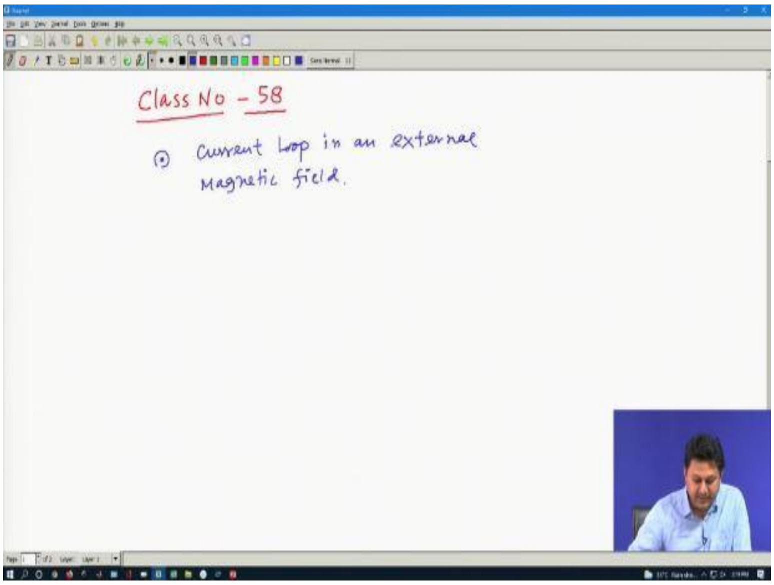
Foundations of Classical Electrodynamics
Prof. Samudra Roy
Department of Physics
Indian Institute of Technology-Kharagpur

Lecture-58
Torque and Potential Energy of Magnetic Dipole, Magnetization

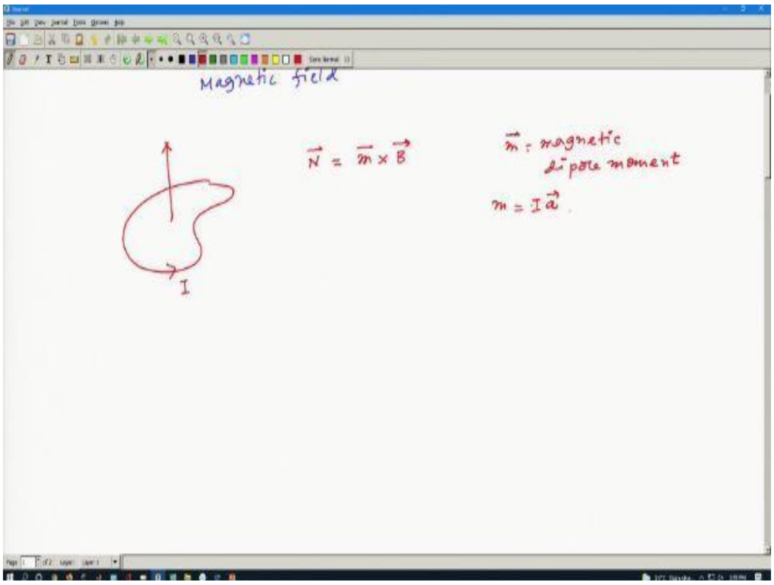
Hello student to the foundation of classical electrodynamics course. So, under module 3 we have lecture 58 today. And we will be going to discuss the torque and potential energy of the magnetic dipole, which we discussed last day also. And then we discuss the magnetization.

(Refer Slide Time: 00:42)



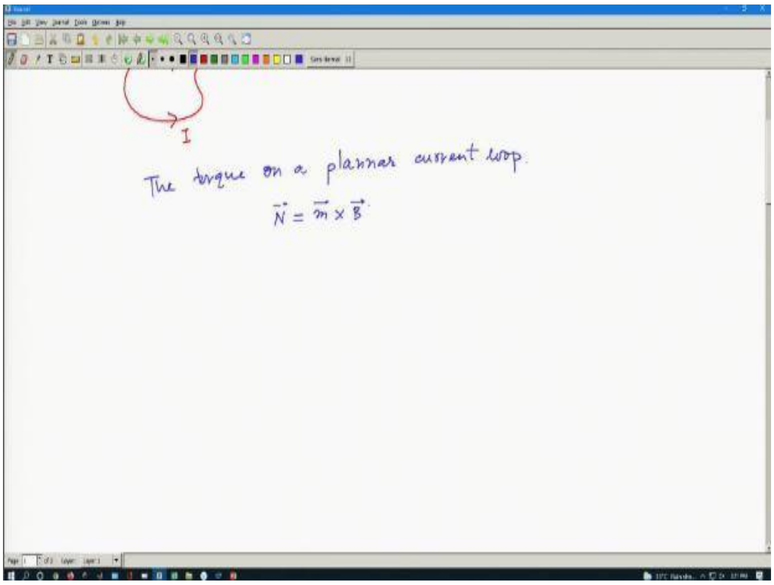
So, we have class number 58. So, in last class, our topic was current loop in an external magnetic field.

