Foundation of Classical Electronics
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Lecture-53
Application of Ampere's Law

Hello student to the foundation of classical electrodynamics course. So, under module 3, so we have lecture 53 today. And in the previous lecture we discussed about the Ampere's law and in today's class we will be going to apply. We are going to apply this law in different cases.

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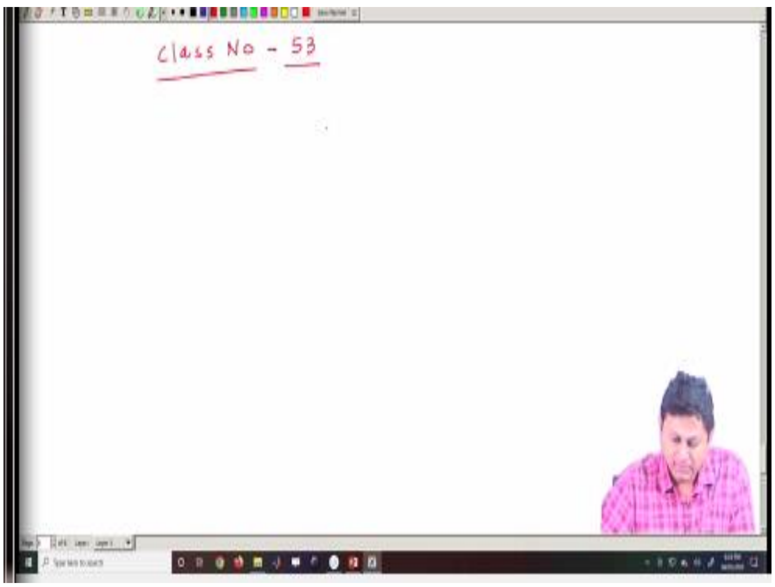

So, we have class number 53.

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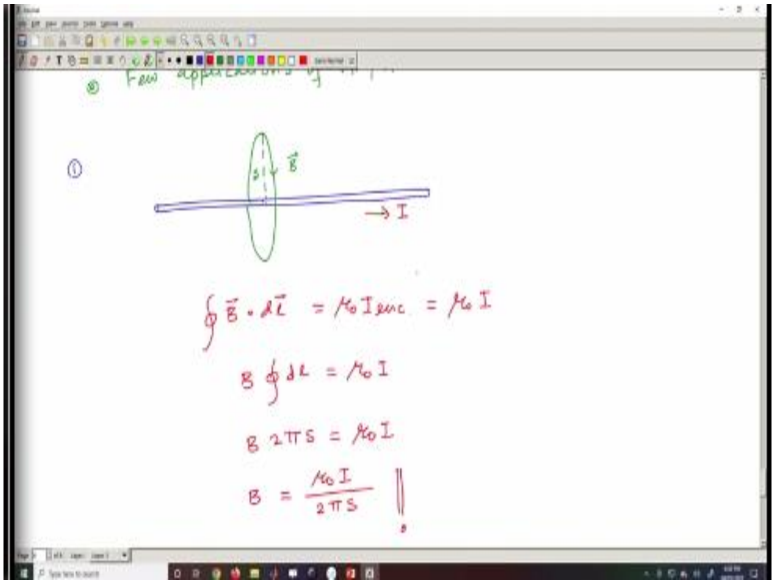

And today's topic is few applications of Ampere's law. So, what kind of applications? The very first is the magnetic field very straight forward few applications, which is also known to you that the magnetic fields at the distance from a long wire that we already calculated using Biot-Savart law. So, that thing again we are going to do here with very simple steps. So, suppose we have a very long wire, which is carrying a current and that current is I and it will going to produce some magnetic field around it.

And this magnetic field at some distance from the core of this fiber, from the center of this fiber, so that is the thing I wanted to calculate. So, magnetic fields will produce in this direction that is the value of the direction of the magnetic field. And if this distance is given suppose this is s the value of the magnetic field over this circle we want to find out and the expression that the equation that we are going to use is simply the Ampere's law.

Ampere's law is suggesting that the closed line integral of $\vec{B} \cdot d\vec{l}$, where this loop whatever the loop I draw is essentially the Amperian's loop = μ_0 and current enclosed enc. Now I know that inside this loop the amount of current is I. So, here I simply write μ_0 I and what is $\vec{B} \cdot d\vec{l}$? Because $\vec{B}$ is constant there will be no change in $\vec{B}$, so it is simply $\vec{B}$ integral of $d\vec{l}$. And then in the right-hand side I have μ_0 I and integral of dl is 2π and the radius here r.

