Foundation of Classical Electronics
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Lecture-72
Tutorial 3 (Magnetostatic)

Hello student to the foundation of classical electrodynamics course. So, today we have lecture 72 and today we will be going to discuss some problem on magnetostatics.

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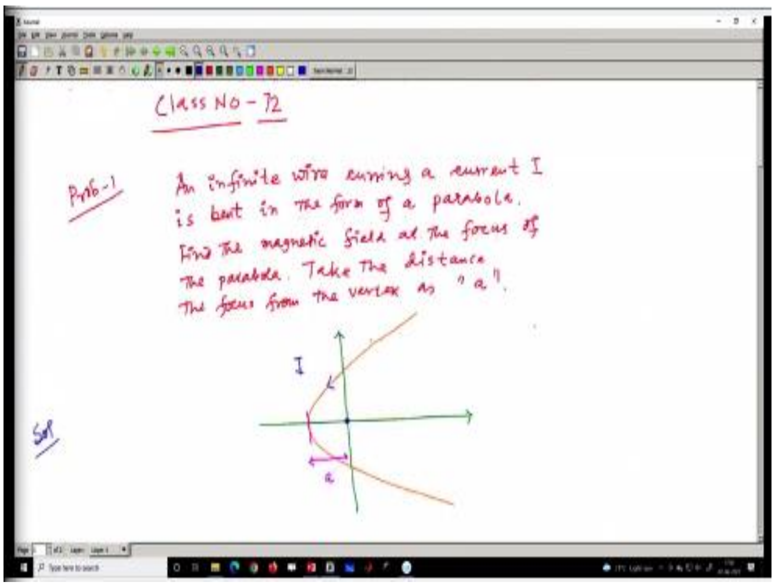

So, today we have class number 72, so let us directly go to the problem, problem 1. So, in problem 1 we have 1 problem like an infinite wire carrying a current I is bent in the form of a parabola**.** The question is find the magnetic field at the focus of the parabola. So, consider, so the another part is given the problem that take the distance of the focus from the vertex as a. So, let me draw that then it will be clear.

Solution, so we have a parabola, so let me draw first the parabola. This is my origin and my parabola if something like this. So, this is the parabola we are having and this is my focus and from vertex to focus this length from here to here this length is a, that is given. And the current is flowing through the wire I, so this is a problem where you directly use the Biot-Savart law. But the thing is that normally we have a simple geometry but here we have something which is little bit complicated we have a wire having parabola.

This kind of problem we are going to encounter in several cases where the design of the wire should be like half circle or it will be square. But the strategy will be same that you need to find out for small element and then you need to integrate over the entire length of the wire to find out what is the magnetic field for the given point.

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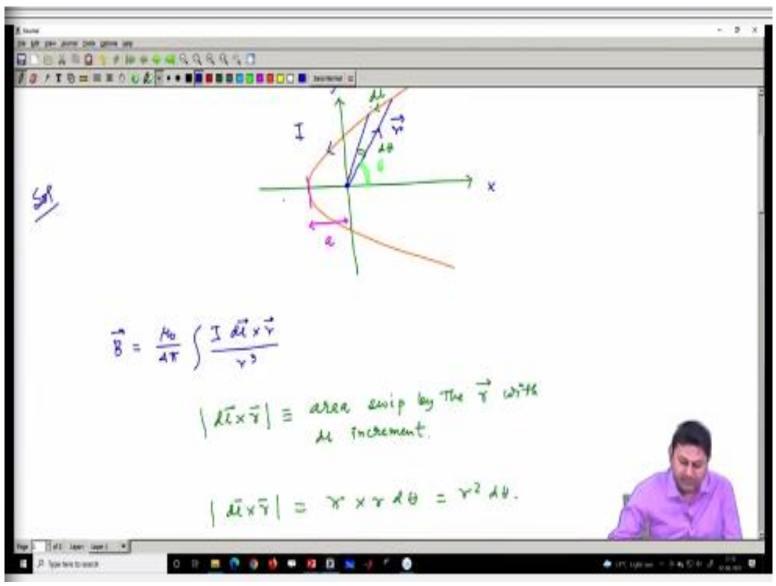

So, here let us consider take a small segment here. So, suppose this is from here to here this is my $\vec{r}$, this angle say θ and this angle is my dθ and here this length is say d$\vec{l}$, so this is the length d$\vec{l}$. So, now we know using the Biot-Savart law if I want to find out $\vec{B}$, my $\vec{B}$ should be $\frac{\mu_0}{4\pi}\int\frac{I\,d\vec{l}\times\vec{r}}{r^3}$. So, here we need to first find out what is $d\vec{l}\times\vec{r}$.