(Refer Slide Time: 01:39)



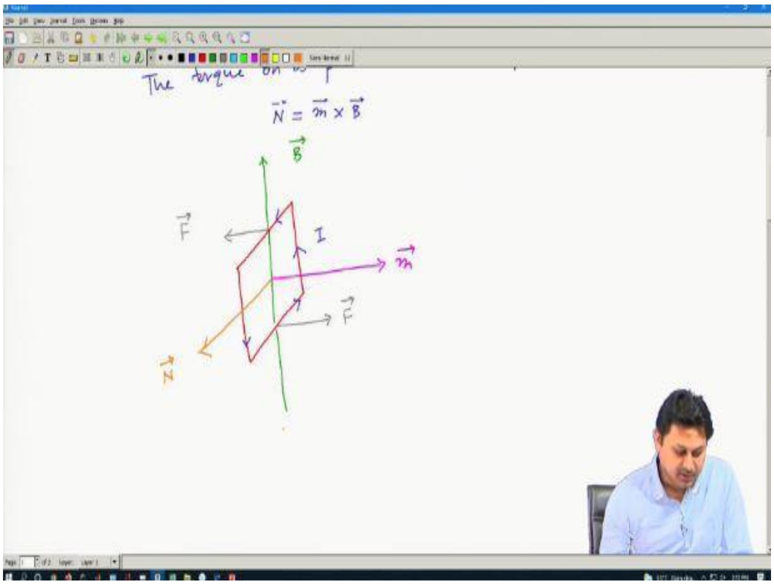
So, suppose we have a magnetic field and we place a current loop like this, where current is flowing I. And say this is the axis here. And the torque that you are going to experience by the system was $\vec{N} = \vec{m} \times \vec{B}$ that we figure out that we derived, where $\vec{m}$ is the magnetic dipole moment is the magnetic dipole moment. And normally it is defined like current into area. So, that is the structure we had last day. So, if I try to understand the torque.

(Refer Slide Time: 02:57)



Then this is basically the torque. So, if I write the torque on a planar current loop then this is the value we already, write it this should be $\vec{m} \times \vec{B}$. Now if I try to understand this.

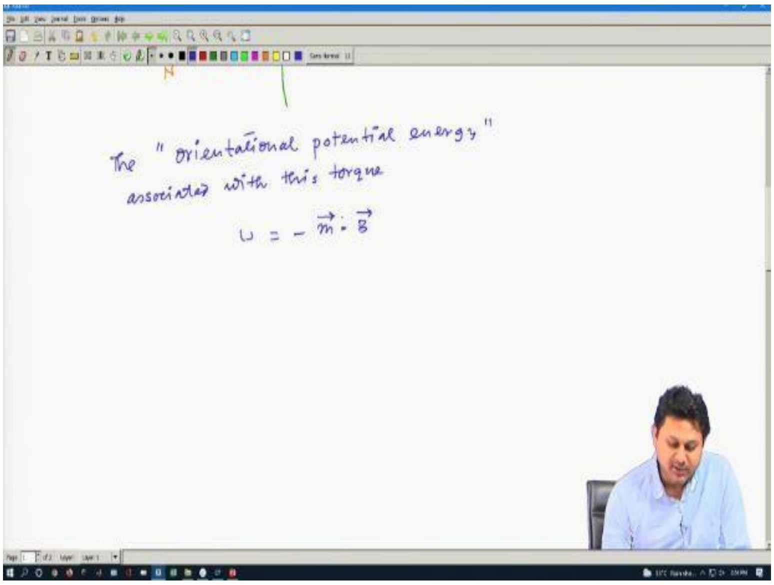
(Refer Slide Time: 03:39)



Suppose this is the applied magnetic field, this is in this direction. So, the current loop suppose it is like this. This is the current loop carrying the amount of current say I. So, since it is carrying a current I the direction of the magnetic dipole moment should be along this. So, this is the direction of the magnetic dipole moment. And the torque the force that will be going to experience by the system should be equal and opposite and this is the force that will experience here.

And since, the current is moving in the opposite direction and this so I will have another force that gives rise to a torque. And the direction of the torque should be along this. This is the direction of the torque vector $\vec{N}$, which is $\vec{m} \times \vec{B}$. This is $\vec{m}$, this is $\vec{B}$ and $\vec{m} \times \vec{B}$ is a perpendicular. So, this is the way I mean the loop, which behave like a magnetic dipole will experience a torque under some external magnetic field. Now what should be the potential energy that we also need to discuss here?

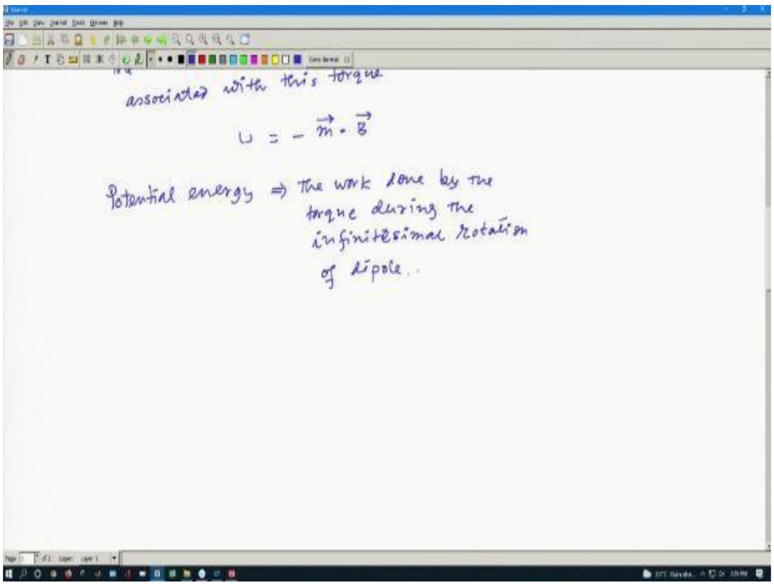
(Refer Slide Time: 05:47)



