Foundations of Classical Electrodynamics
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Lecture-59
Bound Current

Hello student to the lecture of foundation of classical electrodynamics. So, under module 3 today we have lecture 59 and in today's lecture we will go to discuss bound current.

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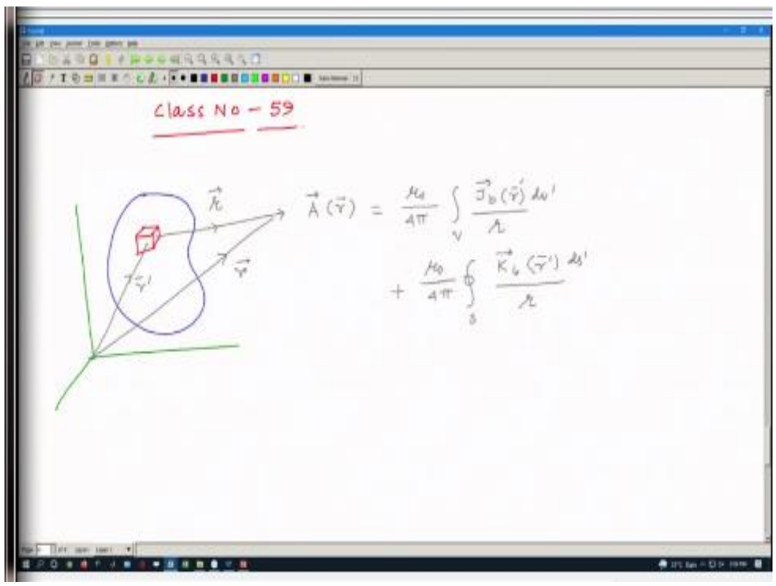

So, we have today class number 59, so in the last class we try to find out suppose I have a distribution of the dipole and if I have a volume element here with some coordinate system then what should be the vector potential and when you calculate the vector potential? We find few terms, so let me first draw the geometry. So, this was $\vec{r}$ and this was $\vec{r}$' and at this point when we calculate the contribution of $\vec{A}$, which is the vector potential.

This vector potential was combined in with 2 terms, so one term was related to say $\frac{\mu_0}{4\pi}$ and that was related to the bound volume current density, we called it $\frac{\vec{J}_b(\vec{r}')}{\text{Л}}$ dv'. And another contribution was $\frac{\mu_0}{4\pi}$ close line integration surface and that is the contribution of the surface bound current density and then $\vec{r}$. So, that I should not put the vector sign here because it is like this.

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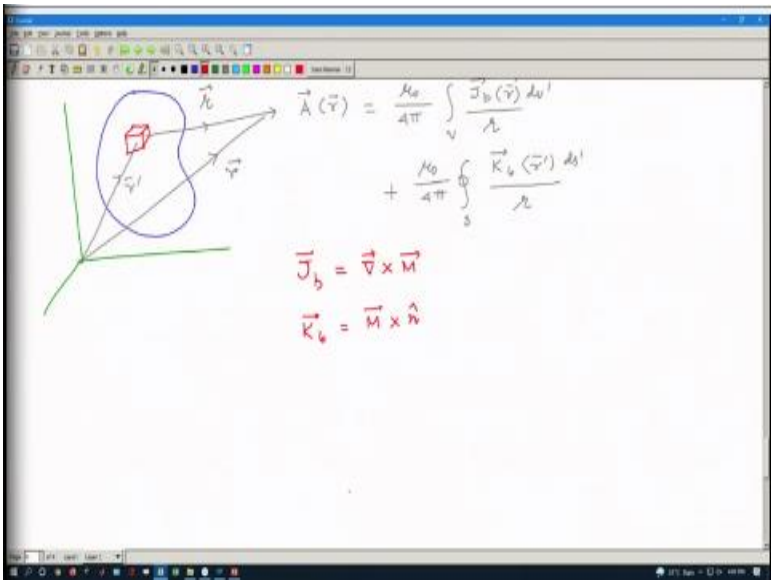

So, today we will be going to discuss by the way where we define bound volume current density is the curl of the magnetization and the surface bound current density is simply $\vec{M} \times \hat{n}$. If you look carefully it is a resemblance with the polarization when we discuss the polarization when we place a dielectric and due to that in an external electric field, some dipoles are created, so we have a polarized material.