In this case it is S, so it is B into 2πS that is equal to μ_0 I and B, the magnitude of B $\frac{\mu_0 I}{2\pi S}$. So, this is a very well established result that we derive for infinitely long wire. But here just exploiting the Ampere's law the derivation becomes very, very simple and no integration and nothing is required, so we will get the same result. Now if I want to find out the direction of the magnetic field in the vector form.

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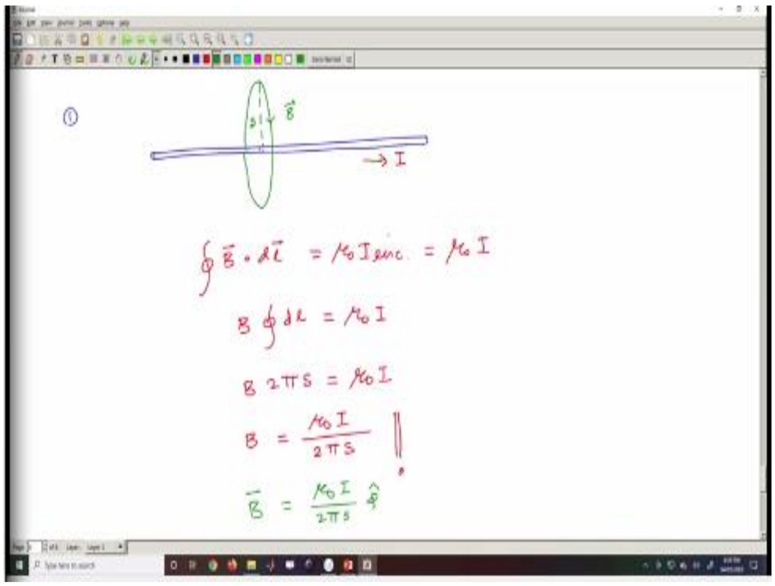

So, the vector form is simply μ_0 I this is the amplitude 2πs and you need vector of φ that should be the direction because it is revolving around the current carrying wire, so it should be something like this. The next thing, the next problem that we do problem 2 is very important and that is the magnetic field of a very long solenoid, let us write it.

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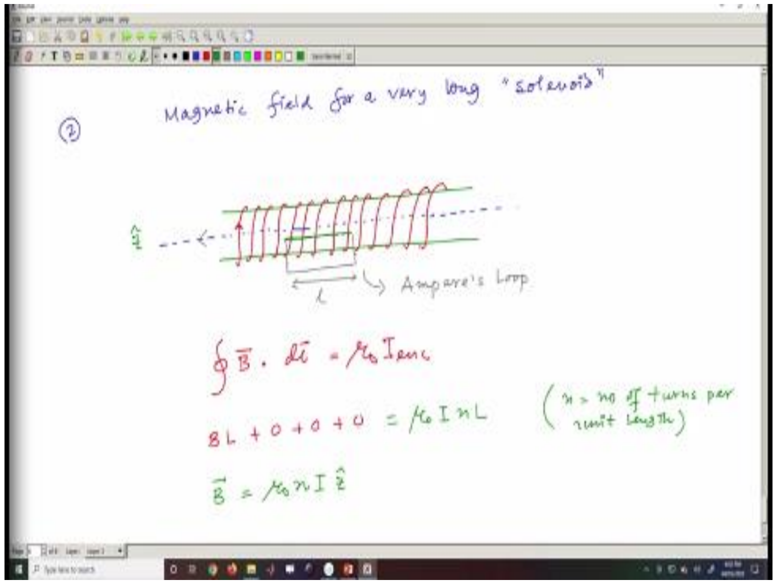

So, I believe all of you are aware of this term solenoid. So, we have say let me draw the solenoid suppose and this is the wire is surrounding like this. So, this is a wire is revolving and that is essentially my solenoid and I have an axis here, this is the axis of the solenoid like this. So, if I now want to calculate the magnetic field inside this long solenoid, then I need to draw the Amperian's loop judiciously.

And if I do that if I choose my Amperian loop like this, so inside over the axis because now it is circular, so the magnetic field should pass inside this through the axis of this solenoid. And this is my Amperian loop or Ampere's loop. And suppose this is of length l and I am going to exploit this expression, which is the Ampere's law that $\vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$ and what is I_{enc} that we need to calculate.