So, if you look carefully to the figure $d\vec{l}$ is this one and $\vec{r}$ is this one, so if I make $d\vec{l}\times\vec{r}$ it is nothing but amount of area that is generated here. So, **so** here $d\vec{l}\times\vec{r}$ this quantity is the area swip by the vector $\vec{r}$ with d$\vec{l}$ increment. So, if that is the case then this quantity simply is r multiplied by r dθ or it is simply r^2 dθ, from the simple geometry we can have this.

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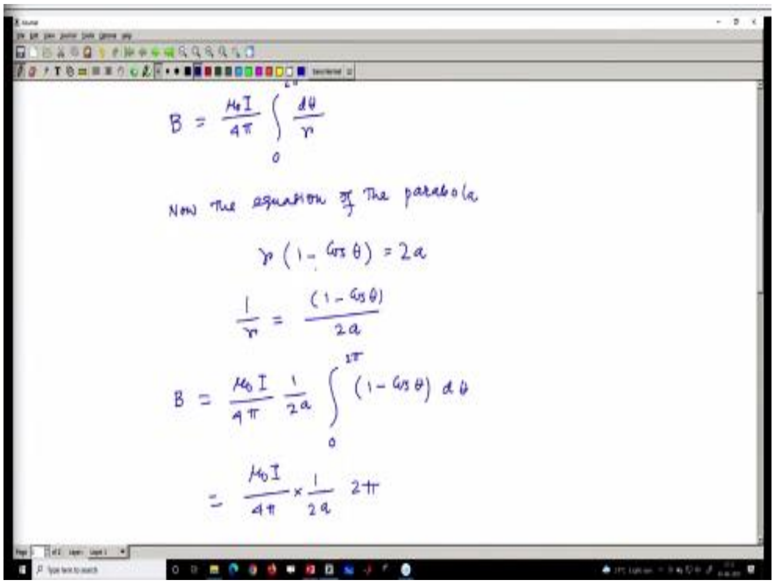

Then my $\vec{B}$ whatever the $\vec{B}$ I am having is $\frac{\mu_0 I}{4\pi}$ and 0, 2π because the length of this parabola is infinite, so my θ range from 0 to 2π and then divided by r. So, I just put $d\vec{l} \times \vec{r}$ here, I I take outside $d\vec{l} \times \vec{r}$ I put r^2 dθ and in the denominator I have r^3, so this r^3 is going to cancel out, this r^2 will be going to cancel out with r^3 and we have remaining 1 r here, so that is the simplified form.

Now what I do? I now equation I am going to use because I need to know what is the relationship between r and θ. So, now the equation of the parabola, in polar coordinate we know it is $r(1 - \cos\theta) = 2a$ if the distance between the vertex and the focus is a then that is the equation we know from the parabola. Now we are going to exploit this equation here because in this equation we know the relationship with r and θ.

So, I am going to replace, so $\frac{1}{r}$ if I replace here, so $\frac{1}{r}$ then simply $\frac{(1-\cos\theta)}{2a}$, that is the value of $\frac{1}{r}$ and that value I am going to replace here. So, this is the B; I should not put any vector sign here because I am calculating only the magnitude. So, my B here, magnitude of B rather is simply $\frac{\mu_0 I}{4\pi}$, which is already there.

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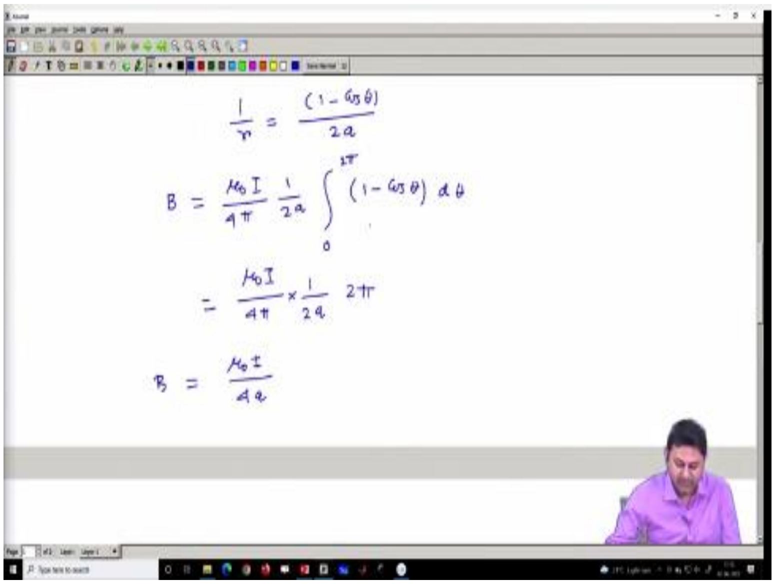

And then $\frac{1}{r}$ term I just write $\frac{1}{2a}$ that should come outside and then we have the $\int_0^{2\pi}(1-\cos\theta)\,d\theta$. So, everything in now in θ inside the integral, so I can simply write B is $\frac{\mu_0 I}{4\pi} \times \frac{1}{2a}$. Now in this integration you can see that the first integral will give me the value 2π and the second integral the cos θ if I integrate 0 to 2π it should be simply 0, so I will not be going to get any contribution of the second part.