And because of that we have some bound current and surface current and the bound current density was the divergence of the polarization. Here like we have the volume bound current density because of the curl of the magnetization and $\vec{M} \times \hat{n}$ for surface. So, that was the definition we put, so today we are going to understand the concept or the physical interpretation.

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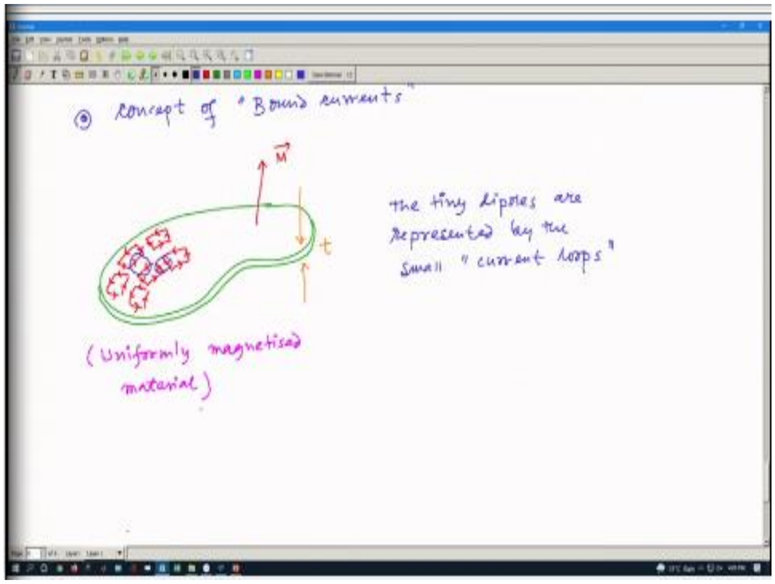

So, today's lecture is will be on the concept of bound currents. So, already I define $\vec{J}$ and let us consider a uniformly magnetized material. So, let me first draw it, so suppose here this is say uniformly magnetized material. So, I have a magnetization then and this magnetization is based on the, so I should have some current loop, let us consider these are the current loops because magnetic dipole moments are defined by current loop.

So, I have these current loops as a representation of the magnetic dipole, these are say the current loops, each current loop are defining as a single magnetic dipole. So, this is the way the current loops are distributed here showing certain magnetization and this is the direction of the magnetization. And we have a thickness here because I have a material, so I should have a thickness and let us have this thickness to be t. So, as I mentioned the tiny dipoles are represented by the small current loop.

This tiny dipoles, there were many dipoles inside the system, so we just represent these dipoles as a small current loops. Now if you look carefully these figures then you find that all the internal currents are cancelling out. So, I can mark here suppose this is one of the internal current that is cancelling because they are moving opposite direction. In the similar way, I can have another current loop here they are cancelling each other. So, you can see that these current loops, which are equal and opposite.

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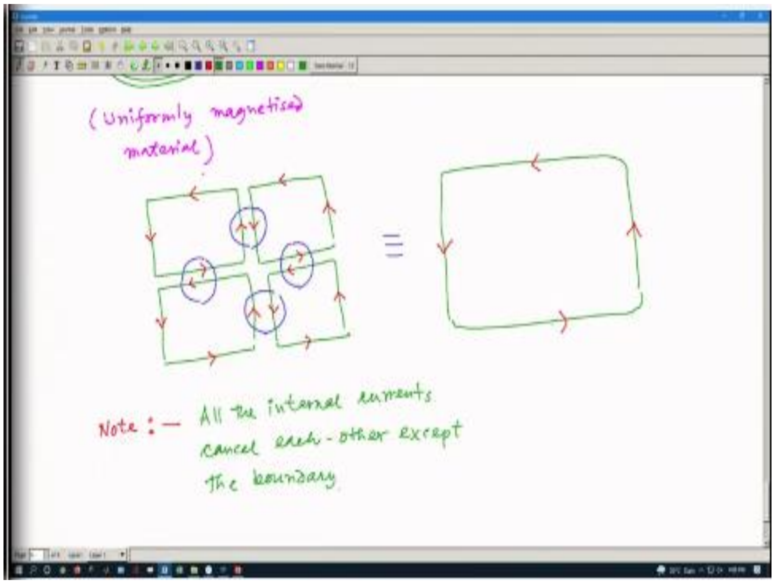

So, I can draw here in a more bigger picture. So, suppose these 2 current loops are present and we have another current loop here and the currents are moving like this. So, all this interlocks here, here, here and here they are cancelling out. Resulting an equivalent system where we have the current only in the boundary because all the internal currents are cancelling out. This is some sort of the drawing that when we proved the Stoke's theorem.