So, the orientational potential energy, what is orientational we will be going to discuss later. The orientational potential energy associated with this torque that is simply U = $-\vec{m} \cdot \vec{B}$. It is exactly the same kind of expression that we have in electrostatic when the dipole is placed in an external electric field and the potential energy that is stored to that dielectric was that amount.

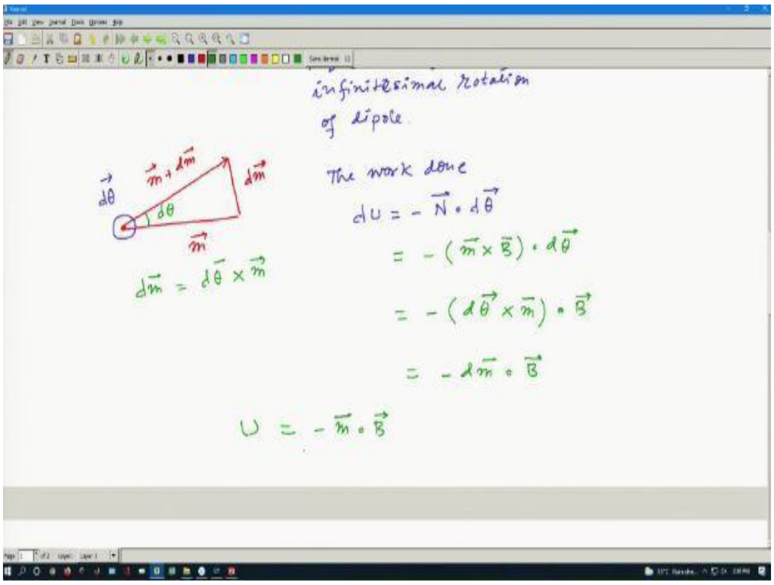
So, you need to do some work to orient these things and that was simply $\vec{p} \times \vec{E}$ and $\vec{p}$ was the electric dipole moment and $\vec{E}$ was electric field. Only you just replace $\vec{p}$ to $\vec{m}$ the magnetic dipole moment and electric field to magnetic field $\vec{B}$ and we will get the result. So, we can discuss this result.

(Refer Slide Time: 07:25)



So, the potential energy is here, the work done by the torque during the infinitesimal rotation of dipole. So, how is that, so let me draw it?

(Refer Slide Time: 08:48)

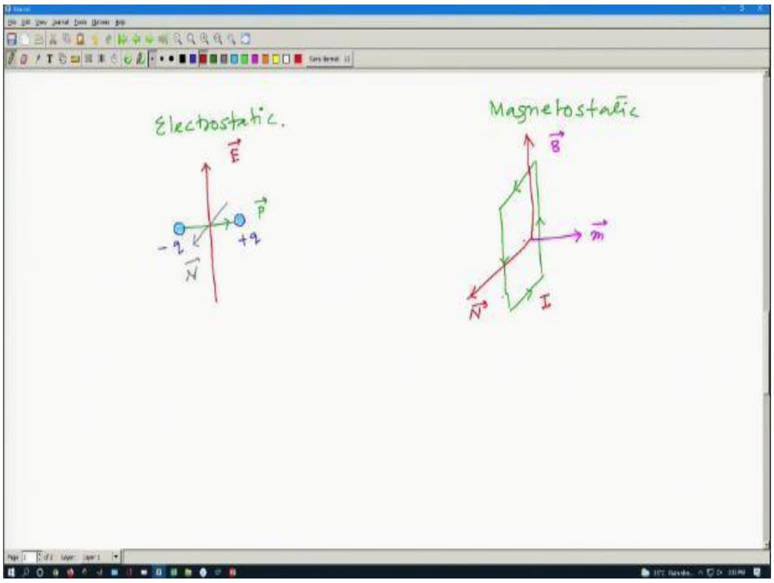


So, this is the top view I am having. So, this is the direction of $\vec{m}$ and if it is rotating to another direction, then this is d$\vec{m}$. And this is say $\vec{m}$ + d$\vec{m}$. And the angle that is rotating is say dθ. So, due to the torque what happened that if you see in this figure? So, these things will be going to rotate, because of the torque. So, this $\vec{m}$ whatever is here now having a different orientation and current loop will slightly move?

And that basically, gives the amount of work done by the torque and that will be stored as a potential energy. So, that we will be going to calculate. So, the dθ here, if I consider this to be a vector, that is perpendicular to the plane. And we can have the work done, which is du is minus of torque and d$\vec{\theta}$, that is the amount of work done by the torque. Now here you should note that this d$\vec{m}$ is simply d$\vec{\theta} \times \vec{m}$. So, I can write here this is as $\vec{N}$ is $\vec{m} \times \vec{B}$ and that should be dθ.

