Foundation of Classical Electrodynamics
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Lecture - 41
Electrostatic Pressure, Capacitor

Hello students to the foundation of classical electrodynamics course, under module 2 today we have lecture number 41 and today we will be going to learn the electrostatic pressure and capacitor.

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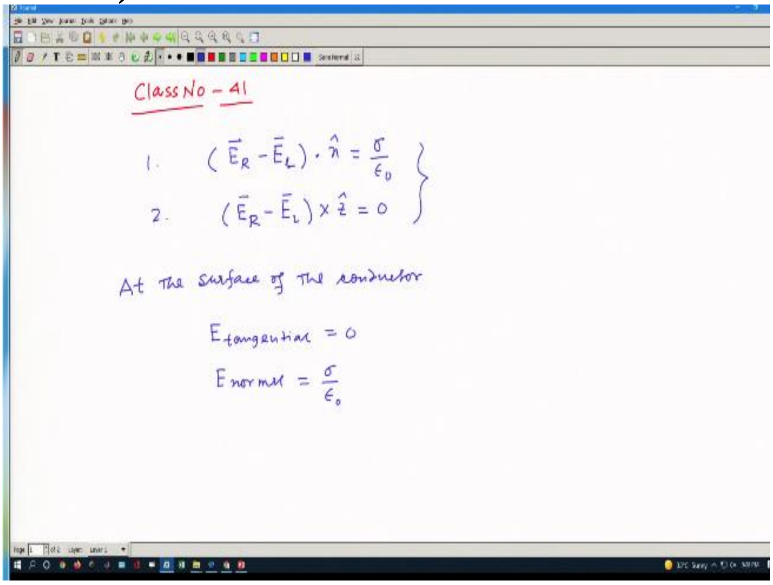

So, today we have class number 41 so, in last class we had a matching condition let me write it once again was this electric field left-hand side electric field right-hand side minus this is discontinuous and this quantity was 0 so, at the surface of the conductor if you remember that means $E_{tangential} = 0$ and $E_{normal} = \frac{\sigma}{\epsilon_0}$. So, that is the tangential component from here we can see that when you make a curl then eventually we are dealing with the tangential component that is continuous.

And when we have the dot n that is the normal component along the direction of z and that is discontinuous. So, these 2 conditions are there for conductor not only that.

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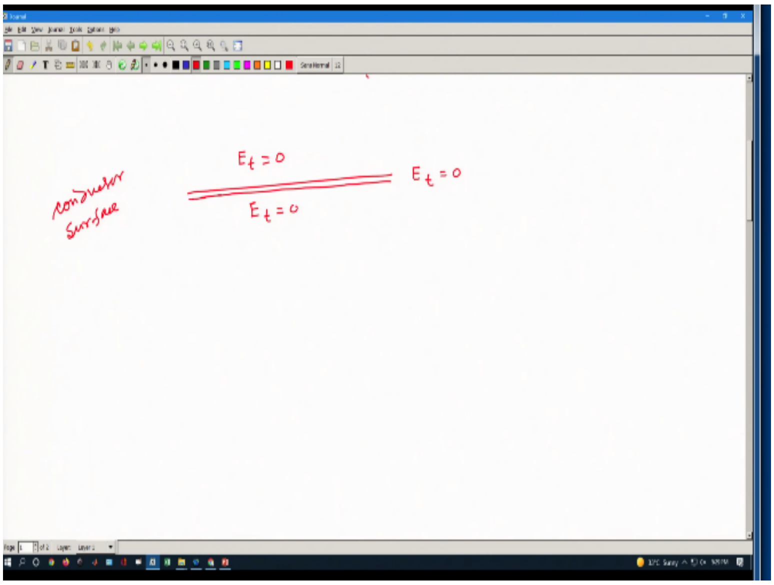

So, suppose this is the boundary of the conductors say and here $E_t = 0$ and E_t, t for tangential and r stands for normal. So, above the conductor and below the conductor that is the tangential component should be 0. So, you should remember that conductor surface that is the surface I just draw here suppose this is a surface.

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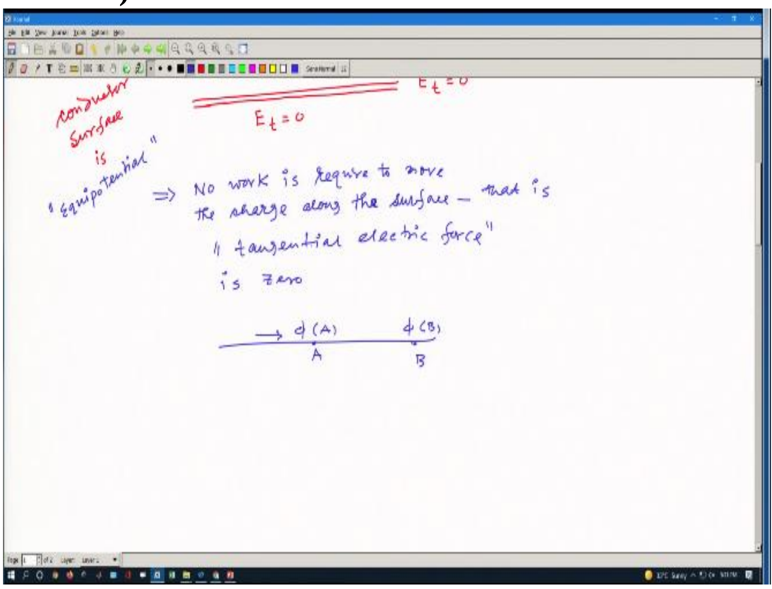

And this surface is equipotential in nature so that eventually means no work is required to move the charge along the surface that is tangential electric force this is 0 because this is a equipotential surface, so, I should not I mean all the points the potential is same. So, if it is the point A and if it is a point B so the potential is same. So, eventually the work done here to move one charge from point A to point B is 0.