But $\vec{B} \cdot d\vec{l}$ if I want to calculate, so please check it so when the wire the current is flowing suppose the current is flowing in this direction? Say current is flowing in this direction; this is the direction of the flow of the current. Now this $\vec{B}$ will only produce inside the solenoid and since it is produced inside the solenoid the $\vec{B} \cdot d\vec{l}$ when we calculate and there are 4 portion of these loop, this is a square loop.

So, this is a square kind of loop, so we have l here and these 2 distances are there but these lengths are perpendicular to the production of the magnetic field. So, if I now calculate this closed line integral I should have 4 integral and this 4 integral will be BL that is inside and rest of the thing should be 0. Why because in one case in couple of cases the $\vec{B}$ is perpendicular because here the $\vec{B}$ is perpendicular to the l and here there is no B at all.

So, I will have 3 consecutive zeros for 3 lines, only the line that will contribute here is this one, this line. So, let me I like this one, so this is the line over which the magnetic field and dot dl will contribute. So, I simply have BL, right-hand side I should have current enclosed, so μ_0 is there, I current is flowing, and so I should be there. But it should not be only I because the current is enclosing by loop, there are few loops are there. In this figure there are 1, 2, 3, 4, 5 loops are there, so that we need to take account.

So, I can take account by just multiplying n into L, where n is the number of turns per unit length and L is the length. So, n is the number of turns per unit length and nL is the total number of turn, so that should be my expression. And then I simply find B = μ_0 n I and l, l will going to cancel out, so that should be the value. And if I want to find out what is the direction? The direction should be along z direction. If this is my say, so if this is my z direction, so this is say z according to the flow of the current. So, this is the magnetic field inside.

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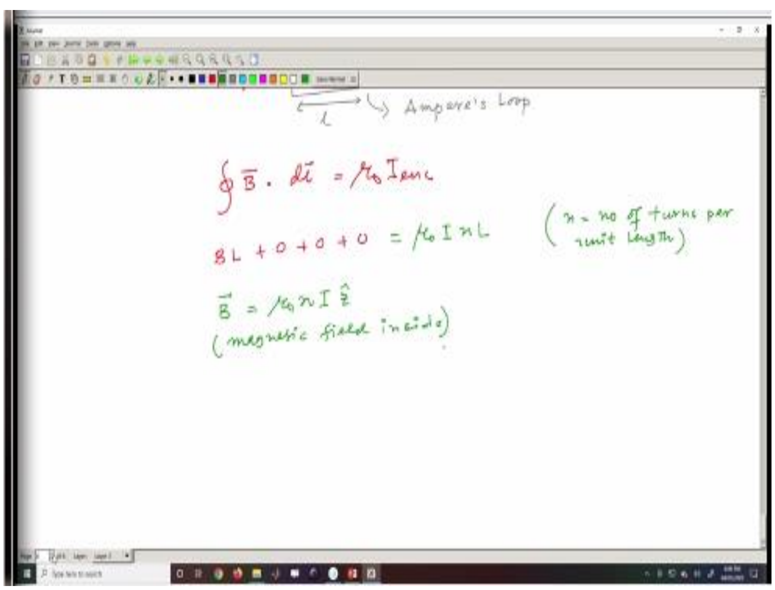

And magnetic field outside I say 0 but we can also demonstrate that. So, for outside let me use.

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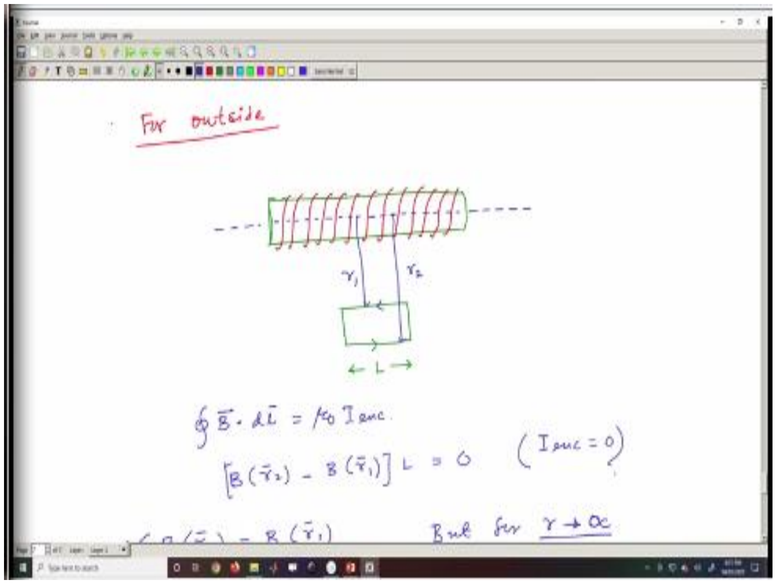

For outside if I calculate, how to calculate this? So, suppose again I have a solenoid like, this is my solenoid and I want to find out. So, this is the axis and from the axis, so let me draw the axis properly and I draw a loop here, say this is length L. And from this axis this length say is r_1 and this from here to here this is r_2.