And I can have this as $\vec{A} \cdot \vec{B} \times \vec{C}$ is $\vec{B} \cdot \vec{C} \times \vec{A}$ so and $\vec{C} \cdot \vec{A} \times \vec{B}$. So, I can write this as -d$\vec{\theta} \times \vec{m}$ and then dot $\vec{B}$. So, this quantity is my d$\vec{m} \cdot \vec{B}$. So, this is du. So, eventually we can have U = $-\vec{m} \cdot \vec{B}$, this is the amount of potential energy that this is this we called orientational potential energy associated with this torque. So, let us let us discuss a bit about the orientational potential energy. What I try to mean is so, before that let us draw the 2 cases. One is so for electrostatic, what we had so let me draw.

(Refer Slide Time: 12:40)

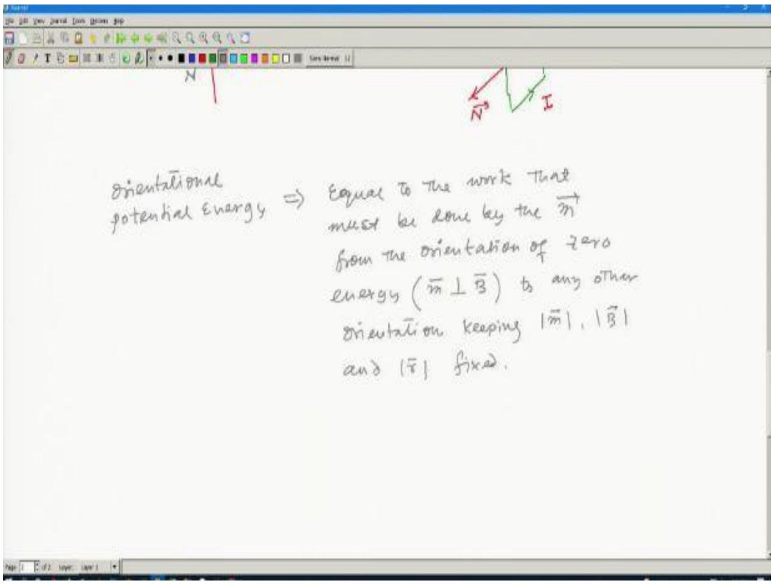


This is say electrostatic and this side I have magnetostatic and side by side I want to discuss. So, the electric field suppose is in this direction for electrostatic. And we have a dipole sitting here with the say - q to + q. And the torque that was generated is along this direction that was the torque. We also calculated the torque in placing the electric dipole in external electric field.

So, this is the electric dipole**.** So, and that is in this direction we had $\vec{p}$. So, this is the direction of $\vec{p}$. So, from here to here, so that is the direction of $\vec{p}$ dipole moment. Now in magnetostatic, what we have? Magnetostatic the magnetic dipole can be replaced by the loop. And this is the current loop and the current is flowing like this, like this, this and this. And the value of the magnetic dipole moment is along this direction, this is $\vec{m}$.

And the $\vec{B}$ is along this direction. This is $\vec{B}$. It should be perpendicular. So**,** let me draw it once again. So, $\vec{B}$ is roughly this and the torque that one can expect is along this direction. And that $\vec{N}$, current is flowing here and that is $\vec{N}$. Now this is the 2 case and for that let us now discuss, what is the meaning of orientational**.**

(Refer Slide Time: 16:04)

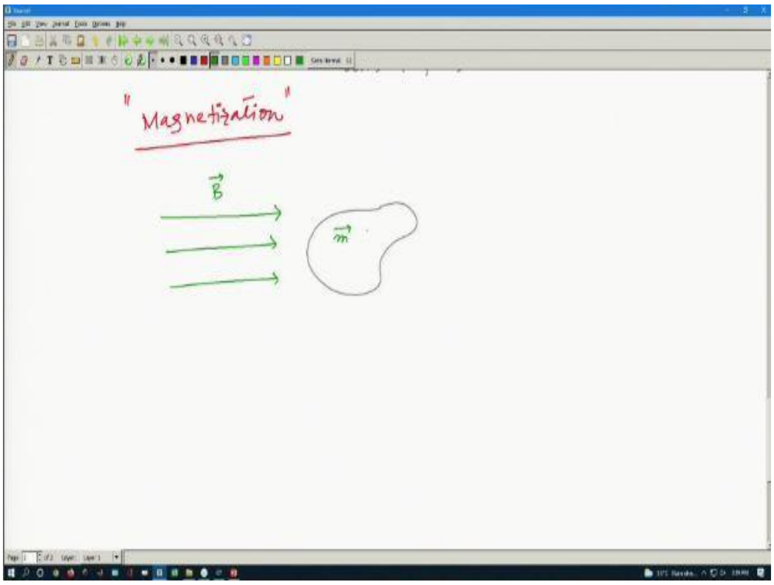


So, in this case we had something called orientational potential energy that eventually mean that which is equal to the work that must be done by the dipole moment. If dipole moment $\vec{m}$ from the orientation of zero energy, when we have zero energy, when $\vec{m}$ is perpendicular to $\vec{B}$ to any other orientation keeping this $|\vec{m}|$ and then $|\vec{B}|$ and $|\vec{r}|$ fixed. So, we just change the orientation by applying the torque, if you just change the torque. So, that there is a change in the orientation, magnitude of $\vec{m}$ should not change.

