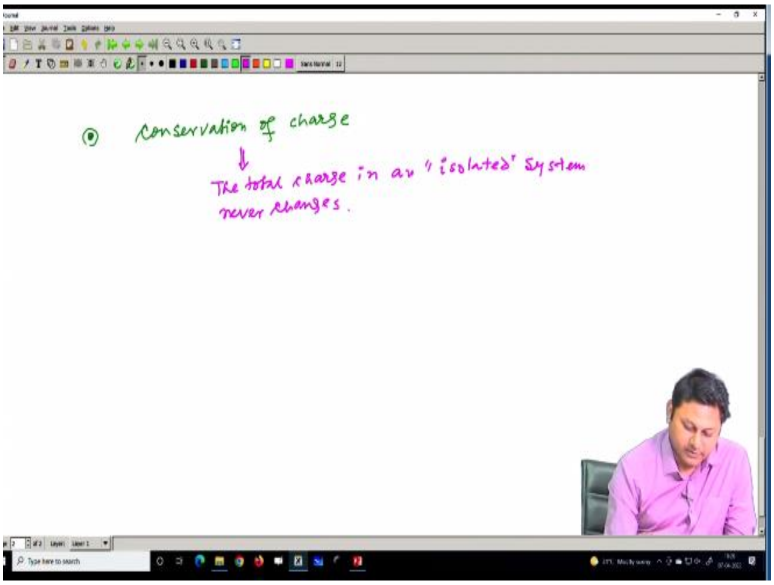

Now, we go back to our initial discussion, which is charge. So, the charge has few properties and now we are going to discuss that. So, 1 is conservation of charge. So, conservation of charge means the total charge in an isolated system never change. So, whatever the total charge we have. So, the total charge in an isolated system never changes whatever the charge is there it should remain.

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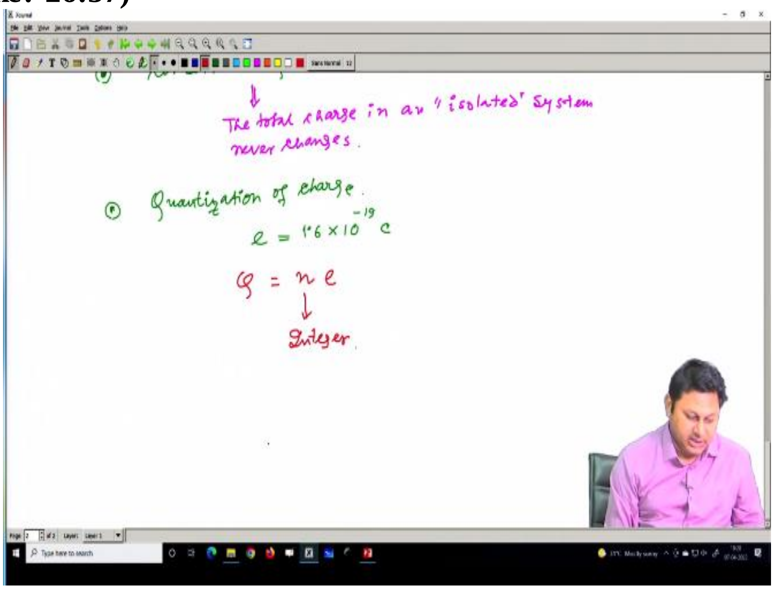

The second important thing is the quantization of charge. So, whatever the charge we are talking about this charge is quantized and the unit charge is nothing but the charge of the electron and this value is around 1.6×10^{-19} Coulomb. So, the minimum charge one can have with this cannot break it. So, whatever the total charges we have that is some multiple of this charge e where e is a positive integer that ensures that there is a quantization of charge. Next thing is the interaction. So, the interaction I already discussed, but since we are making it in a point wise manner so let me do that.

