Foundation of Classical Electrodynamics
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Lecture – 32
Electrostatic Dipole

Hello students to the course of foundation of classical electrodynamics. So, under module 2, today, we will have lecture number 32, where we try to discuss about the electrostatic dipole.

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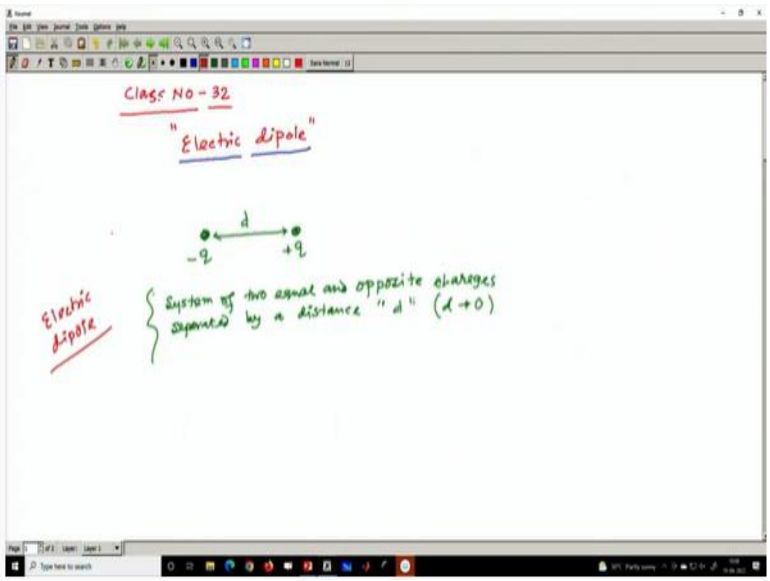

So, today we have class number 32. So, we will discuss the electric dipole, so electric dipole is not a new term. So, the standard definition that we know that if we have 2 charges -q and +q separated by a distance d under the condition. So that means system of 2 equal and opposite charges separated by a distance d under the condition that d is very small, tends to 0 then this is called the electric dipole.

However, we will be going to learn a more detailed discussion; we will do the more detailed discussion about the electric dipole.

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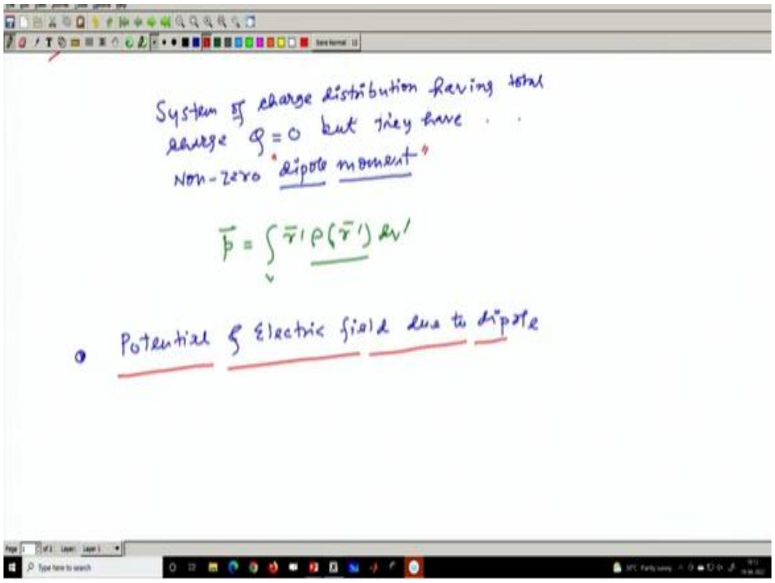

And going to see that a system of charge distribution can also be considered to be a dipole where the system of charge distribution having total charge q = 0 suppose, I have a charge distribution some positive and some negative charges are there and the total charge if I calculate is 0, but they have non-zero dipole moment. So, this is important term dipole moment we will go to discuss this in detail. So, the point is a system of charge distribution having total charge 0.

But, having non-zero dipole moment can also be considered as electric dipoles. So, dipole moment in discrete case here, in discrete case the dipole moment $\vec{p}$ is charged multiplied by the separation, this is the way we define. So, this is my dipole moment. And in this continuous charge distribution, the dipole moment is generally defined in this way, this is the $\vec{r}\,'$ and $\rho(\vec{r}\,')$ dv', this is the distribution of the charge for which I need to use the expression of the charge density and this is the way we define the dipole moment,

So, we will be going to discuss in detail. So, first we will calculate in the classical way the potential and electric field due to a dipole. So, first we calculate this is a very traditional calculation in most of the books you will find. So, we will first do that. So, potential and electric field due to dipole, this dipole structure for example are very important because in different branch of the physics we have this kind of structure where 2 opposite charges are separated by a distance d. So, small distance d; so, the knowledge of the dipole oriented thing is important.