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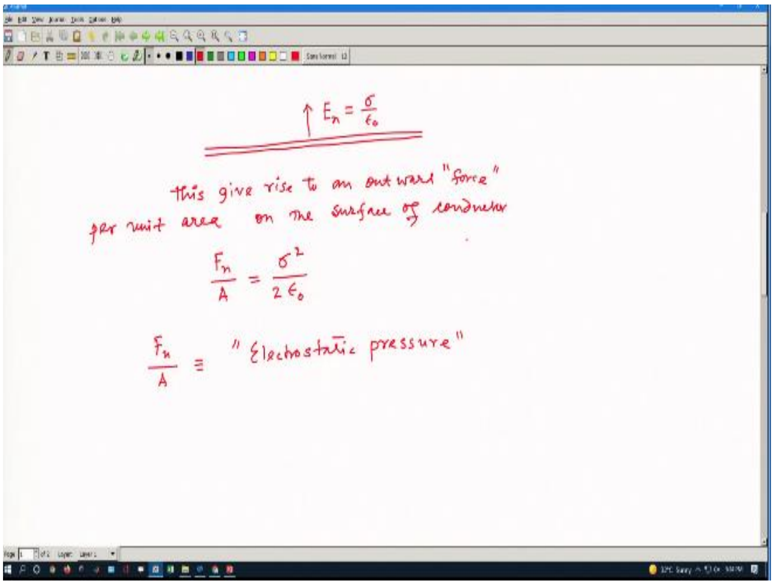

On the other hand, this is my conducting surface and along this direction is the normal component and I write this is E_n and that value is $\frac{\sigma}{\epsilon_0}$ just above the conductor inside the conductor the electric field is 0 but above the conductor I should have the value. So, this gives rise to actually an outward force per unit area on the surface of the conductor. So, we know that this is a conductor suppose this is the surface of the conductor and along this direction perpendicular to the surface.

We have the electric field just above the conductor is $\frac{\sigma}{\epsilon_0}$ and that basically give rise to some force I should say force per area. So, that force per area is σ^2 divided by if we are going to calculate that it is something like this and this force per area n stands for the direction along n this is called electrostatic pressure. So, they are a way to calculate this electrostatic pressure the amount of the electrostatic pressure is $\frac{\sigma^2}{2\epsilon_0}$.

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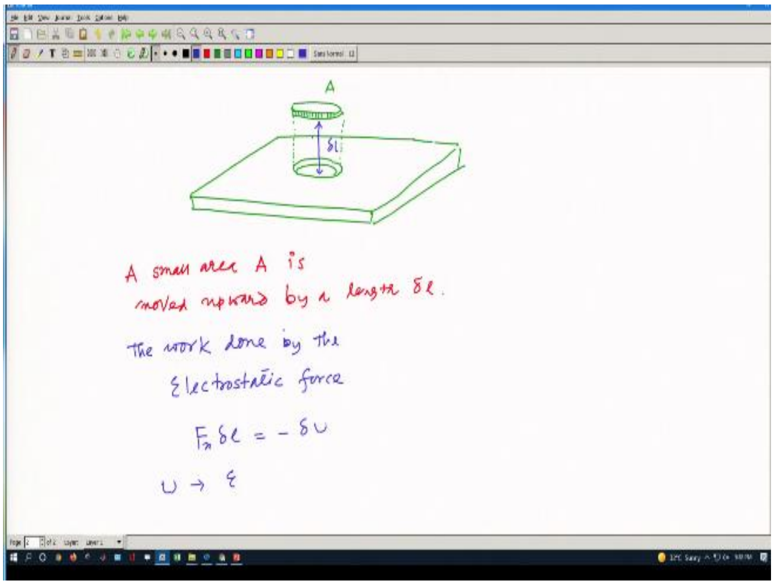

So, there are way to calculate so, let me do that here suppose, I have a surface here like this and it is small area A is moved suppose a small area on the surface let me cut a small area suppose a small area A is moved upward by a length or distance δl and if that is the case it looks like this so, let me draw it properly. So, these areas is moved up I have area that I cut and that disk like area is moved up like this. The area is A and it is moved from this point to this point this is δl.

So, if that is the case then the work done by the electrostatic force because you need to do some work to move this chunk from this from this surface to the length δl. So, the work done by the electrostatic force is simply a F_n δl this is the force and this is the length and that is minus of dU where U is the electrostatic energy so, that energy need to be converted to you know the work done to move this chunk from this place to this place.