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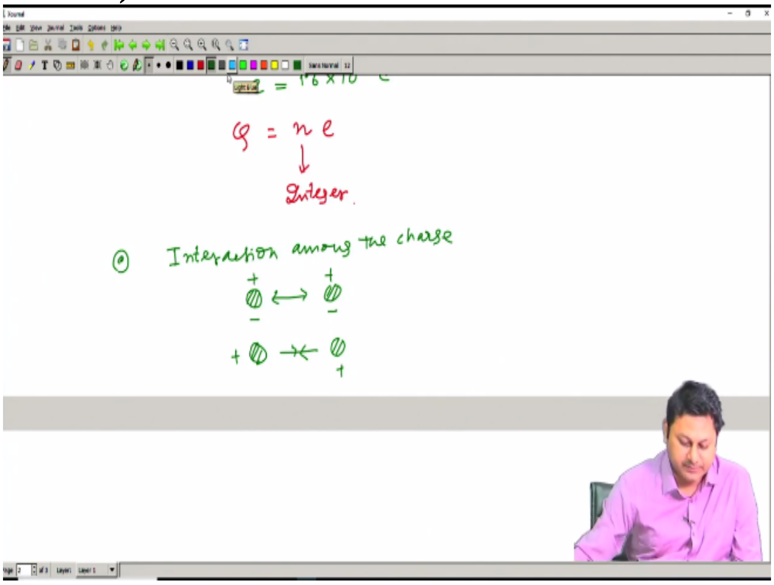

So, the next thing is so interaction among the charges. So, an interaction among the charges is like that the same charge say plus-plus or minus-minus they will ripple and unlike charge that plus and minus they will attract each other. So, like charge the ripple and unlike charge attract what is next?

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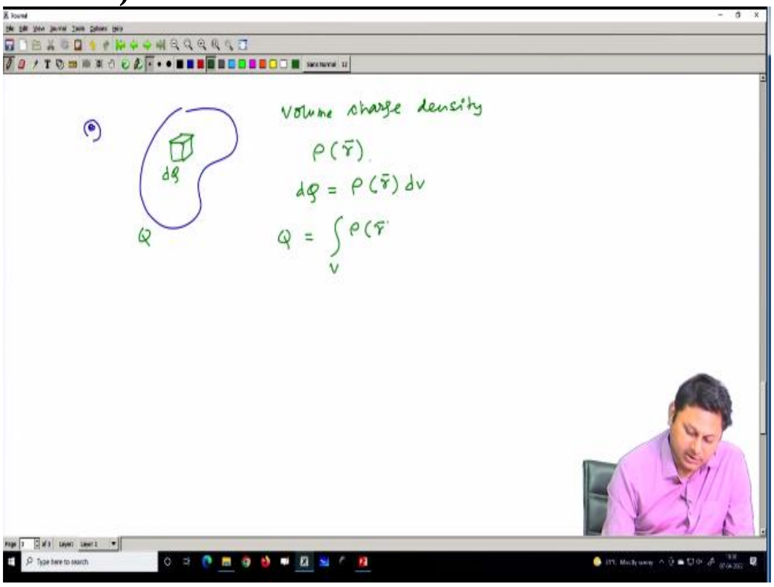

The next is how to define the volume charge? Suppose, we are having an arbitrary volume here and inside that we have a tiny volume element. So, this is a chunk of charge and we have this volume. So, the volume charge density is defined as ρ that should be a function of r. Now, for this tiny region if you have dQ amount of charge then dQ should be equal to $\rho(\vec{r})$, this is a volume charge density over this volume dv.

So, if I want to find out what is the total charge? Then the total charge will be the volume integral of this quantity charge density dv we are going to use this expression several times. After that very important thing that is the classical point charge.

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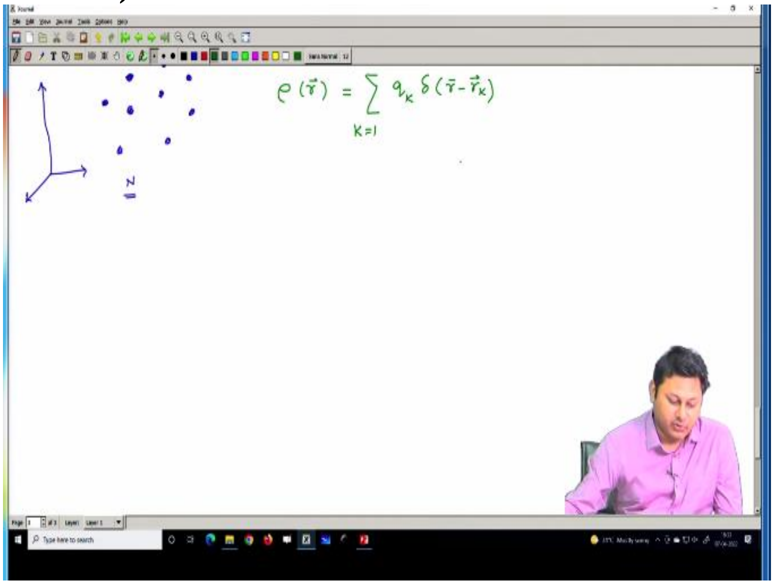