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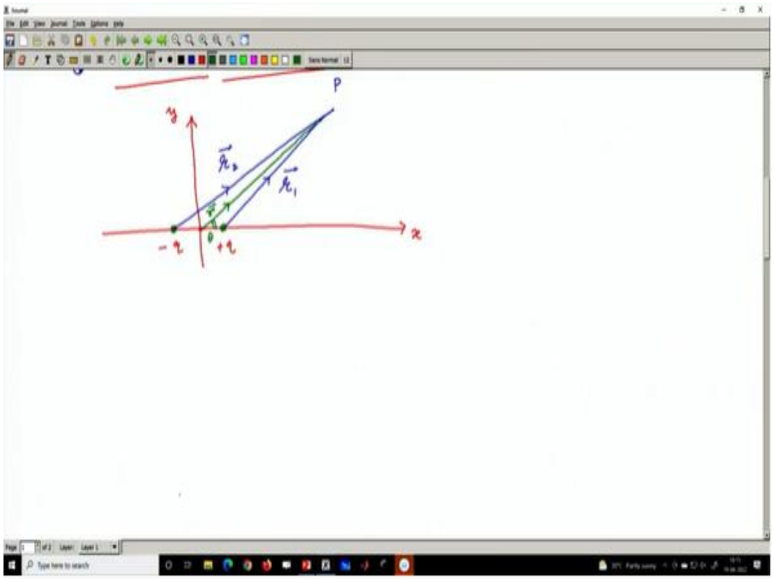

So, that is why we need to learn it carefully because in other branch of the physics not only electrostatic there are a few other cases where this dipole are there I mean this concept of dipole you will find very frequently. So, let us consider a coordinate system where 2 dimensional coordinate system this is x axis, this is y axis and took point q_1 and q_2 charge particular two charge points +q and -q are present here.

So, suppose this is +q and this is -q over the x axis and I want to find out the potential at some other point say I want to find out potential at any point over this xy plane and that point maybe P. So, the vector from the distance from q to P in terms of vectorial notation say this is $\vec{r}_1$ and this is $\vec{r}_2$, this is the position vector for the point P from the point q_1 and q_2 mind it, but I can also draw another line here which gives me the position of P in terms of the origin from origin and say this value is my $\vec{r}$ having an angle say making an angle θ here.

So, this is the structure this is the standard structure we know that we have a dipole here sitting over x axes and try to find out the potential at point P and how to calculate that. So, the potential for these 2 charge point is simply I can use the superposition rule.

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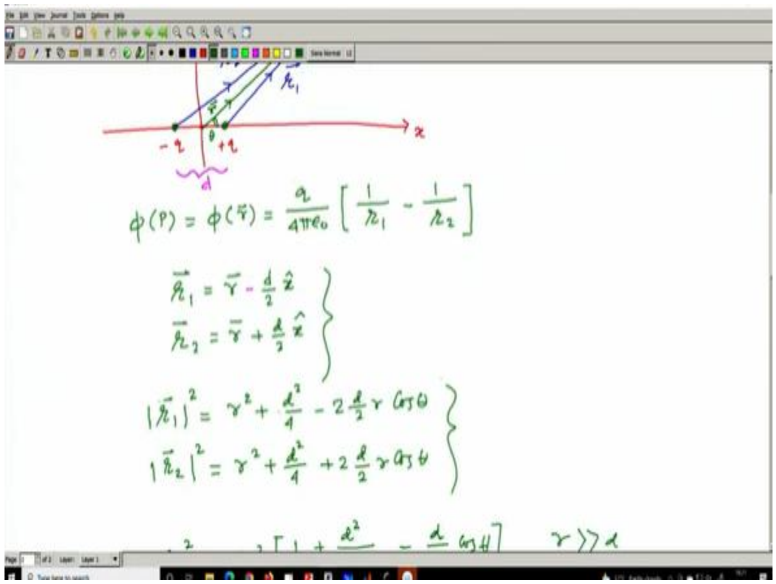

So, the potential ϕ at point P is simply potential that at point $\vec{r}$ that is $\frac{q}{4\pi\epsilon_0}$, q is same for both the cases and the distance is $[\frac{1}{Л_1} - \frac{1}{Л_2}]$ this is minus sign because we have one negative charge and one positive charge and accordingly the distance is $\vec{Л}_1$ and $\vec{Л}_2$, so I write the potential in this way. Now, everything I like to write in terms of $\vec{r}$, because here in the left-hand side, you can see this function of $\vec{r}$.

So, it is expected that whatever the result we have should be in form of $\vec{r}$ because I measured the point where I like to find out the potential in this coordinate system, where $\vec{r}$ is measuring from this origin. So, I can do that. So, vector $\vec{Л}_1$ is simply the $\vec{r}$ and then if now in order to understand in order to write properly, so, I need to define the length between these 2 dipole and that is d, d is the separation between these charge points.