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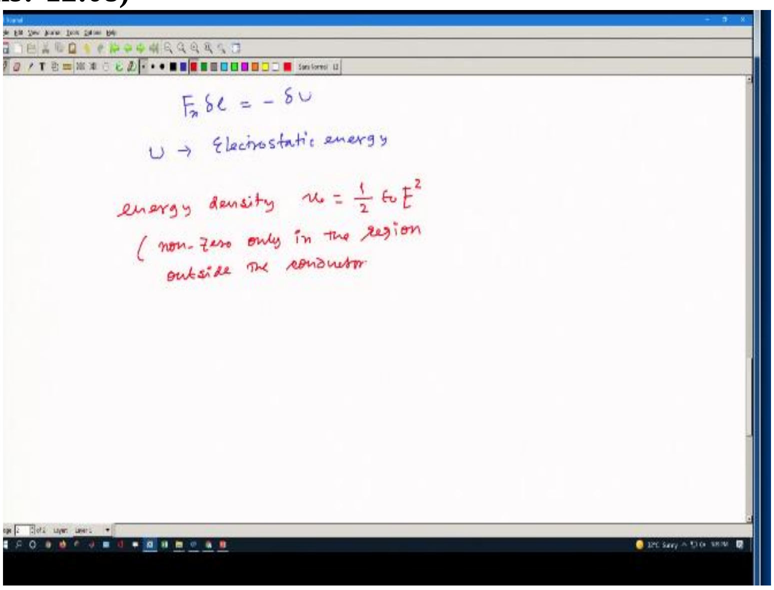

So, this is the electrostatic energy. Now, that we know the energy density normally we write it u is $\frac{1}{2}$ ϵ_0 E^2 that we already calculated earlier E^2 so, that energy density note it is nonzero only in the region outside the conductor because inside the conductor the field is 0 and the field exist only the outside.

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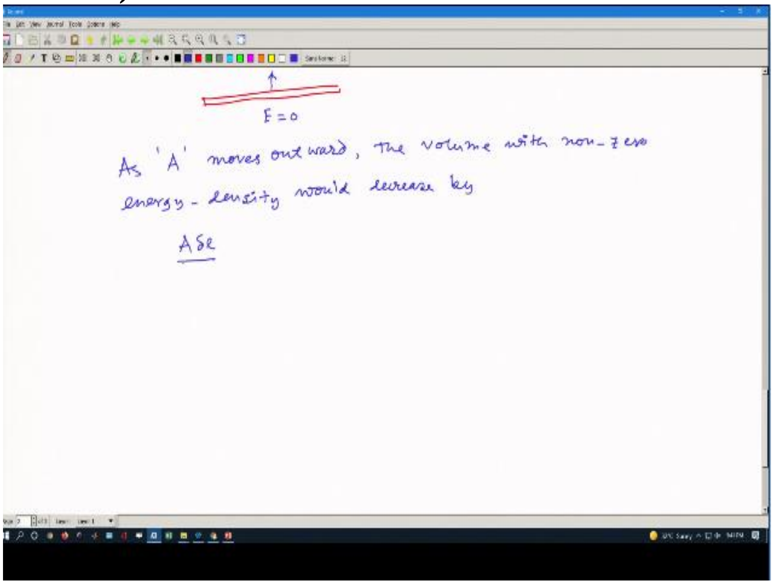

So, if this is the conductor boundary, if this is the surface of the conductor then as I mentioned the E is only here in this region here, this E is 0. So, whatever the energy density we are talking about is just above this conductor, the surface of the conductor and now, what happened that the chunk is moving from the surface to dl. So, that means, it contains a volume of energy that is moving upward.

So, I can say that as the area A moves outward the volume with nonzero energy density so, as the volume is moving upward as you know the area A moves upward, the volume with nonzero energy density would decrease by an amount simply area dl this is you can see that this is the volume that we have when this thing is moving upward, so, this amount of energy will be going to decrease.

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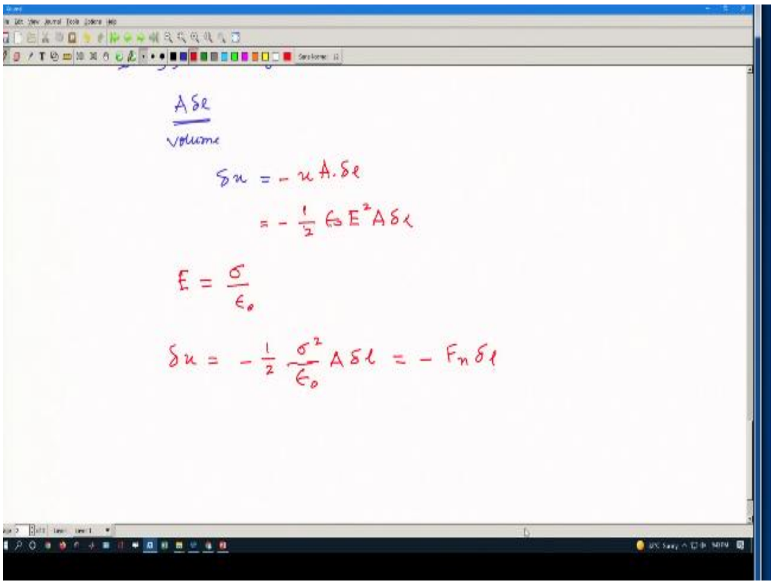

So, δu if this is the amount is going to decrease, then that value should be half into energy density and amount of the volume and that is A • δl, what is the amount of this u that is energy density it is $\frac{1}{2}$ ϵ_0 E^2 and then I multiply A • δl. Now, E for the conductor is $\frac{\sigma}{\epsilon_0}$, and I am going to put that so, δu is simply $-\frac{1}{2}\frac{\sigma^2}{\epsilon_0}$ I should write σ in a better way $\frac{1}{2}\frac{\sigma^2}{\epsilon_0}$.

And then A δl that value should be equivalent to the amount of the work done and that if I write it should be this one that you already calculated. So, the amount of energy that is decreased is equivalent to the amount of the work done to move this chunk A to a distance δl that is the main thing.