So, classical point charges are vanishingly small charge that carries a finite amount of charge. So, these are the scattered classical point charges and suppose this charge is q_k with the k^{th} charge k is integer it is 1, 2, 3, 4, 5, 6, etc. Now, if I want to find out the charge density of this what is the charge density of these discrete point charges now, they are not a continuous like before there is a continuous charge distribution here, but here this is scattered these discrete.

So, suppose there are N charge particles are there total number is N and all the charge particles is having. So, this charge particle Q and its position is say r_k with certain coordinate system here. So, this is the coordinate system and in this coordinate system these charges are distributed with different location and this is discretely distributed and try to find out what is the charge, what is the charge density?

So, charge density for this case, which we defined earlier with ρ can be represented as this k can goes to 1 for N number of charges what we have and each charge associated with a δ $(\vec{r} - \vec{r}_k)$. So, each point at delta function each r point we have a charge and if you make a sum over that then it eventually, we are going to get ρ now it looks quite I mean, it is not quite straightforward, but we can find it quickly how do you find it let us see.

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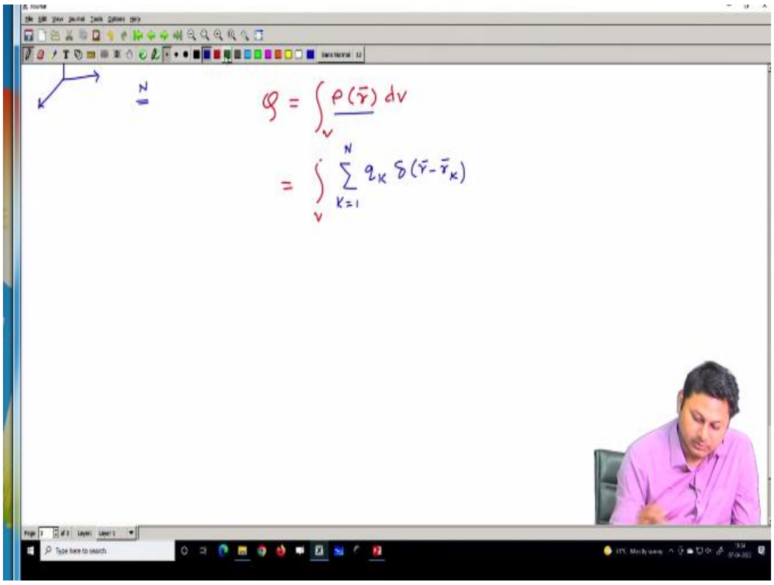

So, our total charge Q should be the $\int \rho(\vec{r})dv$ this is the way we do the integration we find that the total charge can be defined in this way. Now, I know the charge distribution charge density I just define the charge density. So, I will put it here over the volume integral. So, this $\rho(\vec{r})$ I put this value so, this is summation of k = 1 to N. So, these are I am putting and then I put q_k and then I put δ $(\vec{r} - \vec{r}_k)$ and then dv. So, I just put this and then I put dv let us put dv in different colour dv.

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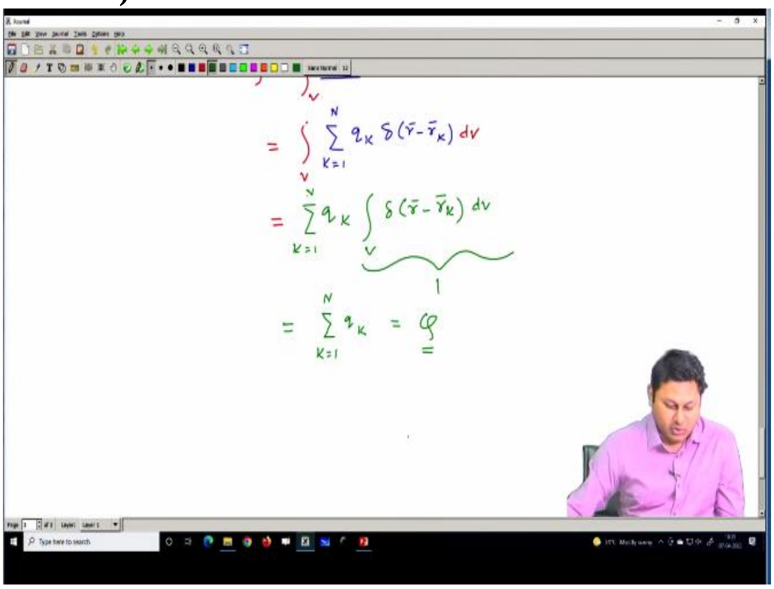