So, the $\vec{Л}_1$ should be $\vec{r}$ - $\frac{d}{2}\,\hat{x}$ and $\vec{Л}_2$ in the similar way it should be $\vec{r}$ + $\frac{d}{2}\,\hat{x}$ this is $\vec{Л}_1$ and $\vec{Л}_2$. Now, I can calculate $\frac{1}{Л_1}$ and $\frac{1}{Л_2}$, but this is the magnitude. So, the magnitude of $\vec{Л}_1$ simply comes up to me because I know $\vec{r}$ - $\frac{d}{2}\,\hat{x}$. So, magnitude of $\vec{Л}_1$ simply r^2 + $\frac{d^2}{4}$ - 2 $\frac{d}{2}$ then r and the angle between these 2 that is cos θ.

This is a very standard way to write down the magnitude of a vector when we know the angle between these 2 vectors the addition of these 2 vectors. And in the similar way I can have the magnitude of $\vec{Л}_2$, which is r^2 + $\frac{d^2}{4}$ and then here we have 2 $\frac{d}{2}$ r and cos θ. Now, r^2 I can write $Л_1^2$. So, this is square it other way I should put it to the power half.

So, r^2 I can write in this way, the reason is very simple that d is very less than r so, I can write in the ratio of $\frac{d}{r}$ to simply neglect that part that we always do and then minus of I take r^2 common so it should be $\frac{d}{r}$ cos θ and this condition is r is very greater than d because d is almost tends to 0 and I try to find out like a point I pull this in compared to this point P this look these 2 d is very small.

So, these 2 are like a point dipole present in the origin so, $Л_1^2$ can we approximated as simply r^2 this term can be neglected. So, it is simply r^2 into 1 - $\frac{d}{r}$ and then cos θ.

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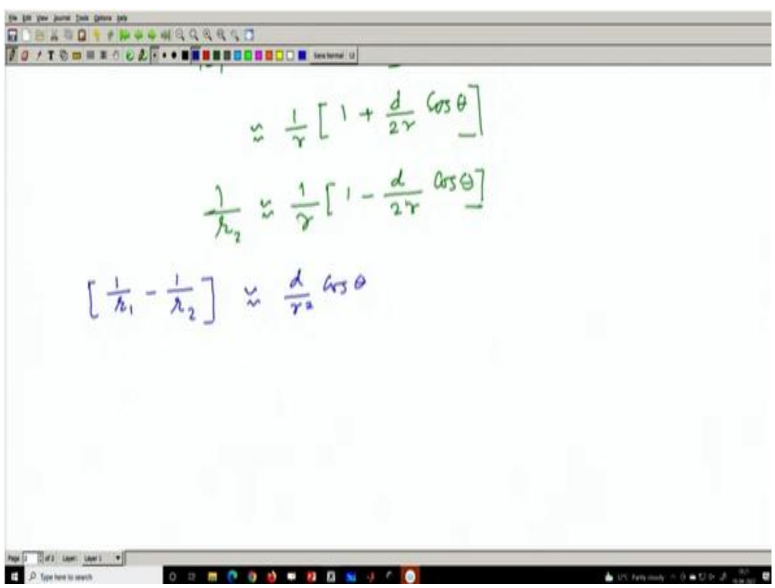

And $\frac{1}{r}$ because $\frac{1}{r}$ is there in the equation, so, $\frac{1}{Л_1}$ is nearly equal to $\frac{1}{r}$. So, and then I should write 1 – $\frac{d}{r}$ and then cos θ whole to the power of minus half. So, now I can make a Taylor series expansion make a binomial expansion and $\frac{d}{r}$ ratio is small. So, I can restrict up to first term and that basically gives me this quantity simply gives me $\frac{1}{r}$ and then it should be one plus because it is a minus sign $1 + \frac{d}{2r}$ and cos θ.

In a similar way, $\frac{1}{\text{Л}_2}$ will be simply $\frac{1}{r}$ and instead of having a plus sign here we have a minus sign $\frac{d}{2r}$ and cos θ. And finally, I need to calculate because in the expression you see that we have, $\frac{1}{\text{Л}_1}$ - $\frac{1}{\text{Л}_2}$. So, that quantity if I calculate here finally, so, $\frac{1}{\text{Л}_1}$ - $\frac{1}{\text{Л}_2}$ that quantity is nearly equal to $\frac{d}{r^2}$ and then cos θ, because I just subtract this from this r will cancel out and $\frac{d}{2r}$ become $\frac{d}{r^2}$ and cos θ.