So, note all the internal currents cancel each other except the boundary. So, as the figure shows all the internal current marked by this blue circles they are cancelling out each other but the boundaries are still there. So, we have the boundary current here. So, this figure whatever the figure I draw is eventually becomes an equivalent figure where a strip is there, not the entire because all the currents are cancelled.

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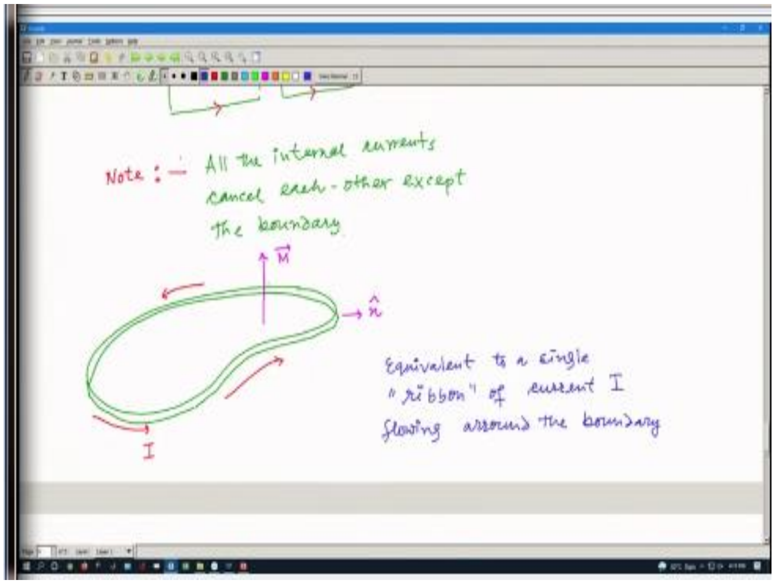

So, this figure whatever I draw if I just try to replace it, it was like this. And now what we are having? So, these are the currents, this is the region where the currents are there, so the current is flowing over these boundaries, so this is the total current I. Magnetization is along this direction and this is the direction of the surface because it is now forming a ribbon. So, this becomes an equivalent, so this is equivalent to a single ribbon like structure of current I. Because this I is flowing through this ribbon surface flowing around the boundary. So, this is flowing around the boundary and it is equivalent to a ribbon.

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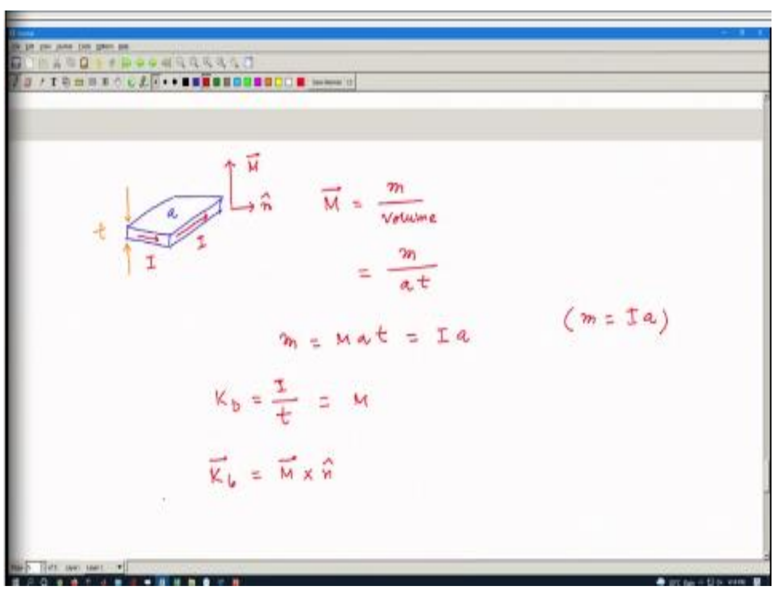

Now, so if I draw here, so how these things are there, let me quickly. So, I have a system here like this, suppose this is my area *a* and the current is flowing over this part, this is my current I, magnetization should be along this direction and the thickness, this is important is still t. So, the magnetization is how much? This is the magnetic dipole moment m divided by the volume. So, that gives me M if a is the area then a into t thickness is my volume.