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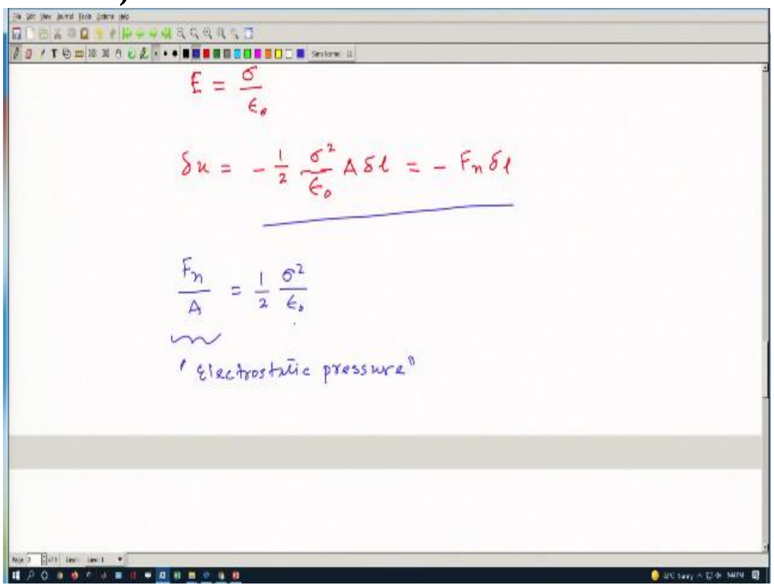

So, from this equation, I can have simply F_n that is a force per unit area is equivalent to $\frac{1}{2}\frac{\sigma^2}{\epsilon_0}$.

So, that is this quantity force per area is eventually the electrostatic pressure this is the amount of electrostatic pressure in terms of the surface charge density and ϵ_0. Let us put few examples to understand this concept. So, let us do this example.

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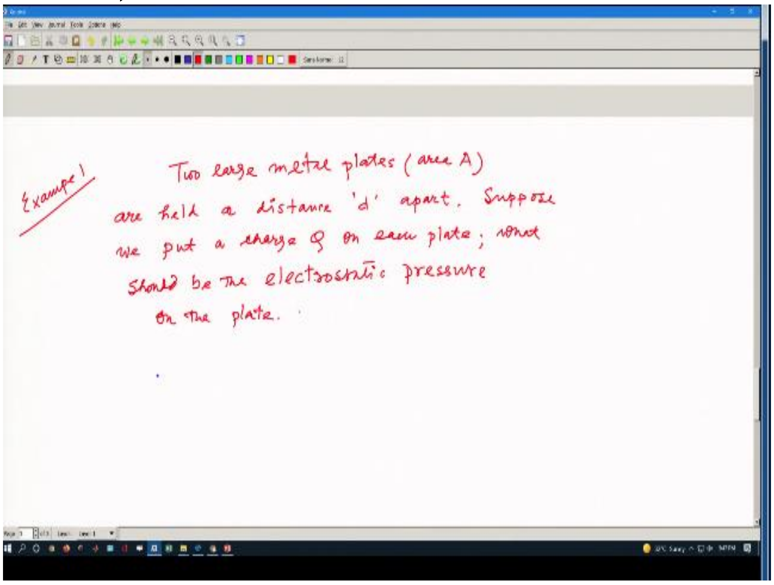

So, let us put example 1. So, example 1 is saying that 2 large metal plates having area say A say held a distance d apart. Now, suppose we put a charge Q on each plate. Now the question is, what should be the electrostatic pressure on the plate?

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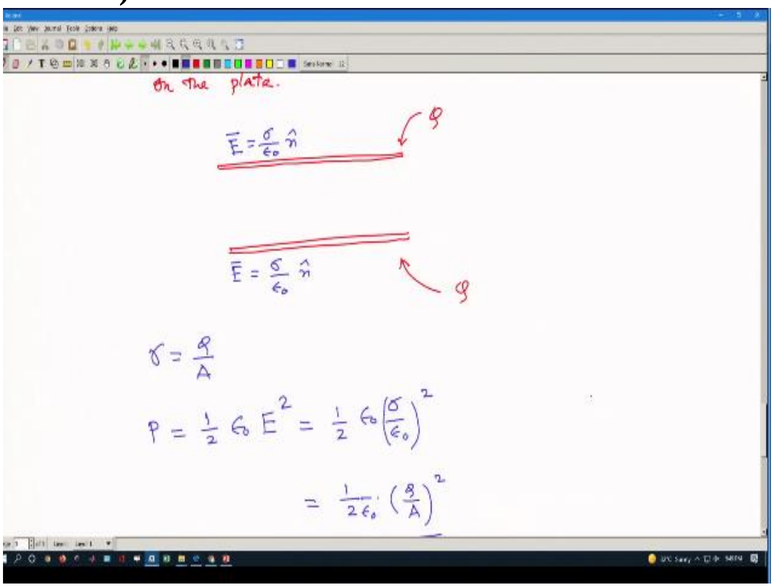

So the system is like that we have 2 parallel plates like this. So, this is a 1 plate and we have another plate like this and I put charge both the plate here and the amount of charge is Q so, the $\vec{E}$ if I calculate since this is a plate the electric field $\vec{E}$ should be $\frac{\sigma}{\epsilon_0}$ with the direction n here $\vec{E}$ should be 0 because all the charge will be here and also here we have $\vec{E} = \frac{\sigma}{\epsilon_0}\hat{n}$, σ is $\frac{Q}{A}$ because the amount of charge.