So, now, I can do in this way the next step I can take this out because q_k is nothing to do with this integration this is discrete charge amount. So, it is simply summation over q_k, k goes from 1 to N and then integration of δ $(\vec{r} - \vec{r}_k)$ dv over volume. Now, this is a well-known information well-known term that if I integrate the delta function over entire volume I should get 1 so, this thing is nothing but summation of k = 1 to N all the charges, which is nothing but my total charge Q. So, this is the way you can define. So, now quickly try to understand what happened for the moving charge, but what happened for the moving charge?

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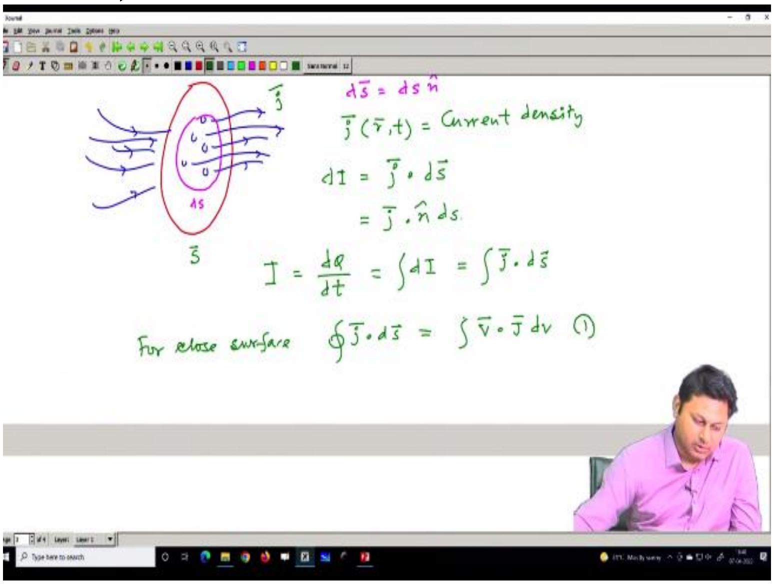

So, next thing quickly discuss about the moving charge, concept of the moving charge so, this leads to the concept of electric current so, now, suppose I have a surface here and the charges are flowing like this so, this is the way the charges are flowing and this is the region I just take this small region. So, ds if I want to find out the area here ds this should be ds $\hat{n}$, n is along this direction $\hat{n}$ and this is the flowing current density I should put $\vec{j}$.

Because charges are flowing here and this is this total surface S and this surface is ds so, $\vec{j}$ here which is a function of $\vec{r}$ and t is called the current density crudely saying the amount of current passing through per unit area. So, dI the amount of current in terms of current density is simply $\vec{j} \cdot d\vec{s}$ or $\vec{j} \cdot \hat{n}$ ds. The total current I is the total charge divided by time $\frac{dQ}{dt}$ and that should be equal to the integration of this dI, which is $\int \vec{j} \cdot d\vec{s}$.

So, now this if I have a close surface for close surface I can have this integration close integration. So, $\vec{j} \cdot d\vec{s}$ suppose the surface is close I have a volume kind of thing then this quantity is eventually the $\vec{\nabla} \cdot \vec{j}$ by using the divergence theorem I can write it this is my equation say 1.