So, that will be our result, so B will be simply $\frac{\mu_0 I}{4a}$. And the direction of the B is perpendicular to the plane of the parabola. So, because this is the way the current is coming and since this is in a plane if it is in x suppose this is in x y plane. So, if it is an x y plane then the B should be perpendicular to that and then it is going to revolve around, so it should be in the φ actually it should be the $\hat{\varphi}$.

But anyway the magnitude will be this one. So, the trick of the problem here again you need to find out the small whenever you have this kind of problem where you are encountering a wire, which is having a different given set, for this case suppose it is a parabola. It can be a rectangular shape or it can be a half circle shape, so different kind of shapes are there. So, you need to calculate for small element dl and then you need to correlate with the when you integrate because at the end of the day you need to integrate you need to correlate the parameters.

So, here I need to correlate with r and θ and I correlate with this information because this is a parabola, so we use this important relation between r and θ, this one and execute the problem. So, this is one typical problem related to the Biot-Savart law then let us go to the next kind of problem.

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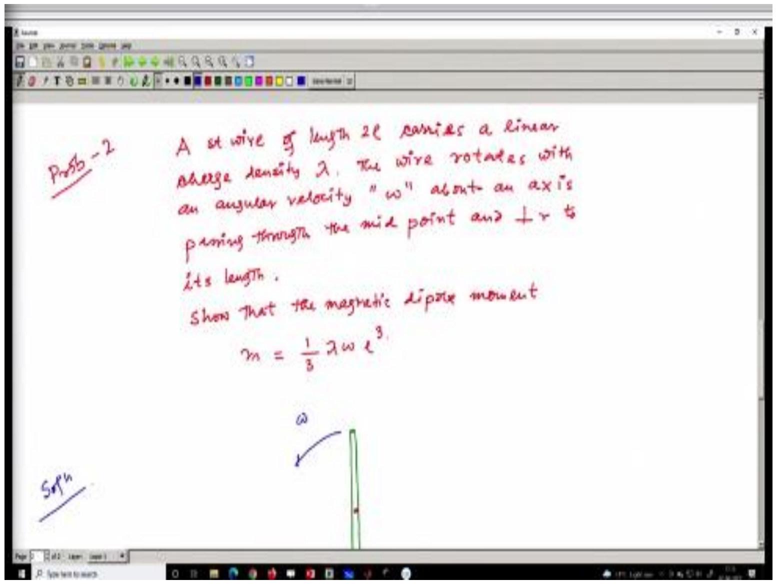

So, the next problem, problem 2 says that a straight wire of length 2l carries a linear charge density cell λ. Now the wire rotates with an angular velocity say ω about an axis passing through the midpoint and perpendicular to it is length. The problem is show that the magnetic dipole moment $m = \frac{1}{3} \lambda \omega l^3$.

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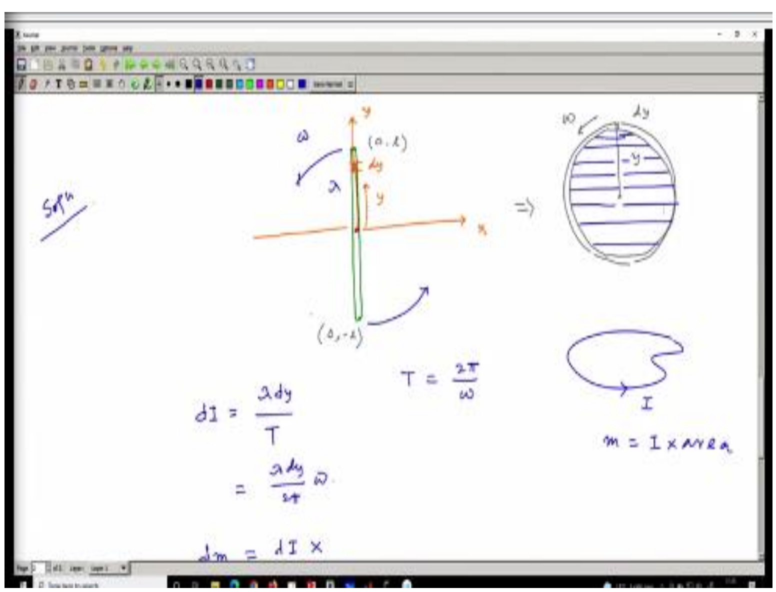

So, let me first draw the picture solution, so we are having a small wire of length 2l suppose this is my wire and this is the middle point. And now an axis perpendicular to this rod and passing through the middle point along that axis it is rotating, so if I draw the, so it is rotating this direction with an angular velocity ω. And also it is having a linear charge density λ, so λ is a linear charge density it is having.