So, the electrostatic pressure if I calculate is we already calculated it is $\frac{1}{2}$ ϵ_0 and because it is also putting a pressure on here so, I should not write $\vec{E}$ = 0 here because the same it is like a capacitor so, you should have a same field here. So, I should not write anything here. So, it is simply $\frac{1}{2}$ ϵ_0 E^2 because this is the formula and $\vec{E}$ I know because this is the amount of $\vec{E}$ that we have here also it should be σ ϵ_0 and E is simply $\frac{\sigma}{\epsilon_0}$.

So, I should have $(\frac{\sigma}{\epsilon_0})^2$ and which is $\frac{1}{2}$ ϵ_0 and then σ^2 I simply write $(\frac{Q}{A})^2$. So, this is a simple problem because I have 2 plates charges are there. So, I know what is the electric field 2 plates are placed parallel. So, what is the pressure that one should experience and that is simply whatever the formula we just derived that $\frac{1}{2}$ σ^2 ϵ_0 with the energy density condition.

And that energy density condition if I write for unit area what should be the pressure what should be the force that is eventually the pressure and that value should be $\frac{1}{2}\frac{1}{\epsilon_0}(\frac{Q}{A})^2$.

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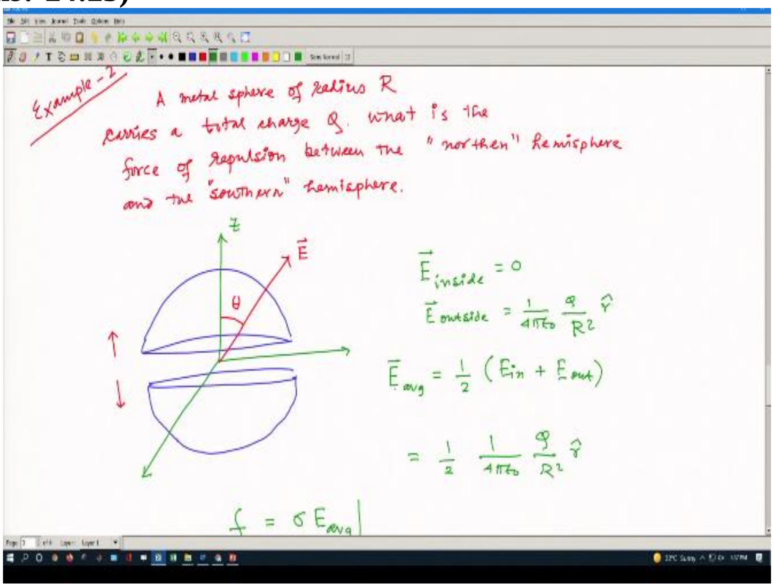

Now, let us go to more non trivial problem, which is example 2 now, in example 2 in this problem it has the problem is saying that a metal sphere of radius R carries a total charge Q if that is the case then what is the force of repulsion between the northern hemisphere and the southern hemisphere very standard problem indeed. So, what we have here? Let me draw so, we have 2 sphere like this.

1 sphere, which is cut from the middle dividing into 2 sphere with the coordinate system line here like this and suppose this is my z direction the question is, what is the you know repulsive force and that force is essentially the electrostatic force or what is the repulsive force that one should I mean this northern and southern hemisphere should put to each other. So, in order to do that first we need to because this is a spherical thing. So, the electric field for these things should be along the direction of R.

So that is in this direction so, that is the field, this the electric field, which is making say angle θ now, the force between these 2 will be along say this direction along the direction of z now, if I divide this electric field into 2 parts 1 is along z and another is perpendicular to the z that means parallel to the surface then this parallel component will no longer come into the picture. So, that is the trick. So of this problem that means, the force you calculate along the direction of z.

So, now we calculate the different I mean the amount of field in different amount of electric field. So, first we calculate the inside and we know that this is a conducting material so inside fairly simply 0. So, what is outside just over the conductor this is a spherical sphere so, we know what is the field just outside and that value should be $\frac{1}{4\pi\epsilon_0}$ the amount of charge that is given divided by R^2 because the radius is given as R. So, this is R^2 and the unit vector along r direction.

So now inside the force is 0 and just outside the force is $\frac{Q}{R^2}$. So we know that in that case, we need to take an average electric field, because when they are repulsing each other. So the force electric field that we take is neither 0 nor outside force. So we are going to take the average one and this average is simply $\frac{1}{2}$ $(E_{in} + E_{out})$ and that is equal to $\frac{1}{2} \frac{1}{4\pi\epsilon_0} \frac{Q}{R^2} \hat{r}$ so that is electric field. So what should be the force? Then the force along the z direction per unit area is σ multiplied by $E_{average}$ over along the z component of these things. So, that is the value.