So, $\vec{B}$ is a constant. So, it will be in this direction only and $\vec{m}$ now in perpendicular to be in this figure. So, that tells us that $\vec{B}$ and $\vec{m}$ are perpendicular, but I can make a torque. And due to that there is a change in the orientation. And this change in the orientation can be considered to be the orientational potential energy that you have in magnetostatic. With respect to electrostatic it is slightly different because, in electrostatic what happened that the potential energy.

I mean in this case what happened that the potential energy is in general it appears because it is equal to the total work that must be done to bring the dipole from infinity. So, here we are not doing that kind of thing rather we are just orient this dipole against this torque $\vec{N}$ and make an orientational potential energy. So, now after that we will discuss magnetization. So, that is another topic today that we need to cover.

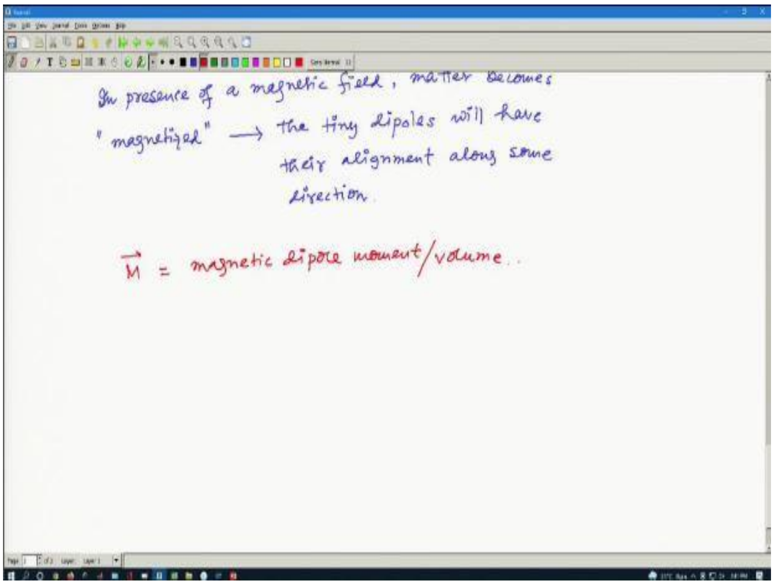
(Refer Slide Time: 19:45)



So, the next topic is magnetization. So, **so** far we are dealing with the problem where magnetic field is calculated mainly in the vacuum. When we use the Biot-Savart law and when you use the Ampere's law whatever then the magnetic field that is produced in vacuum but, if we place a matter in a magnetic field. Suppose, we have a magnetic field here and this is the magnetic field lines and I place a matter here a chunk of magnetic matter.

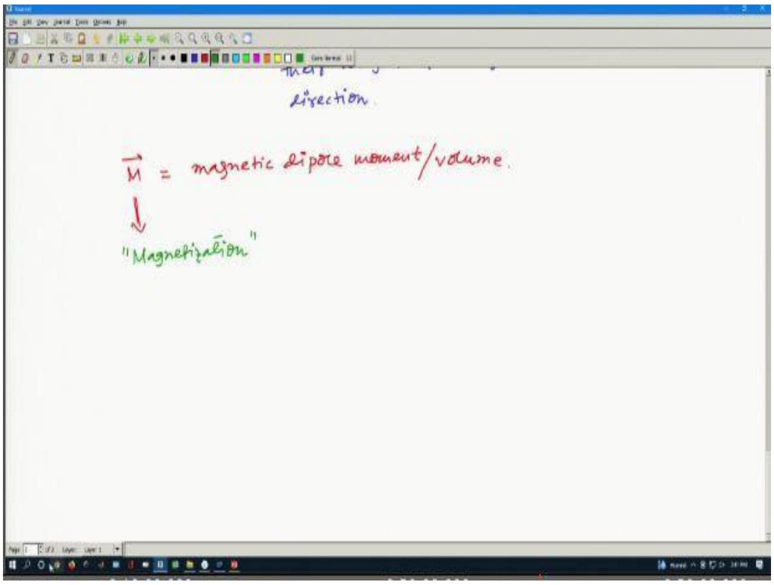
So, some sort of orientation, so the dipole the magnetic dipole will be going to be oriented by this magnetic field. Exactly like the polarization we had in electrostatic problem. So, we have the alignment of this magnetic dipole moment or magnetic dipoles. So, because of that we will have certain property in the material and that is called the magnetization. So, if I want to you know write it in words.