So, from here I can see that m, which is magnetic dipole moment is magnetization multiplied by a t, magnetization multiplied by volume. Again the magnetic dipole moment is current into area as m = current into area. So, the K_b, which we defined as the amount of current, this is the surface current density, so I should write it $\frac{I}{t}$, so from here what I see is it is simply from here $\frac{I}{t}$ it is simply equivalent to M.

Now, so $\vec{M}$ is in this direction, so $\vec{K}_b$ if I write in vectorially then it should be $\vec{M} \times \hat{n}$, $\hat{n}$ is along this direction. So, if I make a cross of $\vec{M} \times \hat{n}$ then it should be the direction of this flow of the current. So, my $\hat{n}$ the surface is along this direction and $\vec{M} \times \hat{n}$ is showing the direction of the flow of the current that we are having here in this simple structure. So, that roughly describe that how the surface current is defined by $\vec{M} \times \hat{n}$.

This is just a rough a crude way to understand pictorially as well as geometrically or physically we can understand that. Then this is for uniform magnetization, so the magnetization is uniform. But if the magnetization is non-uniform then that basically gives rise to something called the bound current density that we are going to show here.

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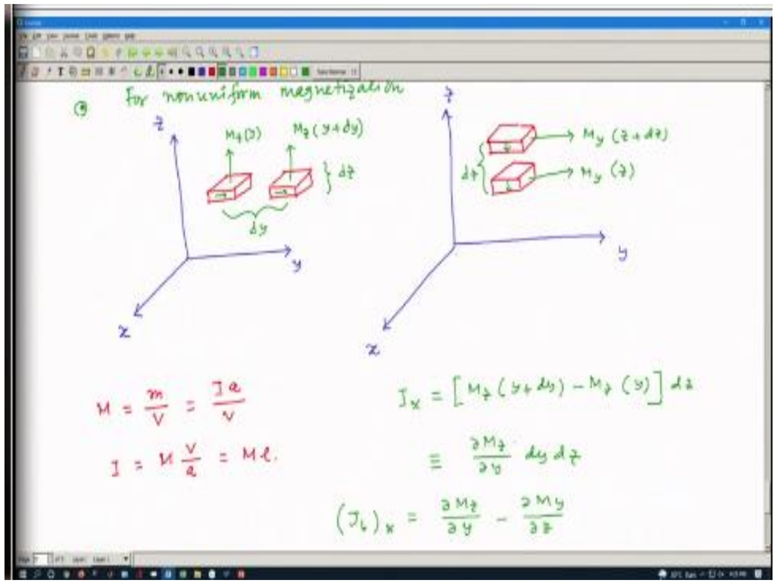

So, next thing is for non-uniform magnetization, what happened? For non-uniform magnetization, so that means specially the magnetization is different, it is not uniform, so let us have a coordinate system then. So, this is a coordinate system say x, y and z and we have a magnetic material here chunk of this magnetic material let me draw it, this and then again we have this and so on side by side.

The current is flowing here and the magnetization is along this direction and this is over z direction, so magnetization I have the z component. And now it becomes a function of y because I am putting this side by side, so it is a function of y now. So, at particular y it is one value and since it is non-uniform in other y say I have this value, this is also a z component of the magnetization but this is a different y.

So, I write say this is y plus say some small distance dy where this distance from here to here it is dy distance because this is along y direction and this length is say dz. So, this is the change the variation of this magnetization because it is non-uniform, so if I go along y direction I can have different magnetization and at y + dy I have magnetization M z and y I have magnetization say M zy, it can also happen along the z direction.

So, if I do that, if I do plot the same figure here, so this is my coordinate system, this is x, this is y, this is z and what I have is this, I have a chunk here. And now the chunk I am having here and the current is now flowing, this is in y direction. So, now the current is flowing along say negative of, so this is flowing along this direction and this direction over surface. The value of $\vec{M}$ then in this direction, this is y component, this is magnetization y component but this is at the value say z + dz and this is the value at z and this distance is dz.

Like here we are having the same thing but now in y direction the magnetization here the z direction we had the magnetization. So, here we can see that the magnetization, let me do quickly one thing, the magnetization is magnetic dipole moment per volume that gives rise to current into area, which is magnetization divided by volume. So, the current I can have magnetization into volume per area and that eventually gives me magnetization into length.