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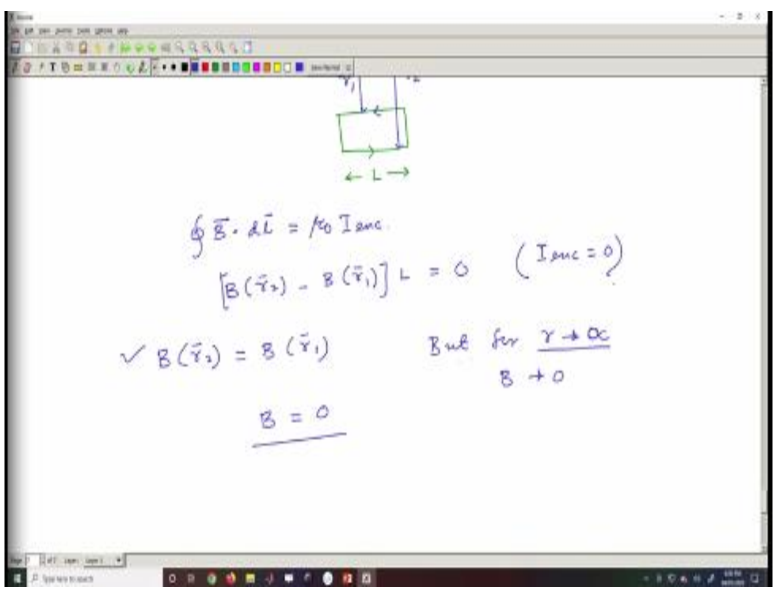

So, I have integration $\vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$. So, from here I can have B at $\vec{r}_2$ - B at $\vec{r}_1$ and multiplied by L is 0 as there is no current enclosed. So, one is plus sign one is minus sign because if I complete this loop it has to be like this, so one is this side and another this side. In one case if it is plus sign and that B is at $\vec{r}_2$ and another is $\vec{r}_1$. Now so this is true this minus this equal to 0, so B at $\vec{r}_2$ = B at $\vec{r}_1$ that the condition we are having.

Now for very, very large distance, so this is the condition we are getting. Now but for r tends to 0 this equation tells me that B is constant because at any r the value seems to be same. Now for r tends to infinity is very large, B has to be 0. So, that simply tells me that B, which seems to be constant is actually 0, which is again a constant value, 0 is a constant, so it does not depend on r and it is a constant and this constant value seems to be 0, why? Because I got this condition from the Ampere's law and it suggests that B at $\vec{r}_2$ and B at $\vec{r}_1$ should be same.

So, that means it is essentially a constant value and what is the constant value? I put the boundary condition that if r tends to infinity B has to be 0, so B itself 0, so outside I find B is 0. Now the next thing is a toroid, so this is a problem 2, now problem 3, case 3.

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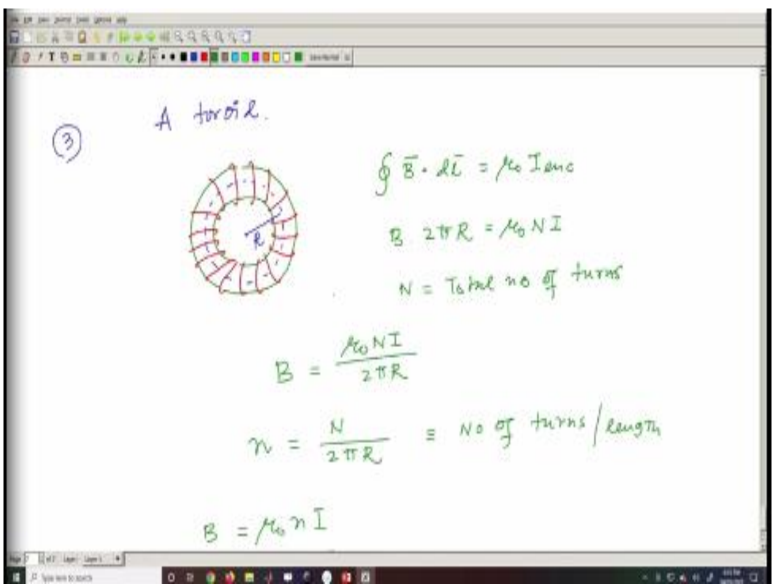

I should have a thing called toroid, so what is toroid? It is just let me draw then you will understand. And the toroid is something where we have, so this is roughly the structure of a toroid. And where we have this wire and this is the axis of this, so from here to here it is say R. Now if I want to find out what is the magnetic field inside this region I make a dotted line there then again I am going to use $\vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$.