(Refer Slide Time: 21:16)



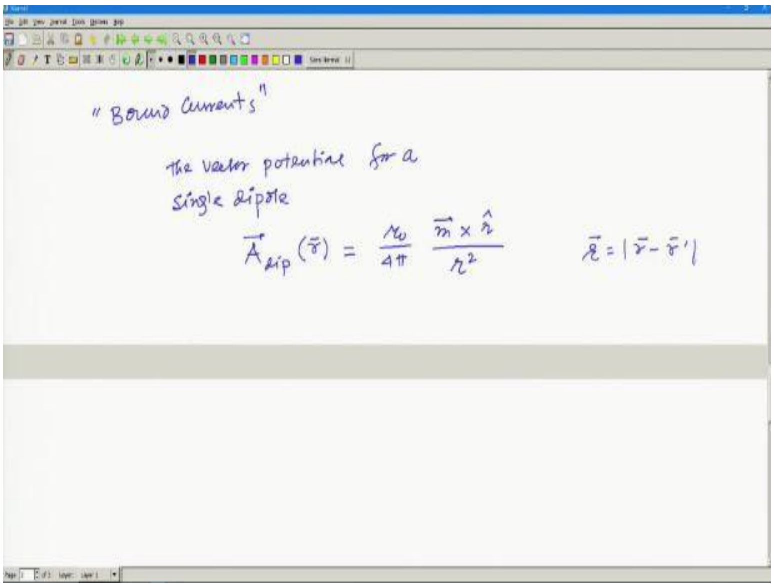
So, in presence of a magnetic field, the matter becomes magnetized. That means the tiny magnetic dipoles will have their alignment along some direction. This is called the magnetization. So, there is some alignment of the tiny magnetic dipole and we have the magnetization.

(Refer Slide Time: 23:08)



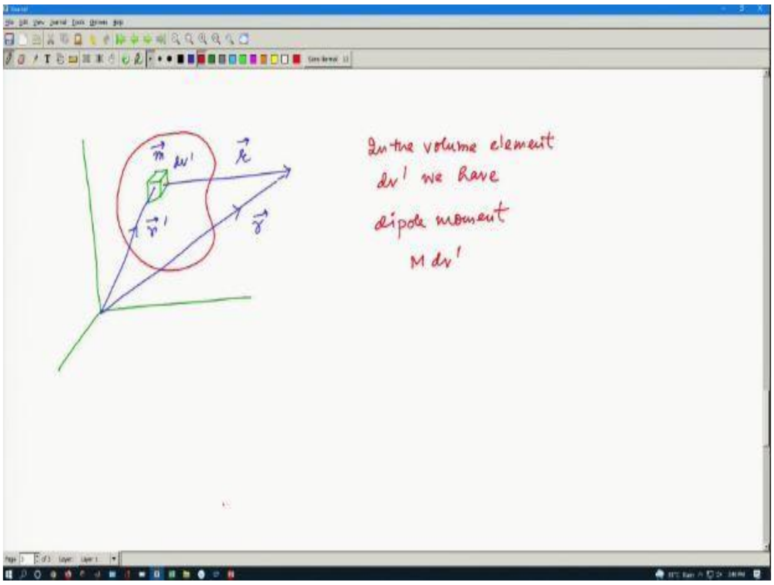
So, we define few things like $\vec{M}$, which we call the magnetization, which is equal to magnetic dipole moment per volume. And this is called magnetization, like polarization it is called magnetization. So, magnetization is the magnetic dipole moment per unit volume like polarization. Polarization electric dipole moment per unit volume magnetization is like that.

(Refer Slide Time: 24:13)



So, here we have an interesting concept called the bound current, like in electric polarization case, so that will going to explore. So, bound currents, so the vector potential for single dipole we calculate that and that is $\vec{A}$ I write dipole to make sure that it is a vector potential for the dipole. And this is $\frac{\mu_0}{4\pi}$ and then it is $\frac{\vec{m}\times\hat{\mathscr{r}}}{\mathscr{r}^2}$ where $\vec{\mathscr{r}}$ is $\vec{r}$ - $\vec{r}$*'* that we know. So, now if I let me draw that, then maybe that these things will be clear.

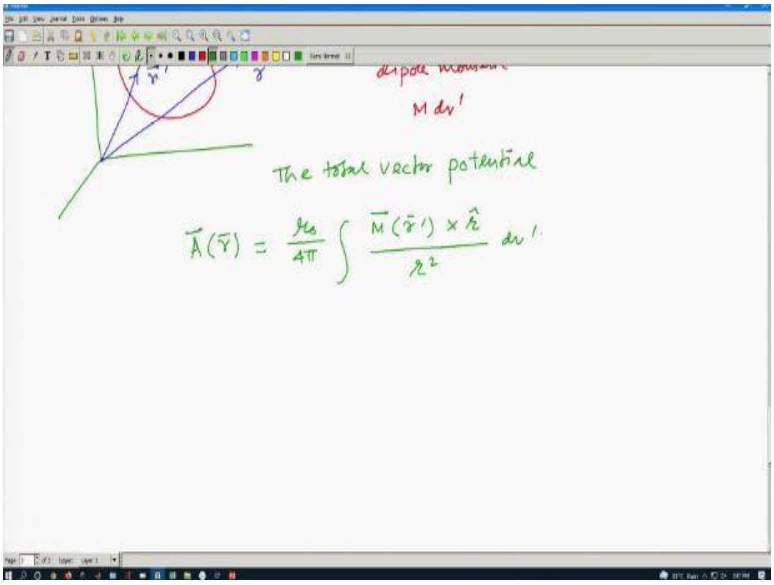
(Refer Slide Time: 25:55)



Suppose we have a distribution of dipole here, have a small element and had a coordinate system. Suppose this is my $\vec{r}$ and this is the location from here to here. This is $\vec{r}$*'* and this point $\vec{r}$. So, this is the dipole moment of this small tiny region and dv*'* is the small volume. So, in the volume element dv*'*, we have dipole moment, so, $\vec{m}$ is a dipole moment of a single dipole, but I have a volume dv.