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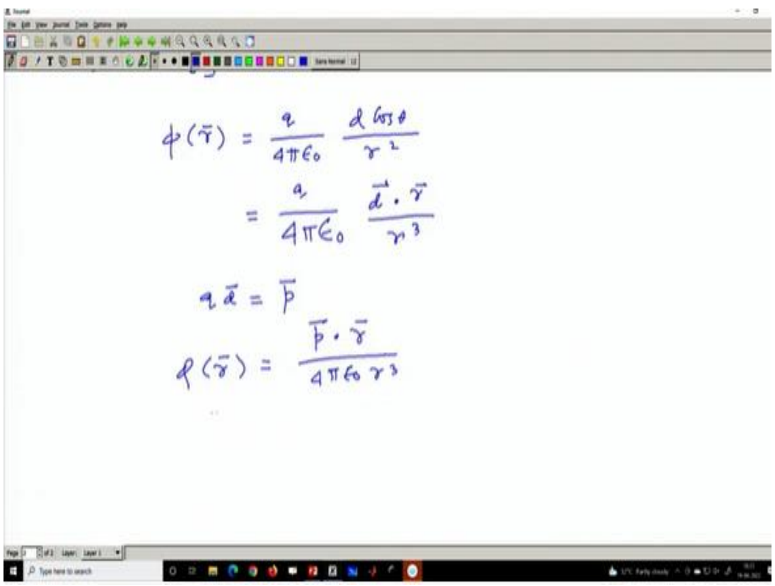

So, the ϕ that I wanted to calculate at point $\vec{r}$ is simply comes out to be $\frac{q}{4\pi\epsilon_0}$ and then I have $\frac{d\cos\theta}{r^2}$. Now, this term I can write as per my convenience. So, I can write in a vectorial form and vectorial form is even though ϕ is scalar quantity, but right-hand side this make use of this cos θ and I can write $\frac{q}{4\pi\epsilon_0}$ and then $\frac{\vec{d} \bullet \vec{r}}{r^3}$.

Now, $\vec{d} \bullet \vec{r}$ gives us the value of d cos θ r and r^3 is there. So, it will cancel out to make r^2. Now, q$\vec{d}$ this quantity I already defined this is called the dipole moment $\vec{p}$. So, my ϕ I can write in terms of the dipole moment, which is the important thing here and eventually I get $\frac{\vec{p} \bullet \vec{r}}{4\pi\epsilon_0 r^3}$. So, at point $\vec{r}$

I know; what is the potential due to the electric dipole whose dipole moment $\vec{p}$ is known. So, that should be the expression of the potential of a dipole at point $\vec{r}$.

Now, the next thing is to find out the electric field due to this dipole, because, starting point we write that potential and electric field. So, electric field you can readily calculate because potential is now known.

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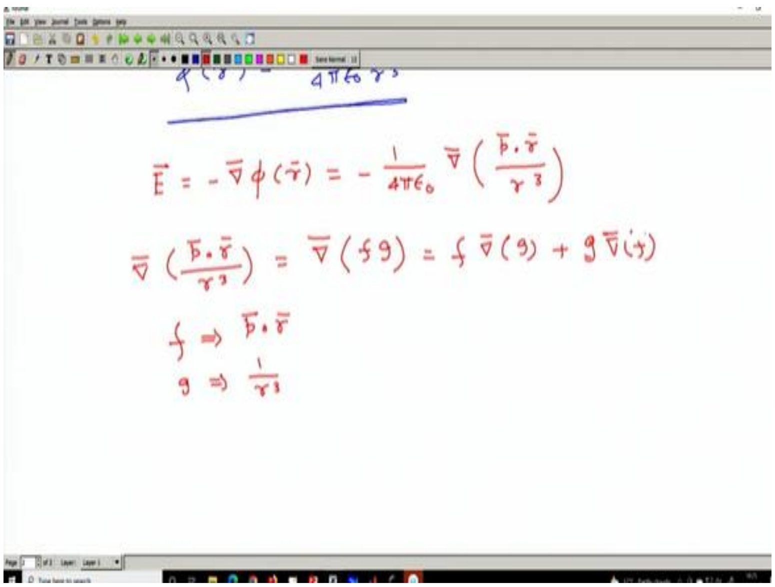

So, my electric field $\vec{E}$ I can write simply by taking the minus of the gradient of this field ϕ ($\vec{r}$) and that is simply $-\frac{1}{4\pi\epsilon_0}$ because this term is constant and then we need to make a gradient of this quantity $\frac{\vec{p} \bullet \vec{r}}{r^3}$ now, this calculation I think I should give you as a homework, because this is a vector calculation and it will be good exercise to find out I can give you the hint. So, eventually whatever this term I am having here $\vec{p} \bullet \vec{r}$ this is a function of $\vec{r}$ and r^3.

So, I should in principle I am having the gradient of 2 scalar field f and g where f here is equivalent to say $\vec{p} \bullet \vec{r}$ and g is equivalent to $\frac{1}{r^3}$. Now, the rest thing is straightforward because you know how to deal with this expression, this is nothing but f $\vec{\nabla}$(g) + g $\vec{\nabla}$(f). So, you know what is your so the next step is to just find out what is $\vec{p} \bullet \vec{r}$. So, $\vec{p} \bullet \vec{r}$ is simply p_x x + p_y y + p_z z and $\frac{1}{r^3}$ is simply $(x^2 + y^2 + z^2)^{3/2}$.