So, what happened that if it is having a linear charge density and if it is moving that means eventually the charge is moving, so it will generate some kind of current. And we know that the magnetic dipole moment is current multiplied by the area, so it will create an area and because of that some sort of magnetic dipole movement will be going to generate, so you calculate that quantity.

So, the strategy is let us take a, so suppose I am having an axis here, this is say x axis, say this is my y axis. So, I can take from here to here a distance y and take a small section here, this section is say dy and we are going to calculate the amount of magnetic moment that is generated due to the current here in dy and then we are going to integrate. The length of the wire is 2l, so this is the coordinate here is say at this point the coordinate is 0, -l and the coordinate of this part is 0, l.

So, that is the structure we are having. So, it will be going to rotate and when it rotates it will generate suppose it is rotating like this, so it will generate a, so this small very small dl you are going to rotate. So, from here to here we have y and this length is dy and when it rotates with a frequency ω, so one current will be going to generate and we already had the area. So, magnetic moment is current into area, so we are going to get the magnetic moment and then we need to integrate over this system.

So, what is the current we are having? First let us calculate this. So, the amount of current that we are having here dI that is the amount of charge that we are having here λ is a linear charge, so it should be charge that we are having in dy divided by the period and that is T, so charge divided by time that should be the current. And T here is the period and I can write T as $\frac{2\pi}{\omega}$, ω is the angular frequency.

So, dI is essentially $\frac{\lambda dy}{2\pi}$ and then ω, that is my dI. So, the magnetic moment for this small area this small length associated to this current element, so this the small amount of current dI I can write this magnetic moment dm it should be current dI into the area. So, which area we are talking here? We are talking about this area, this is the area we are talking, here this is the area we need to calculate.

Because we know that when we have a current then current multiplied loop when you have a loop then this current loop, so magnetic moment let me remind. Suppose we have a current loop when current I is flowing then the magnetic moment is current multiplied by the area of this loop. So, here we are going to use this area, so area will be simply because we know this is y and it is moving in this circular path.

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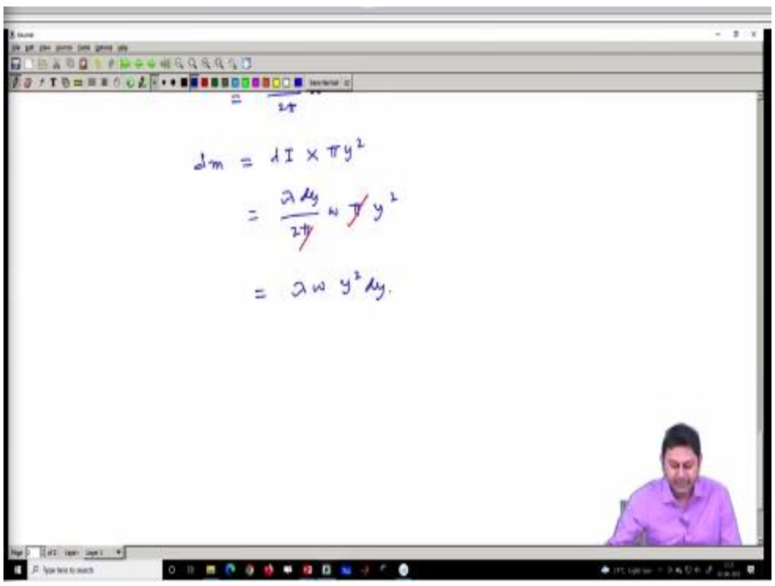

So, it should be simply πy^2, dI we already know, so dI is this $\frac{\lambda dy}{2\pi}$ and then ω then πy^2. So, this π, π will be going to cancel out, so whatever we are having is this, it will be λ multiplied by ω and then y^2 dy. So, when we calculate the total magnetic moment then I need to integrate because I am just calculating for this small.