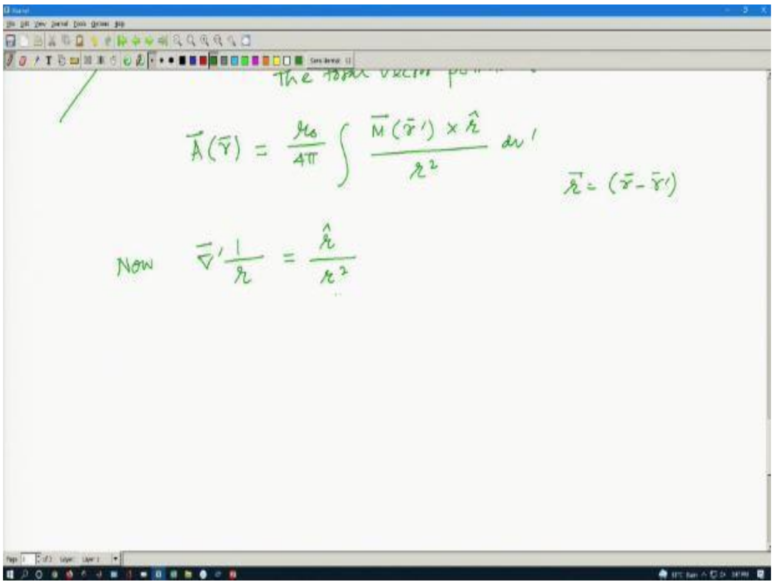
So, in this dv the total amount of dipole moment one can have is simply M multiplied by dv*'*. Because, M is the dipole moment per unit volume and if I multiplied by the volume then we have the total dipole moment is this. So, the total vector potential if I want to calculate here due to this entire structure having the distribution of the magnetic dipole.

(Refer Slide Time: 28:19)



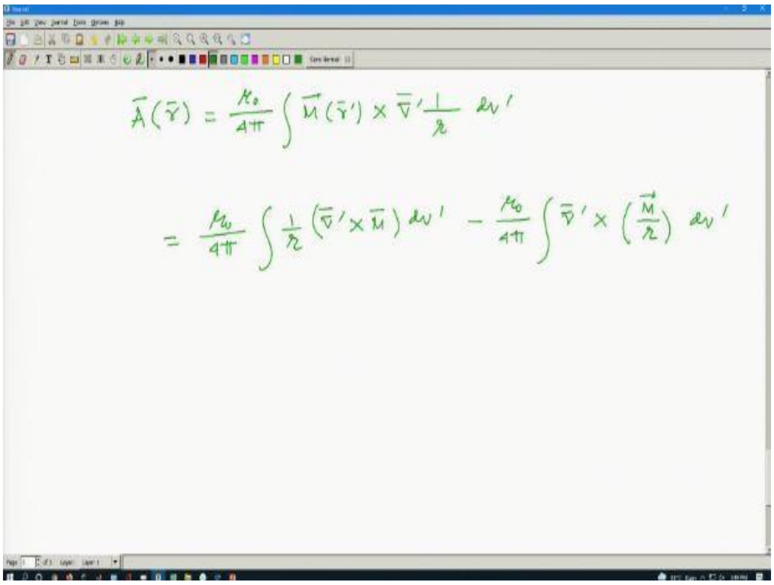
Then the total vector potential can be represented as the total vector potential is how much I need to integrate it because it is a total. So, I have $\vec{A}$ produced at some point $\vec{r}$ is $\frac{\mu_0}{4\pi}$ and then integrate over entire volume, where the small element I have the dipole moment as $\frac{\vec{M}(\vec{r}')\times\hat{\mathscr{r}}}{\mathscr{r}^2}$ and dv' that should be the total. Now I can make use of when $\vec{\mathscr{r}}$ is $\vec{r}$ - $\vec{r}$' according to our notation.

(Refer Slide Time: 29:28)



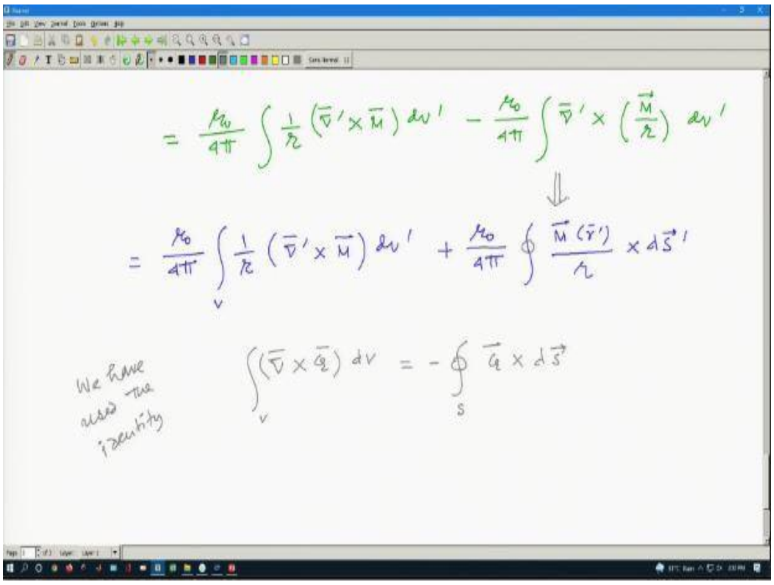
So, now we can have the $\vec{\nabla}'(\frac{1}{\mathscr{r}})$ is equal to $\frac{\hat{\mathscr{r}}}{\mathscr{r}^2}$. So, that I am going to use because I am having a term $\frac{\hat{\mathscr{r}}}{\mathscr{r}^2}$.