So, these 2 functional form, you know and then you just simply calculate, exploiting this you just simply calculate even in Cartesian coordinate, you can calculate the value of $\vec{E}$ and if you do that, then I am giving you the result you should get a value like this.

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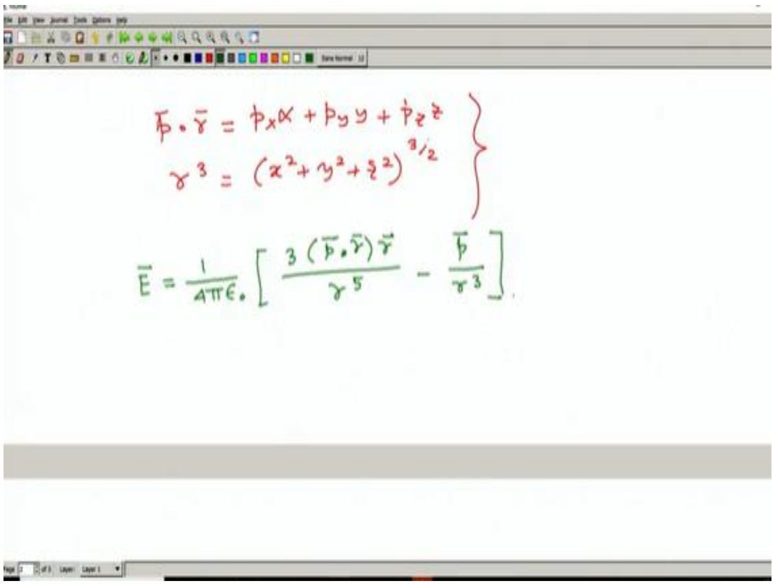

A straightforward calculation that is why I should not do it I keep it as a homework for you. But, I believe you can able to do that then we will get this value $\vec{p} \bullet \vec{r}$ and then $\frac{\vec{r}}{r^5}$ - $\frac{\vec{p}}{r^3}$, follow the standard way and it will not be going to take much time for you to calculate the $\vec{E}$ out of the potential that we calculated. The next thing I can also find out the potential in a interesting manner, so, that I am going to discuss here.

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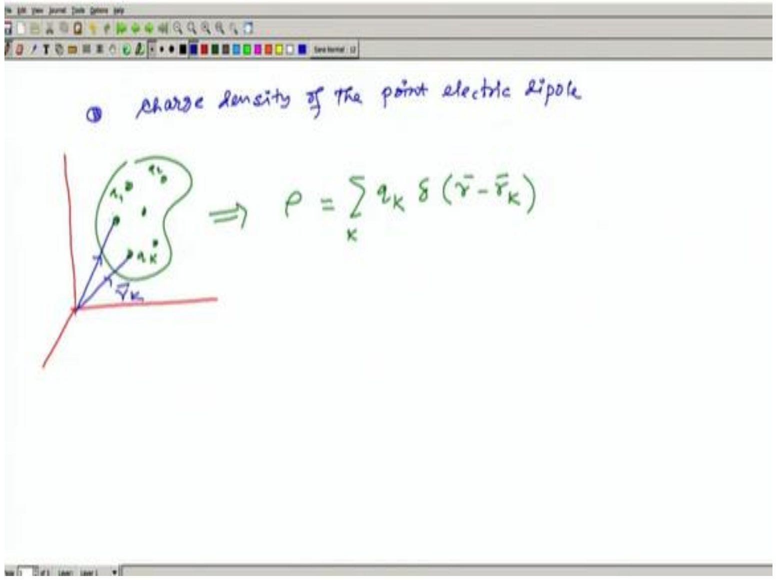

So, first in order to calculate that by exploiting the expression of the potential for continuous charge, this is the discrete way we calculate. So, the charge density for the dipole we need to calculate. So, now I calculate the charge density of the point electric dipole mind it I try to calculate the charge density here for 2 discrete charge and we know. So, let me remind once again how we calculate the charge density suppose, I have a distribution of discrete charge these are the discrete charge is distributed like this, these are the discrete charge q_1, q_2, q_k.

And what was the charge density for that, if you remember the charge density for this discrete charge system ρ was defined like summation of q. So, because I use q summation of q_k and then $\delta(\vec{r} - \vec{r}_k)$ that was the definition that was the charge density for this all these discrete charges and what is $\vec{r}_k$? $\vec{r}_k$ is their position. So, that means this is one $\vec{r}$ and this is another $\vec{r}$ and this is $\vec{r}_k$, this is $\vec{r}_1$, $\vec{r}_2$, $\vec{r}_k$ and so on. So, these are the location of this point charges and I calculate the density and it comes up to be like this.