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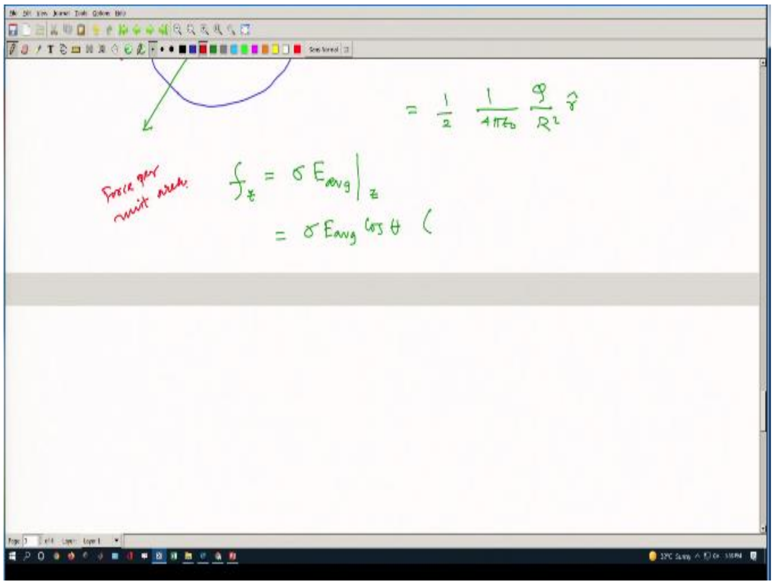

Now, what is this quantity? Quantity if I calculate it is σ and then $E_{average}$ and along z component so, that means I should have a cos θ here and that is as I mentioned this quantity is force per unit area.

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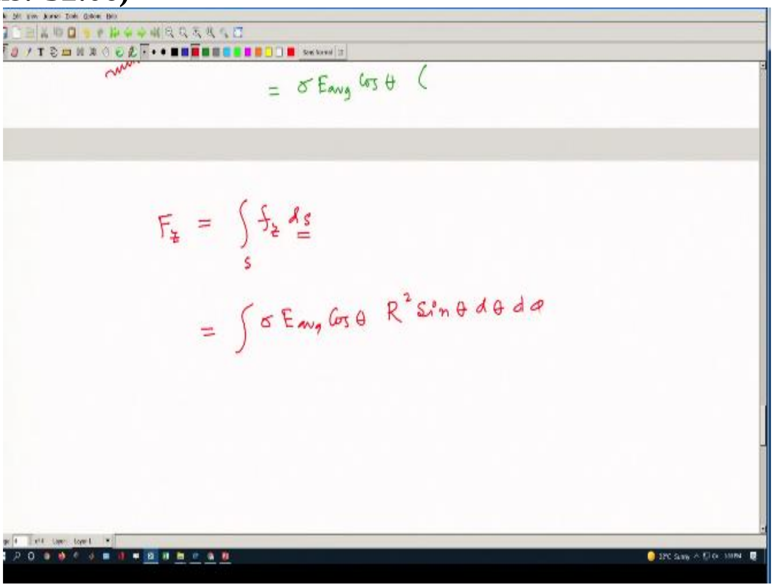

Now, if I want to calculate the total force instead of calculating the per unit area, I need to calculate the force per unit area and that should be integrated over the entire area over the surface this spherical surface and if we do I should simply have integration of σ and then $E_{average}$ then cos θ then the ds and it should be over the surface so, I should have R^2 sin θ and dθ and dφ.

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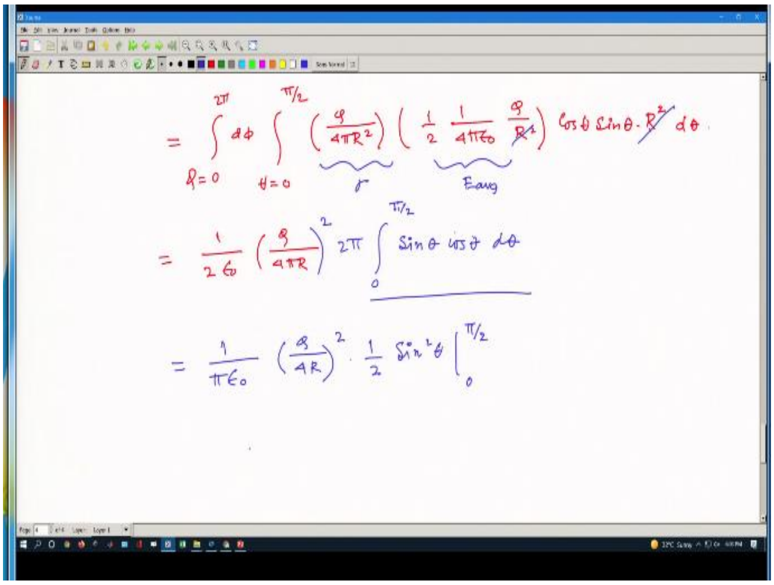

So, now we can write in terms of the unit and under done in terms of the limit of φ should be 0 to 2π as usual and I simply have dφ here, because there is no φ dependency and then I have the average value here and also let me first integrate over θ. θ is 0 to $\frac{\pi}{2}$ because this is a half hemisphere and then I just start putting the electric field here. So, this is $\frac{Q}{4\pi R^2}$ that is the value of the σ first in terms of total charge Q.

And so, this is my σ and then $E_{average}$ is this quantity $\frac{1}{2}$ of $\frac{1}{4\pi\epsilon_0}$ then $\frac{Q}{R^2}$ that is my $E_{average}$ and then rest of the term and that is cos θ and then sin θ also 1 R^2 should be here and it should be over dθ so, eventually I can have $\frac{1}{2\epsilon_0}$ and then $(\frac{Q}{4\pi R})^2$ because 1 R^2 is going to cancel out here I can see that so, 1 this R this R will cancel out so, I should have 1 by and whole square of this quantity.

And then 2π for this integration and then I have 0 to $\frac{\pi}{2}$ of sin θ cos θ and dθ. So, this integration we know and if I simply have this so it should be $\frac{1}{\pi\epsilon_0}$ because one 2 is cancel out and another π^2 I also take it out so, it should be this and then it should be $(\frac{Q}{4R})^2$ and I have half of I just make it 2 sin a half and then it should be 2 sin θ cos θ.