So, the I_x from here if I calculate the I_x that is the change of magnetization. And so if I write here it should be $[M_z(y + dy) - M_z(y)]$ dz, this is the net magnetization. And I am having the net magnetization and that should be this. So, this is equivalent to if I make a Taylor series simply $\frac{\partial M_z}{\partial y}$ and then it is dy and dz. Now in the similar way, I can have so here what I get is. So, if I divide I_x divided by dy dz then I can have $\frac{\partial M}{\partial y}$, so that is simply my bound current because this current is bound.

And if I do the x component of that, that is eventually this divided by this tiny area dy dz and that gives me $\frac{\partial M_z}{\partial y}$ but that is not only the one contribution. Because another contribution is still here because I am taking the y component, so that this variation is still there and if I calculate this variation along y I should have a negative sign same thing. But it should be now $\frac{\partial M_y}{\partial z}$, same calculation but I am having now in the variation along this negative direction.

So, that means now the current is flowing, so here this thing is this means that the current is now flowing in opposite direction. So, the net current that we are having the current density that we are having is this. Now you can see that this is the x component, so I should write it properly. This is the x component of the $\vec{J}_b$, which is the bound current; this is the x component of $\vec{J}_b$. So, you can do that for y component and z component.

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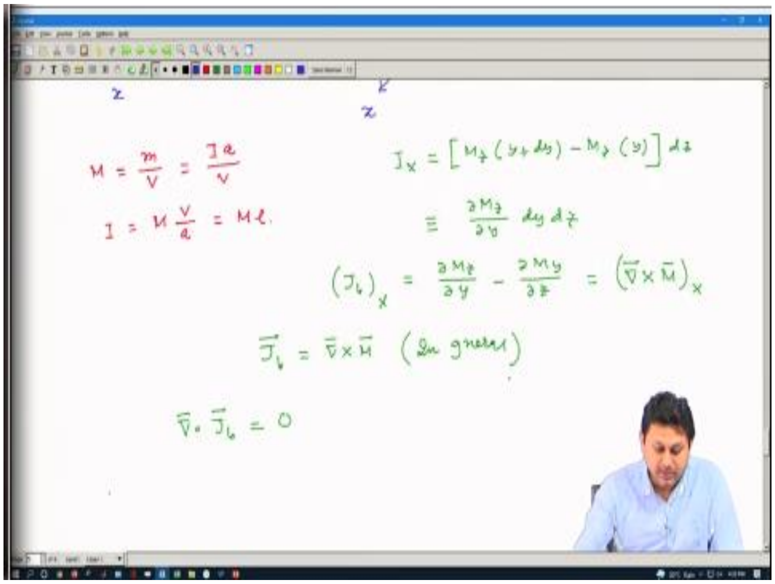

And in general if I write $\vec{J}_b$ in the vector form, then this seems to be, so if I write here this is the $\vec{\nabla} \times \vec{M}$ and x component of that. If you do the y component and z component with the same logic then you eventually get the $\vec{\nabla} \times \vec{M}$ in general. Also if I want to find out the divergence, so you can check it that the $\vec{\nabla} \cdot \vec{J}_b$ is simply 0 with this formalism. So, now we show that how this bound current density and bound surface current density and bound volume current density can be realized physically.

And this is the way one can realize that how the bound current and surface current is there. So, the bound current is bound volume current density is here when we have the variation of the magnetization inside the material if the magnetization is not uniform. Then only we have this curl value non zero otherwise if the magnetization is uniform whenever you do the curl it will always give rise to 0. So, non-uniformity of the magnetization is the reason for having this bound surface current bound volume current density.