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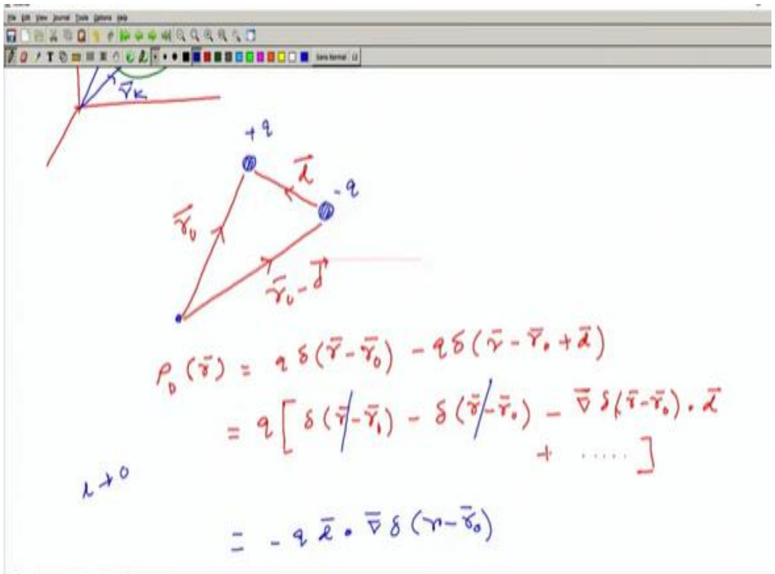

So, this is the same thing for the dipole if I want to calculate the density charged in density, so, I can calculate and suppose this is point say +q and this is -q and suppose we have the origin here and the distance from this point to the charge q the point charge $\vec{r}$ is $\vec{r}_0$ and from here to here I have set d the separation between these vectors. So, these vectors comes up to be $\vec{r}_0$ - $\vec{d}$, this is the vector location for this point charge -q.

Now, my charge density for this system I can write like previously. So, this is the charge density of the dipole system. So, that should be a function of $\vec{r}$ any point $\vec{r}$ and in that is the you know the way I defined here sum over this all the points, I have 2 points here, they simply have q and then the delta function of this location that is $\vec{r}$ and then minus $\vec{r}_0$ and then minus q and the location of the delta function, so, this is $\vec{r}$ and the location is $\vec{r}$ minus.

So, minus of $\vec{r}_0$ + d this is the charge now, this is d is very small so, I can make a Taylor series expansion here d is small tends to 0 almost. So, I can do that because delta function is a function so, any function can be expanded in Taylor series. So, this function I will write simply $\vec{r}$ - $\vec{r}_0$, but this one I can make a Taylor series expansion to the first around this point $\vec{r}$ - $\vec{r}_0$ taking d is small. So, the first q I take comment so I should not write q.

So, this gives us δ ($\vec{r}$ - $\vec{r}_0$) first term and what next then I have minus the derivative of this quantity multiplied by d. So, the derivative means here I have the divergence, the gradient of this delta function and dot d and so on. So, this is a Taylor series I can go infinitely, but I do not need the higher order terms because d is very small. So, I should not go to a very large term because the next time it will be d^2, d^3 and so on. So, that is sufficient.

So, I can see that these 2 terms are going to cancel out and I have a relatively simpler expression and that is under the condition that d is tends to 0 I do not require all the term. I simply have -q and $\vec{d} \cdot \vec{\nabla} \delta (\vec{r} - \vec{r}_0)$ when d is very small that means this quantity is very small d is almost tends to 0. So, that means, this q and q 2 points are located almost at the position at $\vec{r}_0$.

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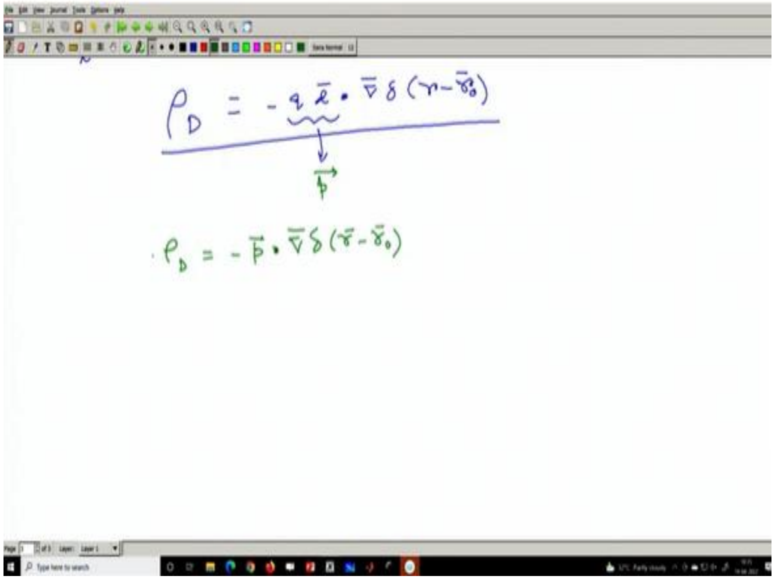