So, that means sin 2θ and if I integrate it should be half of sin 2θ and integrate cos and then if I just integrate it, should be $\sin^2$ θ and then 0 to say $\frac{\pi}{2}$ and because this is sin θ cos θ. So, I can have sin θ = x then cos θ dθ is dx and then I simply have this $\frac{x^2}{2}$ that is the formula I am using here now.

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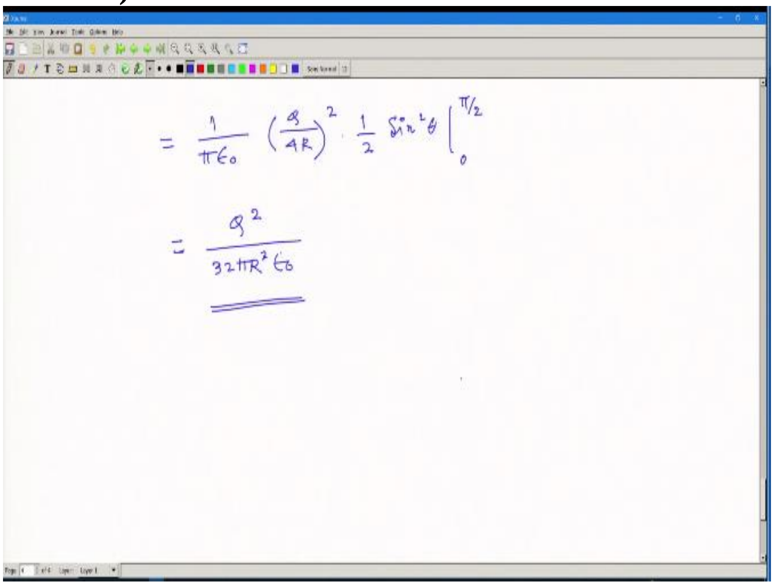

And that value is eventually $\frac{Q^2}{32\,\pi R^2 \epsilon_0}$ so that should be the total force 1 hemisphere should exert on another one this is the value. So, this is the way I believe, you can now understand that how do we deal with this problem I give 2 example, how the electric force this radiation, the electrostatic pressure and the force per unit area you can calculate the force per unit area and then you calculate what should be the force on these 2 plates or these 2 hemisphere etc.

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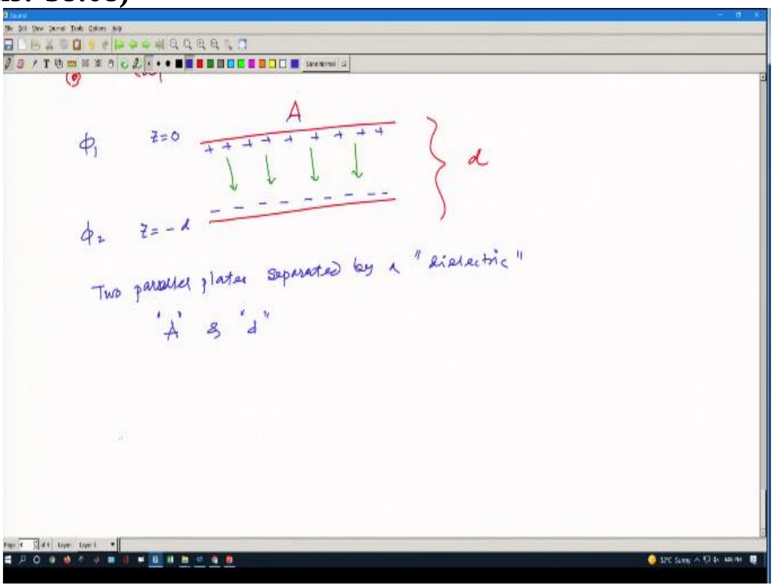

Now, let me move quickly to the concept of capacitor we know that this is not a very new thing. So, capacitor is suppose I have these 2 plates separated by a distance say d and the area is say A and Q_1 is charge say suppose here we have at z = 0 and this point is say z = -d and some charge distribution is there positive charge is here and some negative charge is accumulated here 2 opposite charges are there so that we can have a field here inside this region along this direction.

And if I calculate the potential at this 2 point say this potential is a ϕ_1 and this potentially is say ϕ_2. So 2 parallel plate separated by normally some dielectric we put here in this place what is dielectric we will learn later and area is A and the separation is d that I already mentioned these are the 2 parameters. So, this is an area A and separation is d the electric field in between the plates.

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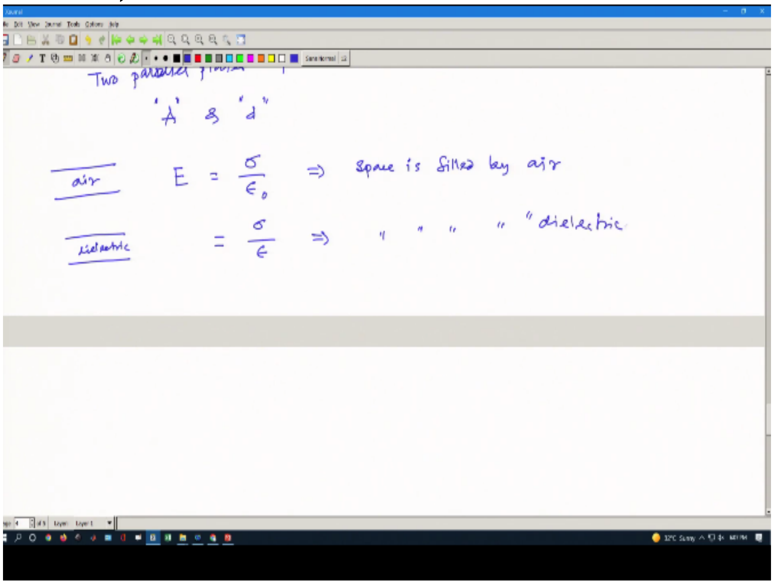