Now B dl will be 2π if it is R, so it is R and it should be μ_0 N I, N is a total number of turn here. So, N has to be total number of turns, so the magnitude of B simply $\frac{\mu_0 NI}{2\pi R}$. Now also it is written N is total number of turn per unit length. Then n then it should be total number of turn divided by 2πR, so this is nothing but number of turns per length. So, it becomes simply B = μ_0 n I, that is the simply the magnetic field for a toroid.

It is exactly something like we have for solenoid but in toroid the 2 ends are merged, so it is like a donut shaped thing and inside that if you have a magnetic field the value will be something like this. Now after having all this let us now start another important topic, another important part of the magneto static and that is the concept of magnetic vector potential.

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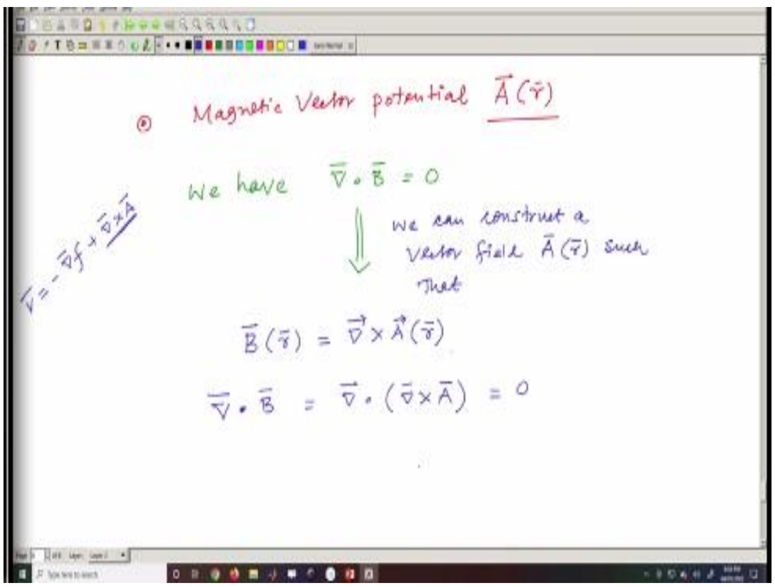

And normally it is written as $\vec{A}(\vec{r})$. So, so far we have the $\vec{\nabla}\cdot\vec{B} = 0$, so when we have $\vec{\nabla}\cdot\vec{B} = 0$ that essentially means that we can construct a vector field such that. So, from here so we can construct a vector field say $\vec{A}(\vec{r})$ such that $\vec{B}$ can be written in this way, $\vec{B}$ as a function of $\vec{r}$ can be written as a curl of that vector field. So, this is to some extent the statement that we already had when we are discussing about the Helmholtz theorem.

Any vector field can be represented in terms of $\vec{A}$, so in general any vector field $\vec{V}$ it was shown that one can write as scalar field + curl of a vector field, in general. So, for magnetic field I am just taking this one because for magnetic field we mentioned that the scalar field, there is no scalar field for magnetic field. So, scalar potential is not there, only the vector potential is there.

So, I can write it and if I write it automatically follows the $\vec{\nabla} \cdot \vec{B}$ is 0. Because if I make divergence both the side after defining $\vec{B}$ in this way, so the $\vec{\nabla} \cdot \vec{B}$ is simply divergence of some quantity, which can be written in terms of curl. And we know that divergence of curl of something is identically 0, so that follows. So, that means I can always write $\vec{B}$ in terms of curl of some field and this field is our vector potential.