So, this is the position of the $\vec{r}_0$ become the position of the dipole and based on that, I can write an expression for the charge density for the dipole. So, this is the expression for charge density, but you can see carefully one can see that this value is known q multiplied by d is nothing but the dipole moment $\vec{p}$. So, in dipole everything we write in terms of dipole moment. So, that is why I can write the charge density in terms of dipole moment and it comes out to be $-\vec{p} \cdot \vec{\nabla} \delta (\vec{r} - \vec{r}_0)$. So, that we know, now the point is what we need to do when, what we will do with this?

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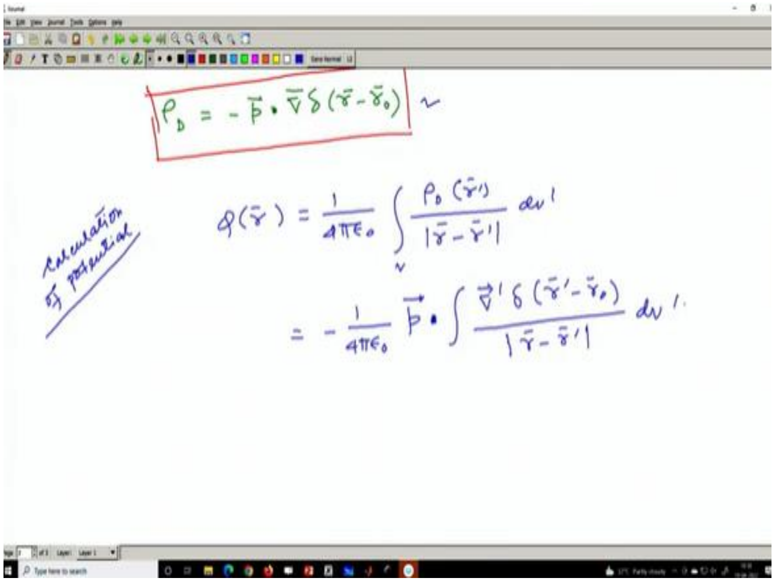

I know that this is an expression for the charge density for a dipole. Now, after that what we will do? So, we can calculate the potential in a tricky way. So, let me do that. So, the potential we have already calculated that thing we want to calculate once again. So, the potential that we calculate here is this quantity $\vec{p}$ dot $\vec{r}$ divided by. So, this is already calculated using the traditional way.

So, we will go to use the expression of the potential directly and that we know that calculation of the potential I am doing the same thing here, so the calculation of the potential here you know this potential at any point $\vec{r}$ is simply $\frac{1}{4\pi\epsilon_0}$ and if you know the charge density according to the Helmholtz theorem I can write it and for dipole it should be simply this, so now this ρ d I know so that value is here so I will put it and I can write it as $-\frac{1}{4\pi\epsilon_0}$ $\vec{p}$ is a constant so I can take it outside dot.

And then after that we have this integration and it suggests that it is integration and this is over $\vec{r}$ so it over $\vec{r}\,'$ the operator should be primed and then $\frac{\delta(\vec{r}\,'-\vec{r}_0)}{|\vec{r}-\vec{r}\,'|}$ and dv' over the entire volume.

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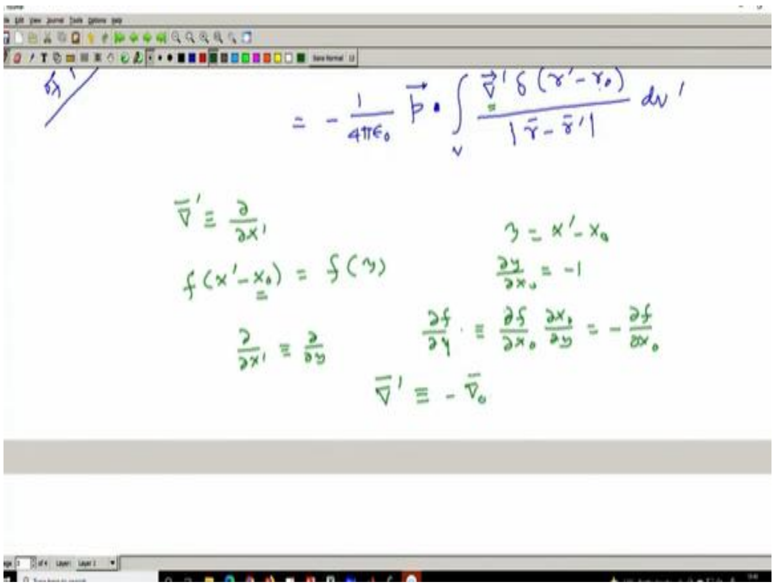

Now I make a trick here because this operator is $\vec{r}$' so it is like I am going to use it like a chain rule like this operator is essentially del say del x and if I make a prime so that is $\frac{\partial}{\partial x'}$ in one dimension I am just writing in one dimension so that means and the function whatever I have delta function is like x' - x_0. So, I can write this function as a whole y so $\frac{\partial}{\partial x'}$ is eventually $\frac{\partial}{\partial y}$ and if I now write in terms of this y_0.