(Refer Slide Time: 29:54)



So, I can write my $\vec{A}$ then function of $\vec{r}$ in this way $\frac{\mu_0}{4\pi}$ and then I integrate it $\vec{M}(\vec{r}')$ and then cross the $\vec{\nabla}'(\frac{1}{\text{Л}})$ and then dv'. Now I can make use of the relation that is that curl of so, this I can write in this way, that $\frac{\mu_0}{4\pi}$ and then I can have like $\frac{1}{\text{Л}}$ and then this cross $\vec{M}$ dv' and - $\frac{\mu_0}{4\pi}$ integration cross $\frac{\vec{M}}{\text{Л}}$ dv'. If I expand this quantity then I will get this term plus this term. So, that this term is this minus this. So, that I simply write here and then I can have this.

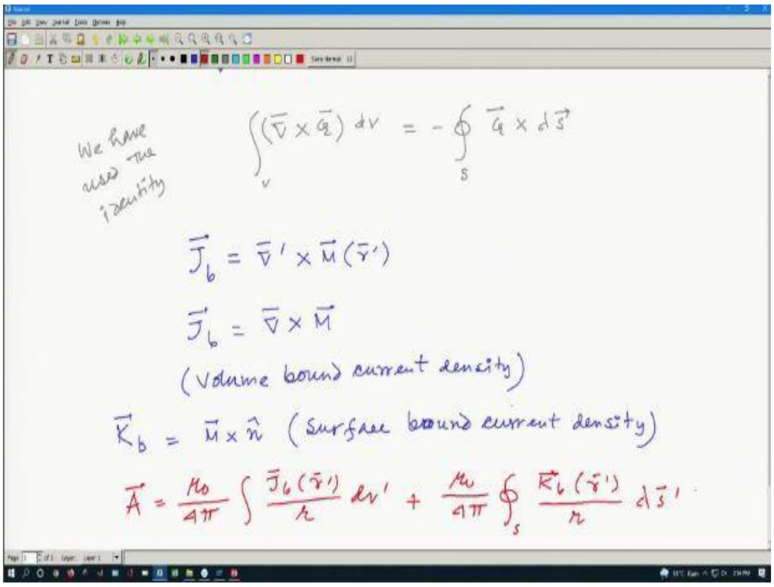
(Refer Slide Time: 32:04)



So, next I will write like $\frac{\mu_0}{4\pi}$ and then this is the volume integral and $\frac{1}{\text{Л}}$ then $\vec{\nabla}' \times \vec{M}$ dv' the same term. But here I am going to use something a vector identity. Let me first write and then I will show what is the vector identity and that is $\frac{\vec{M}(\vec{r}')}{\text{Л}} \times$ ds'. So, the identity we use is this one. So, this is one of the problems in Griffith's book.

So, we have used the identity and the identity is if I have a vector field say $\vec{G}$, $\vec{\nabla} \times \vec{G}$ dv over v is equivalent to minus of close surface integral. So, I should have a closed surface integral here and then if then $\vec{G}$ cross not $\vec{A}$, I am using the vector field $\vec{G}$ so, $\vec{G} \times d\vec{s}$. So, I use this vector identity to get from this to this, this expression to this one.

(Refer Slide Time: 34:16)



So, then after doing that I define these currents bound current. One is the bound current density and that is $\vec{J}_b$ and that I defined at this cross $\vec{\nabla}' \times \vec{M}$ which is function of $\vec{r}'$. So, simply I can write it is a $\vec{\nabla} \times \vec{M}$ just dropping this prime notation. So, that is one definition and this is called the volume bound current density. Similarly, I can have another term $\vec{K}_b$, which we have like $\vec{M} \times \hat{n}$ by definition and this we call the surface bound current density.

So, based on this definition I can write my $\vec{A}$ in this way. My $\vec{A}$ should be now $\frac{\mu_0}{4\pi}$ and then integration volume current density that should be function of $\vec{r}'$ divided by Л dv' + $\frac{\mu_0}{4\pi}$ and the total surface integral. Then surface current bound current density function of $\vec{r}'$ divided by Л $d\vec{s}'$. So, I think I should stop here, because I do not have much time.

So, in the next class what we do that we will discuss more about this bound current density and surface bound current density, volume bound current density and surface bound current density and the physical significance of that. So, with that note I like to conclude today's class. See you in the next class.