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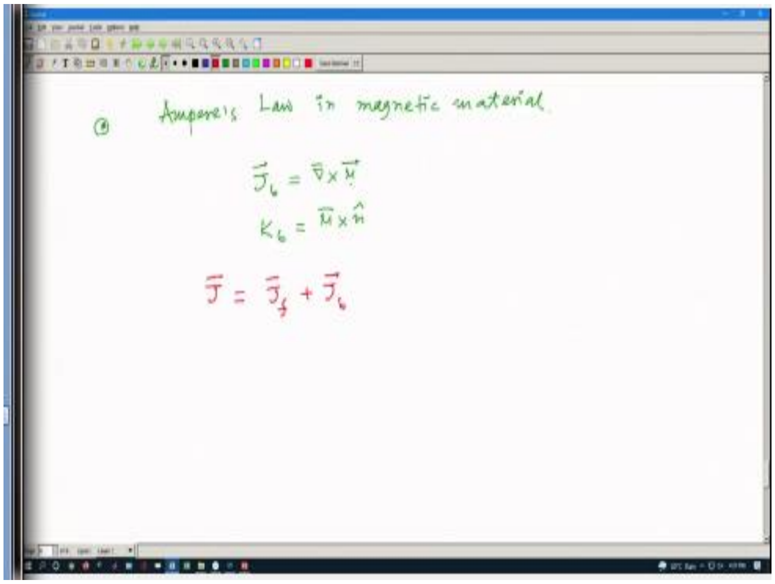

So, now quickly do few things and that is what should be the form of the Ampere's law? So, let me write it Ampere's law in magnetic material. So, how we can modify this law in magnetic material because now magnetic material we have this bound current, which is related to the magnetization. Let me write it and this is $\vec{M} \times \hat{n}$, so now total current density is now composed of 2 parts, one is the free current and another is the bound current.

Because this bound current arises because of the non-uniformity of the magnetization as I mentioned and it is inside the material where, if there is no material, so there is no question of magnetization, only we are dealing with the free current density. But here in the material we should now include this $\vec{J}_b$.

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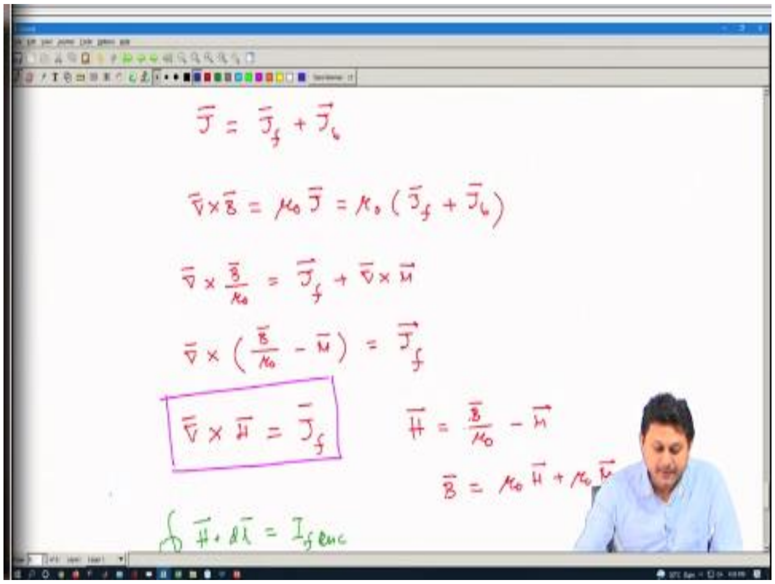

But according to the Ampere's law $\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$, now I should write $\vec{J}$ as $\vec{J}_{free} + \vec{J}_b$ because now it is the combination of 2. Then I can have curl say I write $\frac{\vec{B}}{\mu_0}$ that is equal to $\vec{J}_f$ and then plus $\vec{\nabla} \times \vec{M}$ because $\vec{J}_b$ is $\vec{\nabla} \times \vec{M}$. And I can have an expression like $\vec{\nabla} \times (\frac{\vec{B}}{\mu_0} - \vec{M})$ is $\vec{J}_f$.

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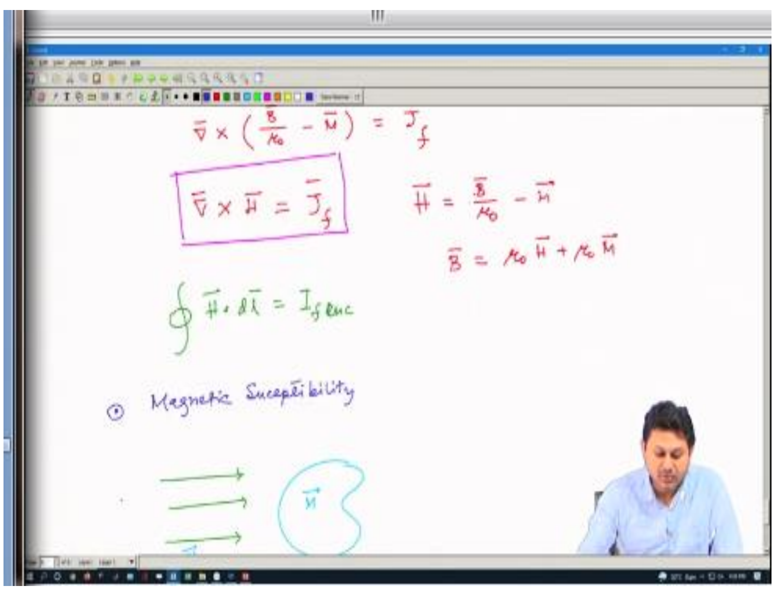