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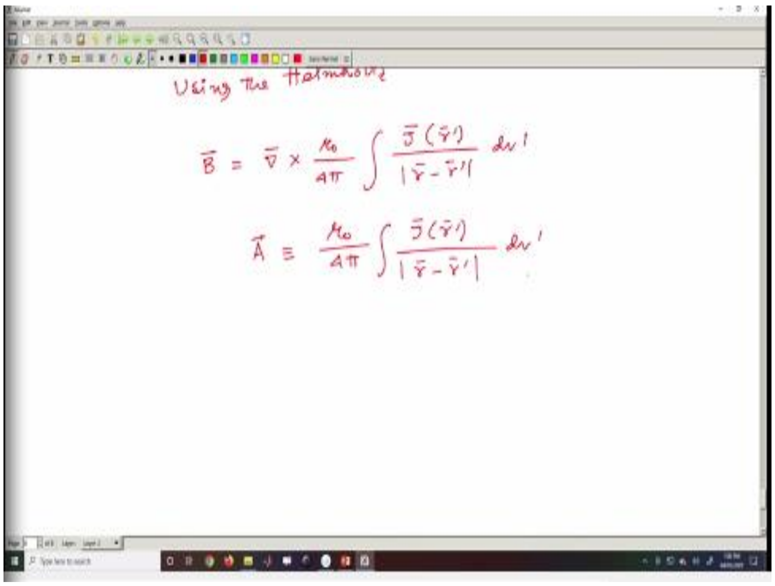

Now from the Helmholtz theorem since we mentioned that using the Helmholtz theorem we can have the $\vec{B}$ as the curl of that quantity. So, $\frac{\mu_0}{4\pi}$ and then $\int \frac{\vec{J}(\vec{r}')}{|\vec{r}-\vec{r}'|}$ dv'. So, from here I can write that $\vec{A}$ is simply $\frac{\mu_0}{4\pi} \int \frac{\vec{J}(\vec{r}')}{|\vec{r}-\vec{r}'|}$ dv'. So, if you forget the Helmholtz theorem, so let me quickly remind that what was that for magnetic field.

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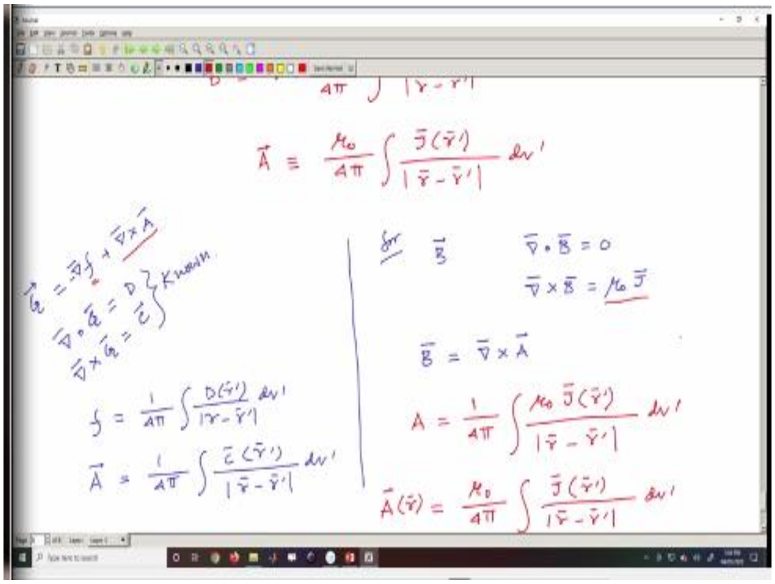

So, any vector as I mentioned can be represented in terms of minus of this plus $\vec{\nabla} \times \vec{A}$. Now if I want to construct $\vec{A}$, so now it is the given thing is this one, it was given that the divergence of these things. For magnetic field it is given and $\vec{\nabla} \times \vec{G}$ is if I write in generalized way, so say $\vec{\nabla} \cdot \vec{G}$ I put it like D and it is $\vec{C}$. So, if these two are known then the point is can I construct my, so here I should write $\vec{G}$ here, so $\vec{G}$ can be always constructed in this way, this is the unknown field and this is the recipe.

So, I can always write f in this way $\frac{1}{4\pi}$ integration of divergence whatever we had divided by $\vec{r} - \vec{r}'$ dv', D has to be function of $\vec{r}'$ that was one thing. And $\vec{A}$ this is a scalar potential, and $\vec{A}$ I can write as $\frac{1}{4\pi}$ integration of $\vec{C}$ it is a function of $\vec{r}'$ and then $\vec{r} - \vec{r}'$ dv', that was the statement of the Helmholtz theorem. Now if I tally these 2 side by side for $\vec{B}$ what we get?