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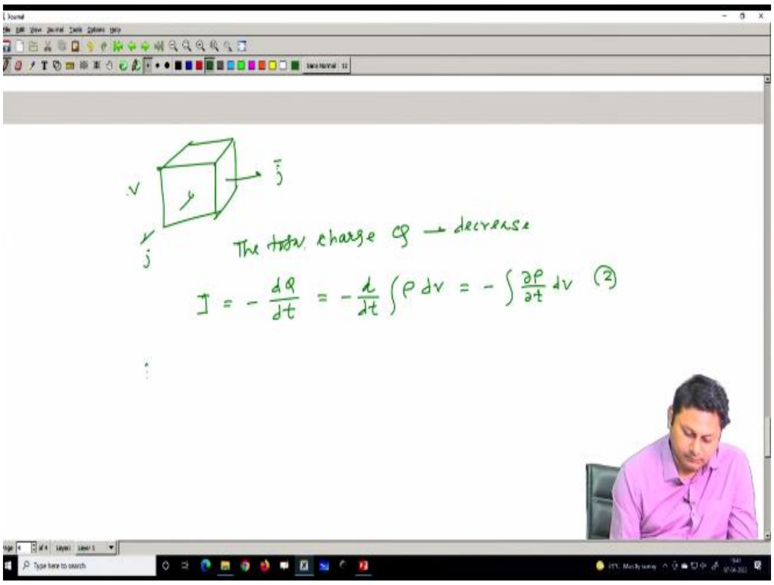

So, now, if this is the volume I am having suppose and total charge are flowing out and I am taking the close surface this is the $\vec{j}$ flowing out, $\vec{j}$ flowing out. So, the total charge Q should decrease in the volume whatever the volume you have to should decrease, so, the current also I can write here that since it is decreasing so, it should be $-\frac{d}{dt}$ and the charge density we know if it is decreasing, so, I can write it in this way. So, this is simply $-\frac{d\rho}{dt}$ and dv this is equation 2.

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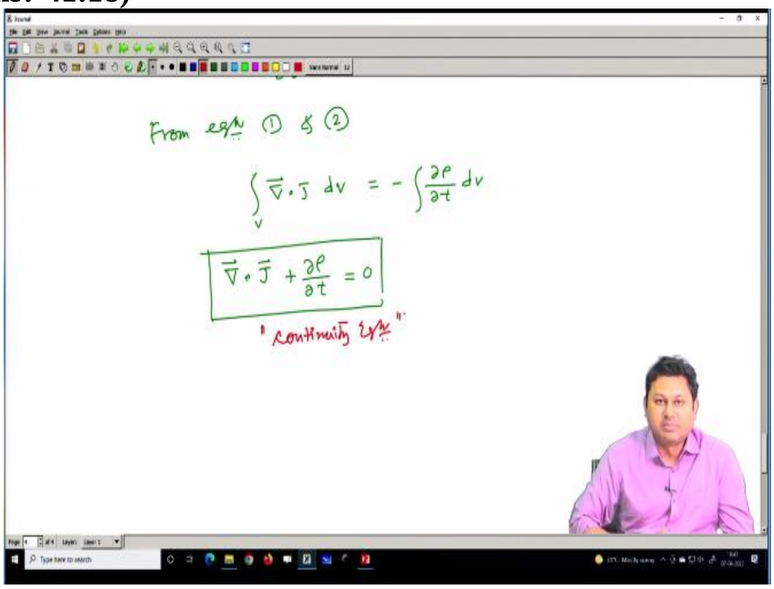

So, if I compare equation 1 and equation 2 from equation 1 and 2 I find a very interesting relation that is the divergence of the current density over a volume dv is equal to minus of this quantity $\frac{d\rho}{dt}$ over dv. So, from this equation I can compare and find that $\frac{d\rho}{dt}$ = 0 this is a very important equation in electrodynamics also in quantum mechanics you will have this equation it is called the continuity equation.

So, this continuity equation will be very important because it basically tells us when the light current is flowing outside, so that should be equivalent to the divergence of that should be equivalent to the rate of change of the divergence of the current density is equal to the negative of the rate of change of the charge density. So, that is why this quantity is called the continuity. So, in order to maintain the continuity of the charge flow, we need to have this equation.

So, later we will be going to discuss about this equation also when we deal with Maxwell's equation, but at the beginning it is better to have a note that how charge flow give rise to the current density and then if I try to understand in terms of the charge density, then what is the relationship between the current density and charge density so this is the constant relation between the charge density and current density, which we call the continuity equation. So, with this note I like to conclude here, thank you for your attention. See you in the next class.