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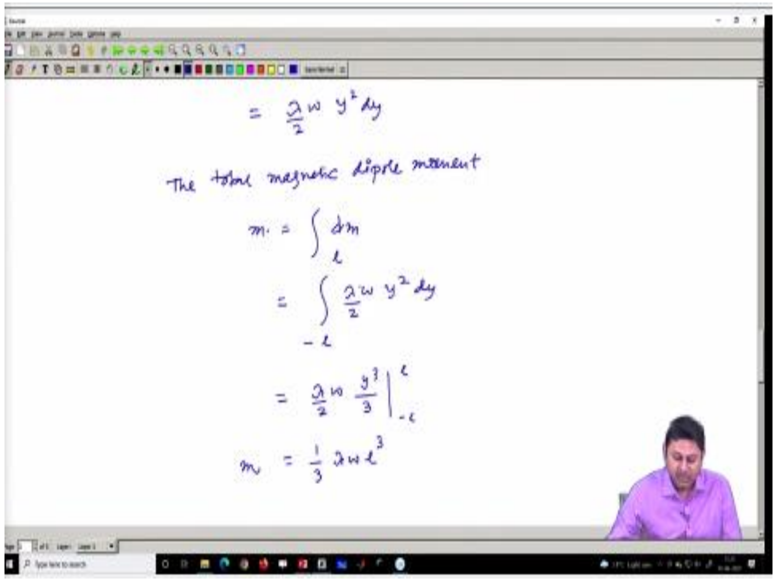

So, the total magnetic moment m, so the total magnetic dipole moment that should be m, this is the integration of dm. So, that quantity is simply $\int_{-l}^{+l}\frac{\lambda}{2}\omega y^2 dy$. So, $\frac{\lambda}{2}\omega$ it value should be y^2 simply I write $\frac{y^3}{3}$.

So, it should be $\frac{\lambda}{2}$ and that quantity we calculate for -l to l. So, if we put this -l to l it should be simply $\frac{1}{3}\lambda\omega\ l^3$, that is the amount of magnetic moment that we are going to generate and in the question that is the value that is given. So, this way you calculate the magnetic moment for a distribution of the charge and when it is rotating under certain angles, rotating in certain direction.

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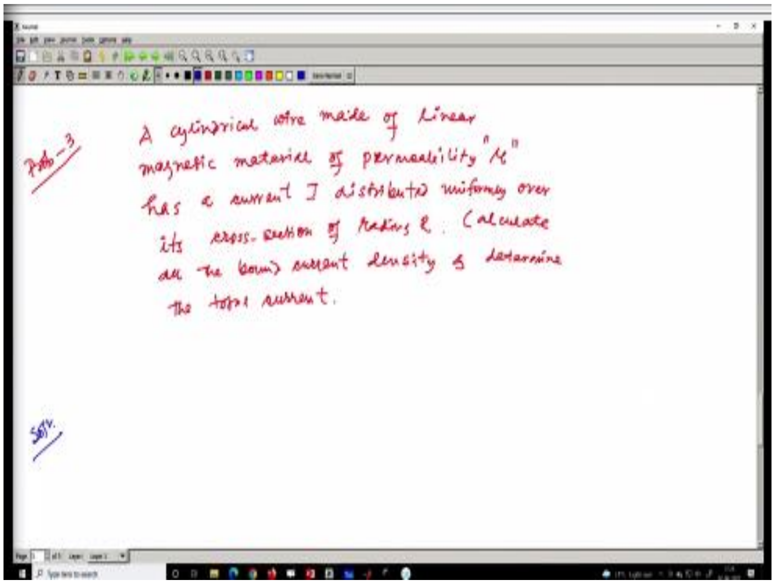

After that let me go to the next problem, the problem 3 saying that a cylindrical wire made of linear magnetic material of permeability μ has a current I distributed uniformly over its cross section of radius R. Question is calculate all the bound current density and determine the total current. So, what is the problem? Let me draw that and then I am going to understand the solution.

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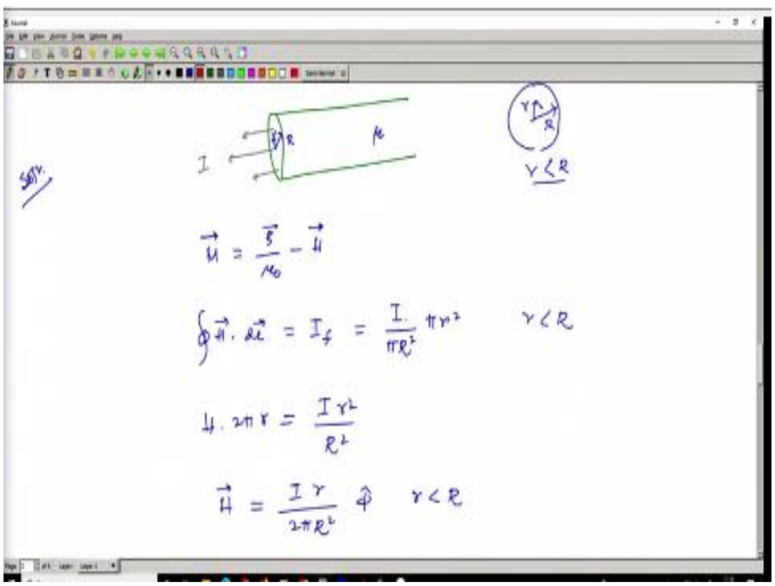

So, we have a wire made of linear magnetic material. So, here we are having a wire, suppose this is my wire and the material is magnetic material. So, we have a μ here and the current we are having here is uniformly distributed, the current I is flowing. Now if a current is flowing then we should have a magnetic field here inside, so $\vec{B}$ will going to generate and that give rise to magnetization and from that you can calculate the current density, so let us do that first.