For $\vec{B}$ we had $\vec{\nabla} \cdot \vec{B} = 0$ and $\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$, that we know. So, if I want to construct $\vec{B}$ since the divergence is 0, I have to construct my $\vec{B}$ in this way, curl of some vector potential $\vec{A}$. If I tally here, so I am taking only this part because f is 0 for $\vec{B}$. Now if that is the case I know what is my $\vec{A}$ and my $\vec{A}$ is simply $\frac{1}{4\pi}$ integration of $\vec{\nabla} \times \vec{B}$, which is $\mu_0 \vec{J}$, so μ_0 I have $\frac{\vec{J}(\vec{r}')}{|\vec{r}-\vec{r}'|}$ dv'.

So, $\vec{A}$ is essentially $\vec{A}$, which is a function of $\vec{r}$ comes up to be $\frac{\mu_0}{4\pi}$ integration of $\frac{\vec{J}(\vec{r}')}{|\vec{r}-\vec{r}'|}$ dv', in the light of Helmholtz theorem. And that is precisely I wrote here the same thing. So, I know what is $\vec{A}$ and now before concluding, so this is a very important thing we write that.

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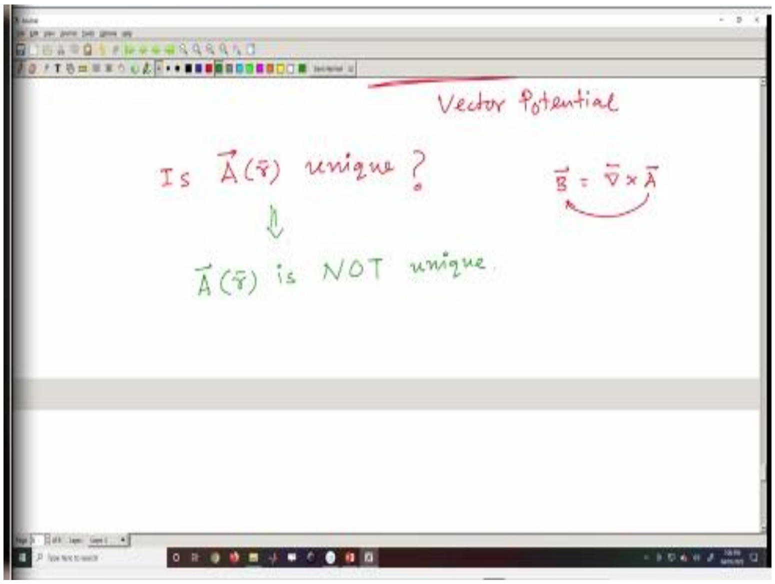

This is called the vector potential and since it is related to the magnetic field it is also called magnetic vector potential. So, that we wrote here the magnetic vector potential. So, now the question is okay, I have a vector potential, I know the expression, is it unique? So, I should comment here something because this is important. So, the question is, $\vec{A}$, which is at $\vec{r}$, is $\vec{A}(\vec{r})$ unique?

So, can I construct a unique $\vec{A}$ to define my vector $\vec{B}$? Because $\vec{B}$ can be defined, $\vec{B}$ can be constructed from $\vec{A}$, so $\vec{\nabla} \times \vec{A}$. So, if I know $\vec{A}$ then eventually from that information I should know $\vec{B}$. So, if the $\vec{B}$ is given the corresponding $\vec{A}$ is unique or not? That is the question. But here we can readily see that $\vec{A}$ is not unique, the answer here should be $\vec{A}$ is not unique, why?

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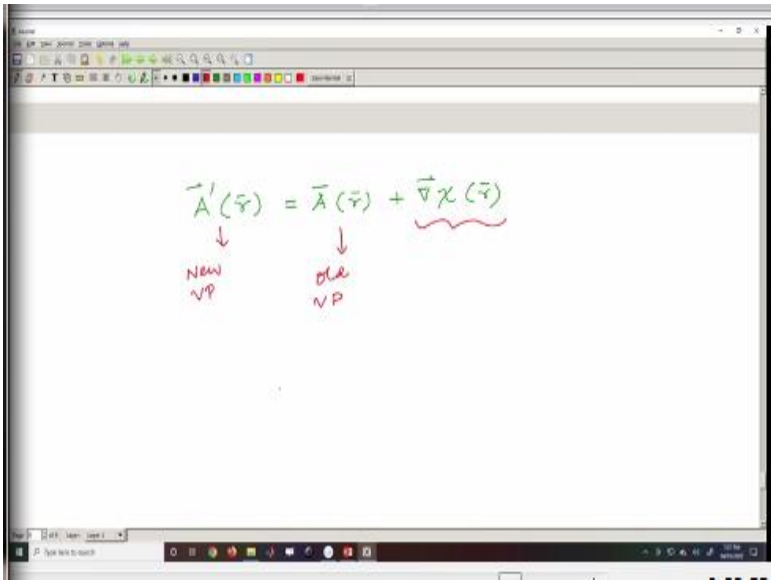