Or I can have $\vec{\nabla} \times \vec{H} = \vec{J}$ this is another form, when this $\vec{H}$ is simply $\frac{\vec{B}}{\mu_0} - \vec{M}$ or in another notation $\vec{B}$ is $\mu_0 \vec{H} + \mu_0 \vec{M}$, whatever. So, $\vec{\nabla} \times \vec{H}$ is $\vec{J}_f$, this is the new form of Ampere's law under the material. And if I want to find out this integral form it should be $\oint \vec{H} \bullet d\vec{l} = I_f$ enclosed enc that I should write. Now magnetic susceptibility, we can have another term here which is relevant which is called the magnetic susceptibility like the electric susceptibility.

So, the magnetic susceptibility that thing we so what happened? When we have the external magnetic field and place a magnetic material here, so the magnetization that is produced, so this is the external magnetic field let us write it that $\vec{H}$. So, what happened?

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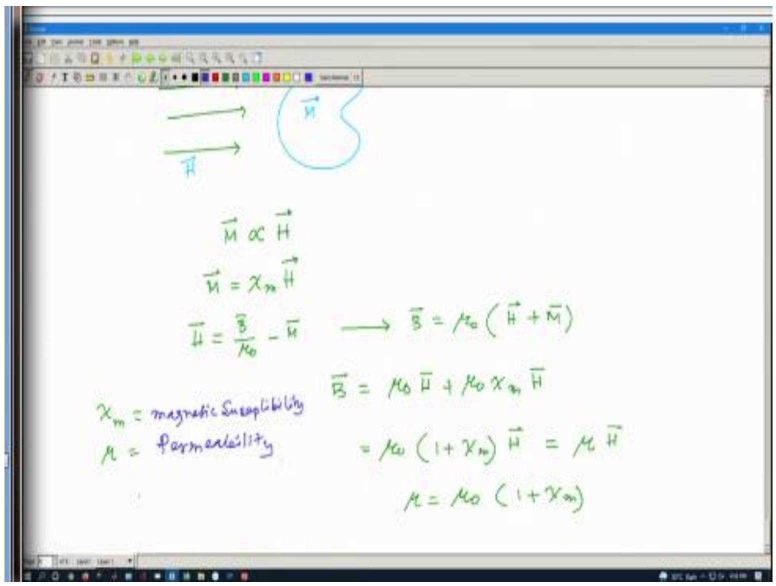

The magnetization that is produced is proportional to $\vec{H}$; the magnetization is proportional to the magnetic field $\vec{H}$. So, if I now have reduced, so that means it is some constant multiplied by $\vec{H}$, so this χ_m is called the magnetic susceptibility. Now $\vec{H}$ is again $\frac{\vec{B}}{\mu_0} - \vec{M}$ that gives me as I already wrote that $\vec{B} = \mu_0 (\vec{H} + \vec{M})$. And $\vec{M}$ is χ $\vec{H}$, so I can have a relationship with $\vec{B}$ and $\vec{H}$, so $\vec{B}$ is $\mu_0 \vec{H} + \mu_0 \chi_m \vec{H}$ or $\mu_0 (1 + \chi_m) \vec{H}$.

And we normally write it as $\mu \vec{H}$, where μ is μ_0 (1 + magnetic susceptibility), χ_m is called the magnetic susceptibility and it is permeability μ. So, today we have our time up, so I like to conclude here. So, in today's class we learn about the magnetization and then we try to understand that how the bound current and the surface current can be realized, the bound magnetic current and bound surface current density can be realized.

And then try to understand the concept of magnetic susceptibility and permeability etcetera. So, with that I like to conclude my class here, so thank you for your attention and see you in the next class.