So, we have the expression of the magnetization is $\frac{\vec{B}}{\mu_0}$ - $\vec{H}$ that we know, this is my magnetization. So, first we need to find out what is $\vec{H}$ because the current is flowing. So, we are going to exploit this expression close line integral, so this step $\oint \vec{H} \bullet d\vec{l}$ that is the free current that is flowing here. And $\vec{H} \bullet d\vec{l}$, so if I have Amperian so current here what is the current that is flowing. This is I and then I should have πR^2 is the total current, so this is the current per unit area and the current that is enclosing is this one when r is less than R.

So, here that means from here to here it is R, so I calculate the current somewhere which is here and so if I make this figure that will be better. So, this is my R and I find at some r, which is less than this one inside this wire. So, I simply have H $\times$ 2πr and in the right-hand side we have I $\frac{r^2}{R^2}$. So, $\vec{H}$ simply comes out to be $\frac{Ir}{2\pi R^2}$ and if I want to find out the direction it should be $\hat{\varphi}$ for r less than R.

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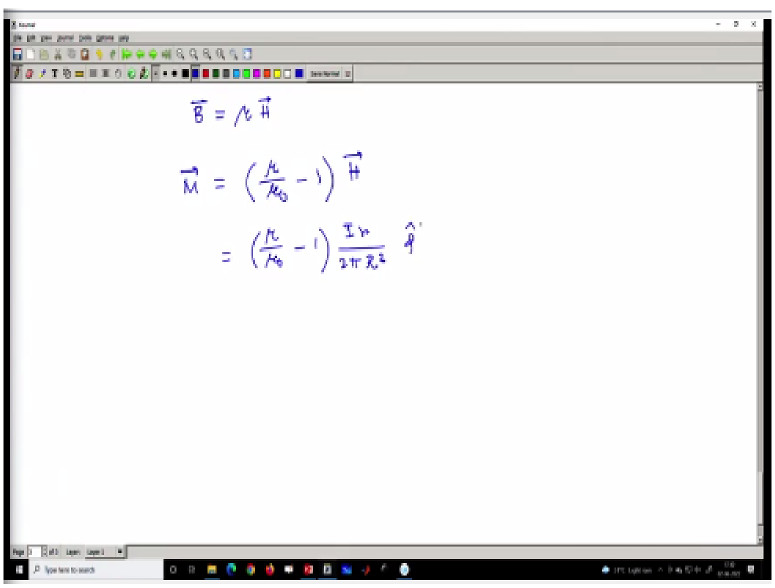

So, again $\vec{B}$ I can also write it is $\mu\vec{H}$ because the permeability of this material is μ. So, then my magnetization simply comes out to be if I replace this $\vec{B}$, so $(\frac{\mu}{\mu_0} - 1)$ and this $\vec{H}$. So, this $\vec{H}$ we generate, so it is simply $(\frac{\mu}{\mu_0} - 1)$ and H is $\frac{Ir}{2\pi R^2}$, which is a function of r with a direction say $\hat{\varphi}$ direction.

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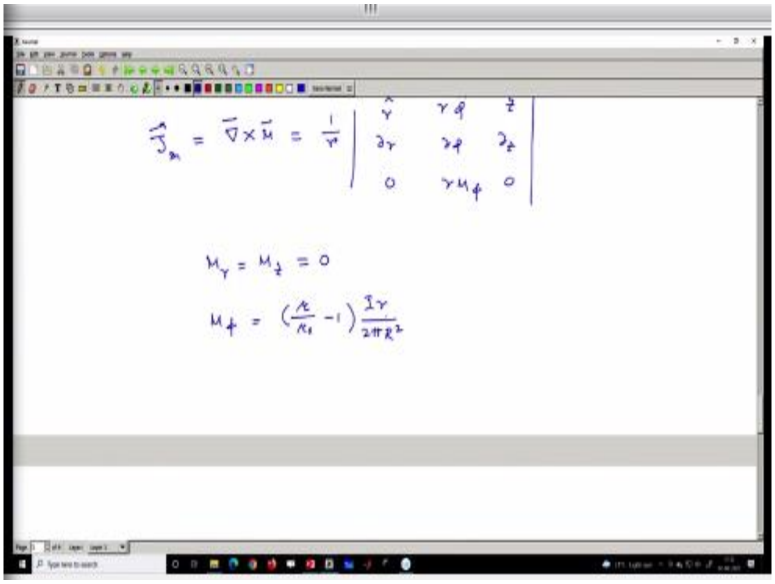