If I calculate the electric field in between the plate E that is simply $\frac{\sigma}{\epsilon_0}$ if the separation between 2 plates is filled by air that is otherwise it should be this quantity. So, $\frac{\sigma}{\epsilon}$ depends on if the space is filled by air that means, this is the capacitor and here we have air and in this case this is the capacitor we have some dielectric and if the space is filled by some dielectric.

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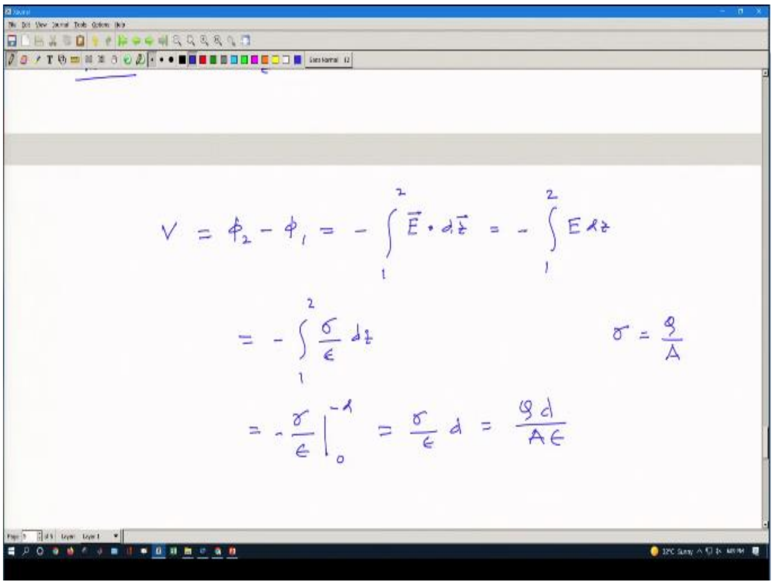

Now, the potential difference V is simply ϕ_2 - ϕ_1 and that is this quantity 1 to 2 if I go from 1 to 2 and then if I do some work on that so, this is along z direction or simply E dz 1 to 2. So, E we know this is - 1 to 2, E is $\frac{\sigma}{\epsilon}$ dz and that quantity is σ, -ϵ from 1 to 2 point means 0 to -d and that gives me $\frac{\sigma}{\epsilon}$ d, σ I can write in terms of this is total charge divided by area. So, I can have $\frac{Qd}{A\epsilon}$ that is the potential difference we have.

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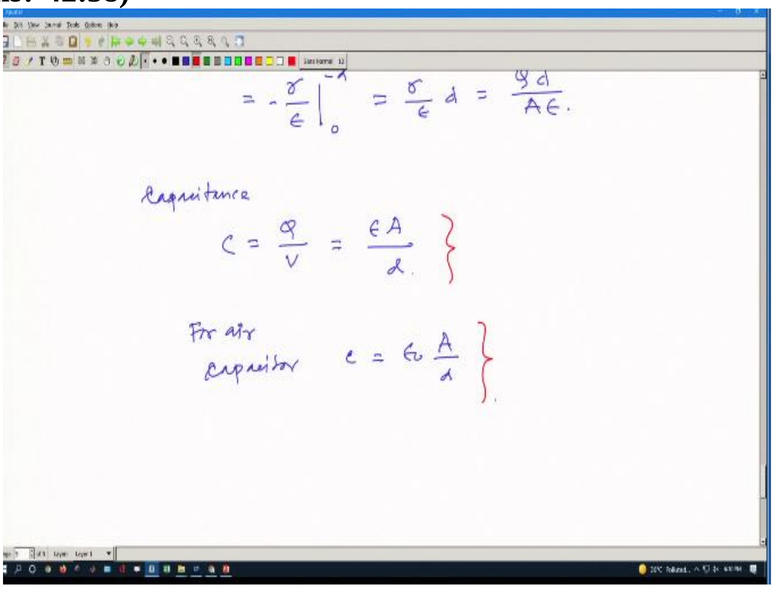

Now, the capacitance is the quantity that we define. So, the capacitance we define like C = $\frac{Q}{V}$ this is the way we define the capacitance and from that, we find that for the capacitance comes out to be $\frac{\epsilon A}{d}$. Now for air capacitor we have simply C = $\frac{\epsilon_0 A}{d}$ and this is a very well-known you know expression for capacitance and we know that how by reducing d or by increasing A how we can increase the capacitance or by changing this material or how we can increase the capacitance?

So, this is the way one can understand the capacitance very simple way and the next class today I do not have much time to discuss more in the next class we will try to calculate the amount of energy that we have in the capacitor and then try to understand what is the meaning of dielectric because here we mentioned that the capacitor in between the capacitor we put some kind of dielectric to increase, but what is dielectric, what are the properties?

And if the electric field is there inside the dialectic how it should behave. So, these kinds of things we will go to discuss in the future class. Thank you for your attention and see you in the next class.