Because let us construct a new $\vec{A}$, which we call $\vec{A}'$, $\vec{A}'$ is my new vector potential and which is related to the old vector potential $\vec{A}$ in this way plus I add an arbitrary vector quantity as a divergence as a gradient of something like this. So, this is the additional term I add, this is my old vector potential, this is my new vector potential, I propose that okay I am proposing a new vector potential.

And old vector potential and new vector potential are related to this way by having an additional term I just add this quantity with the old vector potential. The question is, should the new vector potential produce the same V? And the answer is yes.

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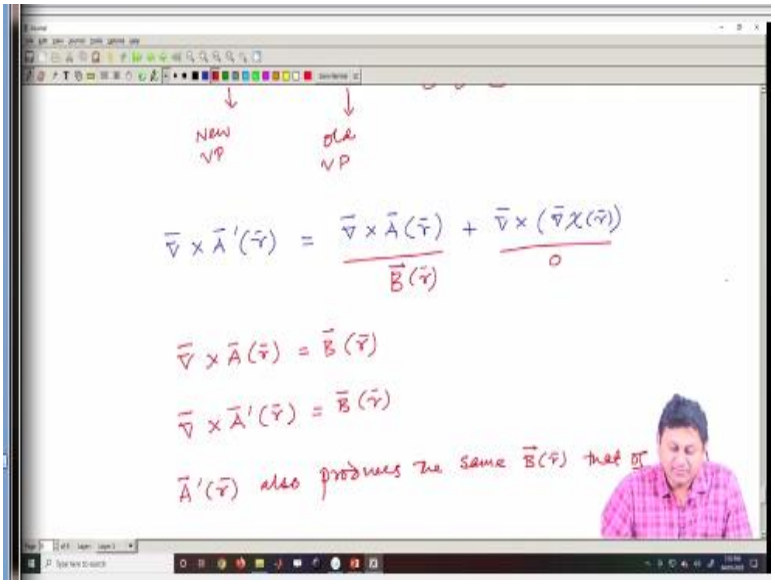

So, if I want to calculate that I can do it readily, if I do $\vec{\nabla} \times \vec{A}'(\vec{r})$ = I take curl both the side $\vec{\nabla} \times \vec{A}$, this is my old vector potential + curl of gradient of some scalar field say χ. Now you can see that this quantity is $\vec{B}$ because old vector potential can give rise to the value of V and that is the known expression. What about this quantity? This is 0 that means I can write my old vector potential give rise to the value of $\vec{B}$ like this.

And my new vector potential, this is not the old one but new I construct also give rise to the value of same $\vec{B}$ it is producing. So, that means $\vec{A}'$ are also produces the same $\vec{B}$ that of the old one, which is $\vec{A}$. So, today we learnt that what is vector potential, so I will like to conclude here because I do not have much time. So, today we understand the vector potential, we define the vector potential.

And we see that the vector potential from the Helmholtz theorem we know what is the value of the vector potential that is first thing. Second thing we check that this vector potential is not unique, so this vector potential is producing some kind of vector field like $\vec{\nabla} \times \vec{A} = \vec{B}$. But this $\vec{A}$ whatever the $\vec{A}$ we are talking about can be manipulated, you can write your own $\vec{A}$, some $\vec{A}$ is given and that $\vec{A}$ is producing the given vector field.

Now you can add another term with that like we did here to show that you make a $\vec{A}'$ and that $\vec{A}'$ and also producing the same magnetic field. So, that means these $\vec{A}$ and $\vec{A}'$ both are eligible to produce the same vector field $\vec{B}$, that means the vector potential is not unique. So, with this note I will like to conclude in the next class we see that to some sort of constant we are going to put on this vector potential.

And this constant is called the Coulomb gauge and then we try to calculate the vector potential $\vec{A}$ for few cases. So, it is exactly like the electrostatic problem where we find the scalar potential ϕ and then calculate the scalar potential for a given distribution of charges. Here also we will do the same thing, instead of calculating magnetic field we calculate the vector potential and from that you can extract the magnetic field by just making a curl over that. So, with that note I would like to conclude here, thank you for your attention and see you in the next class.