Now we know what is the magnetic current density $\vec{J}_m$ and $\vec{J}_m$ is replaced by that $\vec{\nabla} \times \vec{M}$. So, if I calculate here the $\vec{\nabla} \times \vec{M}$ because $\vec{M}$ is there, so I need to use the cylindrical coordinate here of the curl then I have $\hat{r}$ r$\hat{\varphi}$ and $\hat{z}$ then the operator ∂_r and then ∂_φ and then ∂_z. And the component of the M 0 rM_φ because M_φ component is only there and 0. And you note that $M_r = M_z = 0$ and the φ component that we calculate it is $(\frac{\mu}{\mu_0} - 1)$ and then $\frac{Ir}{2\pi R^2}$.

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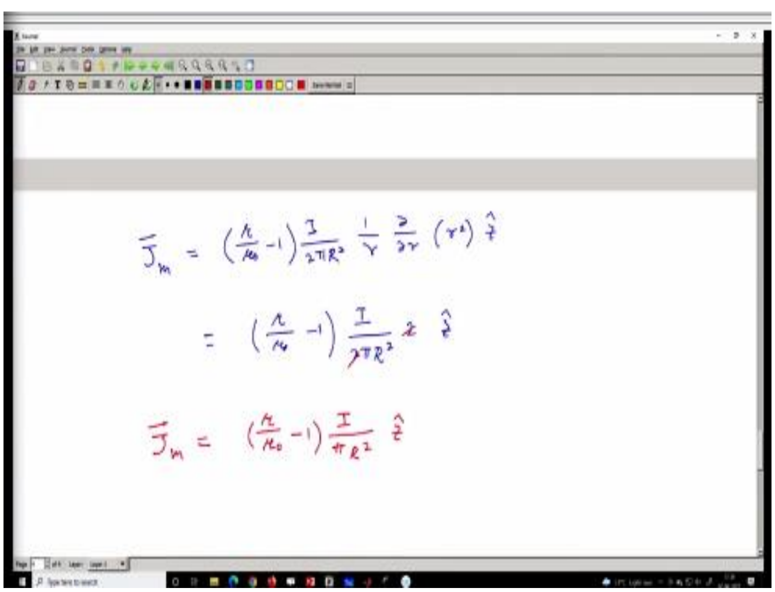

So, my $\vec{J}_m$ then simply becomes $(\frac{\mu}{\mu_0} - 1)\frac{I}{2\pi R^2}$, these are constants and it will be operated by this way $\frac{\partial}{\partial r}$ and then I simply have $\frac{\partial}{\partial r}$ this operator will be going to operate one r is there. So, it should be r^2 and then $\hat{z}$ direction. So, that quantity is simply $(\frac{\mu}{\mu_0} - 1)\frac{I}{2\pi R^2}$ and if you calculate it should be 2R and this R will calculate will be there.

So, it will be 2 and $\hat{z}$, this 2 will be going to cancel out here and here. So, finally we have the value of my $\vec{J}_m$, this is the magnetic and that value is $(\frac{\mu}{\mu_0} - 1)\frac{I}{\pi R^2}\hat{z}$. So, that is the value of the magnetic bound current density due to the magnetization. And why the magnetization is there? Because the current is flowing and that is why the magnetic field is going to generate and that magnetic field this since the material is magnetic material, so we have a magnetization.

And this magnetization is now function of r, so that is why curl of this magnetization is not equal to 0, so that means we should have a non-vanishing curl and that is equivalent to the value of this $\vec{J}$, which is the magnetic bound current. And what about the bound surface current? And that we also know.