So, $\frac{\partial}{\partial x_0}$ can be written as I can make it this $\frac{\partial}{\partial y}$ using the chain rule I can do that del say this is my function so I can do using the chain rule that it is over x_0 and $\frac{\partial x_0}{\partial y}$ but here y is x' - x_0, so $\frac{\partial y}{\partial x_0}$ is simply -1. So, this operator if I now change in terms of $\vec{r}_0$ then I should only have a minus one so these things is $-\frac{\partial f}{\partial x_0}$, so whatever the operator we have in prime that is equivalent to minus of 0 this 0 stand for this $\vec{r}_0$ over that I am operating and that is the trick here.

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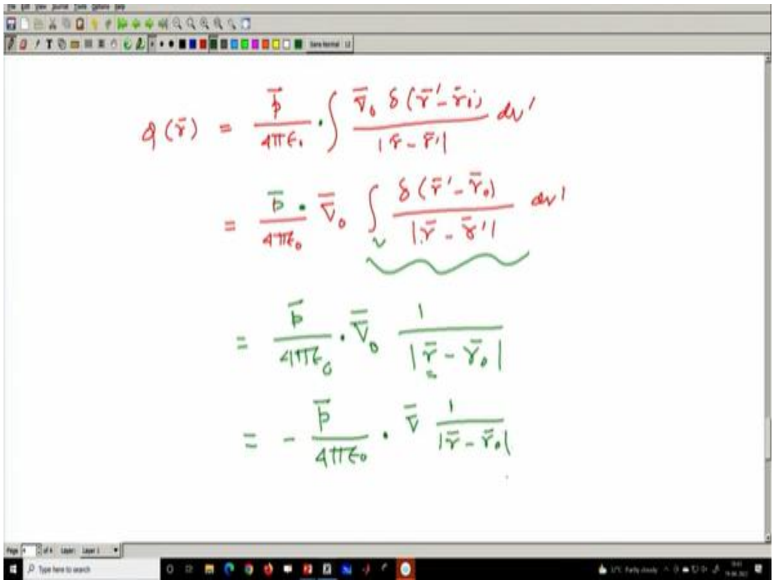

If I do that step my ϕ ($\vec{r}$) will be simply $\frac{\vec{p}}{4\pi\,\epsilon_0}$ and I can take this operator outside because now this operator so let me first write because this negative sign absorbed and I have this operator over $\frac{\delta(\vec{r}'-\vec{r}_0)}{|\vec{r}-\vec{r}'|}$ and then dv' now this is over $\vec{r}_0$ and inter integration is over prime so I can take safely I can take this operator outside the integral like this.

And then my integral should be simply $\frac{\delta(\vec{r}'-\vec{r}_0)}{|\vec{r}-\vec{r}'|}$ dv' now this delta function associated with the integration over entire volume this is known value right and this value is simply $\frac{1}{4\pi\,\epsilon_0}$ and then we have over 0 and that quantity is a delta associated with delta and I can execute this integral and eventually I have 1 divided by in place of $\vec{r}'$ I will simply have $\vec{r}_0$ so it is $\vec{r}_0$ and that is operator over operator $\vec{r}_0$.

Now I can simply so 1 p is missing here so I need to write a $\vec{p}$ here so $\vec{p}$ is missing, so now these things again I go back to you know to write in terms of original operator $\vec{r}$ in the same way and then again I replace back my previous negative sign and it should be $4\pi\epsilon_0$ and then it should be here we have a dot sign because it is throughout it is a $\vec{p}$ dot that thing so I have dot then $\vec{\nabla}\frac{1}{\vec{r}-\vec{r}_0}$.

So, this is precisely the value we had here if I go back and check what is the potential I get this is the value because $\frac{\vec{r}}{r^3}$ you can execute by just making $\vec{\nabla}\frac{1}{\vec{r}}$. So, once you do that we are going to get the same result so this is another way to you know find out the potential of this dipole by exploiting the density of the system of dipole, charge density of the system of the dipole.

So, this is another way to just show that people can also do in this way. So, we do not have much time today our time is almost finished, so I like to conclude the class here so see you in the next class and in the next class we will extend more different thing related to dipole we try to understand what happened if the dipole is placed some external electric field, what should be the energy, what should be the torque etcetera. So, thank you for your attention and see you in the next class.