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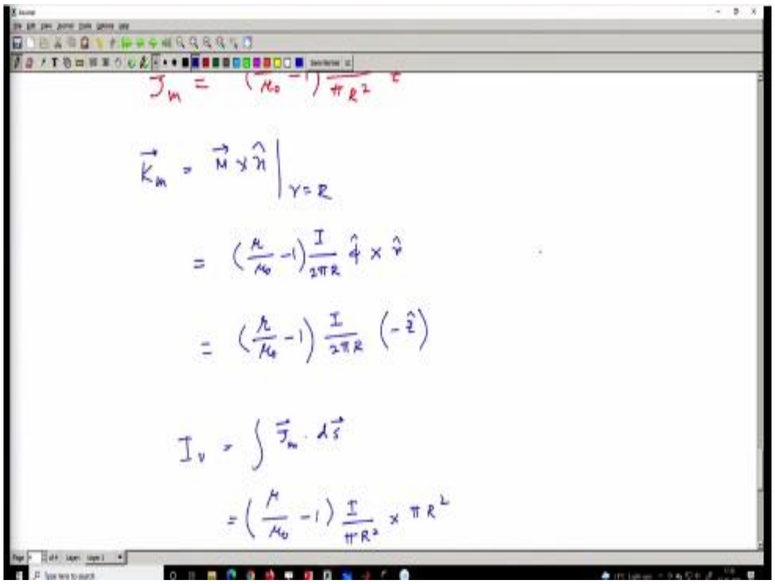

We represent is $\vec{K}_m$ and that is $\vec{M} \times \hat{n}$ at the surface and if you do that because this value we know $(\frac{\mu}{\mu_0} - 1)\frac{I}{2\pi R}\,\hat{\varphi} \times \hat{n}$ here = $\hat{r}$. So, that value $\hat{\varphi} \times \hat{r}$ gives me something like $(\frac{\mu}{\mu_0} - 1)\frac{I}{2\pi R}$ and this $\hat{\varphi} \times \hat{r}$ is essentially $-\hat{z}$. So, that value is my bound magnetic surface current.

Now if I want to find out the total volume current I_v, I need to integrate because I need to calculate the 2 term current now $\vec{J}_m$ over this surface and that is simply $(\frac{\mu}{\mu_0} - 1)$ that value is there. And we have $\frac{I}{\pi R^2}$ multiplied by πR^2, whatever the $\vec{J}_m$ value I had here is $\frac{I}{\pi R^2}$, so and then the total area is πR^2.

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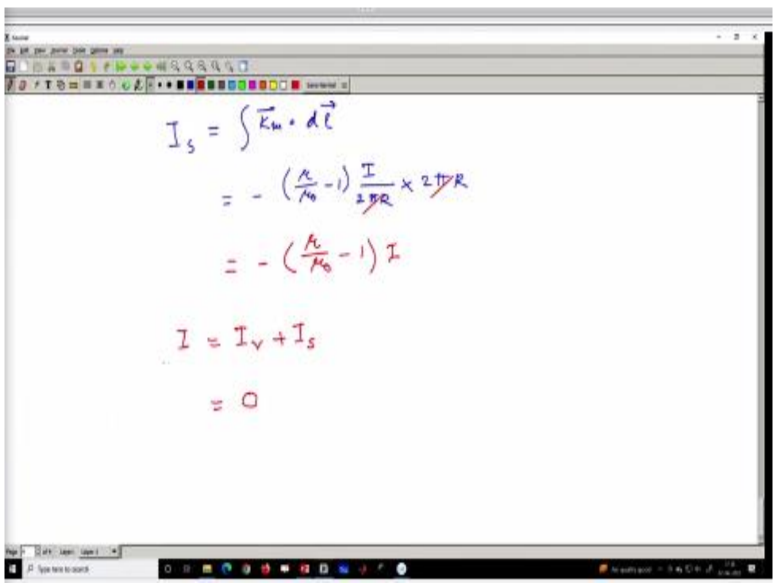

So, these things will simply be $(\frac{\mu}{\mu_0} - 1)$ I this is not the current density but the total volume magnetic current, so I simply write it as I_v. In a similar way what is the total surface current I should have I_s is simply $\vec{K}_m \cdot d\vec{l}$ and that is $-(\frac{\mu}{\mu_0} - 1)$ and then $\frac{I}{2\pi R}$ multiplied by $2\pi R$. Because this came whatever I get is $\frac{I}{2\pi R}$ here and with a negative sign and the total circle is $2\pi R$ over this periphery.

So, this will be going to cancel out, so I have here something is $-(\frac{\mu}{\mu_0} - 1)$ I. Now if I calculate the total current I it should be $I_v + I_s$, this is the current due to the magnetization and if you see it this value is 0, which should be. Because this is a bound magnetization surface current, this is a bound magnetization volume current. So, if I add these 2 together then I am going to get a 0 result. So, ok, we do not have much time today to discuss more problem.

So, in another class to do few other problems. So, with that note I am going to conclude, so hopefully this typical problem will help you to understand the subject in a deeper way. So, please try to do the exercise for taking the problem from different books, these are few typical problems I have given. Next class I will be also going to continue, we will do maybe some couple of problems related to electromagnetic wave.

And then try to understand how or related to thing etcetera. So, with that note let us conclude, thank you very much for your attention and see you in the next class.