Foundation of Classical Electronics
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Lecture-73
Tutorial 3 (Magnetostatic and EM Wave)

Hello student, so for the fundamental of classical electrodynamics course. So, today we have lecture 73 and we will be going to continue the tutorial where we discuss the problem on magnetostatic and electromagnetic wave.

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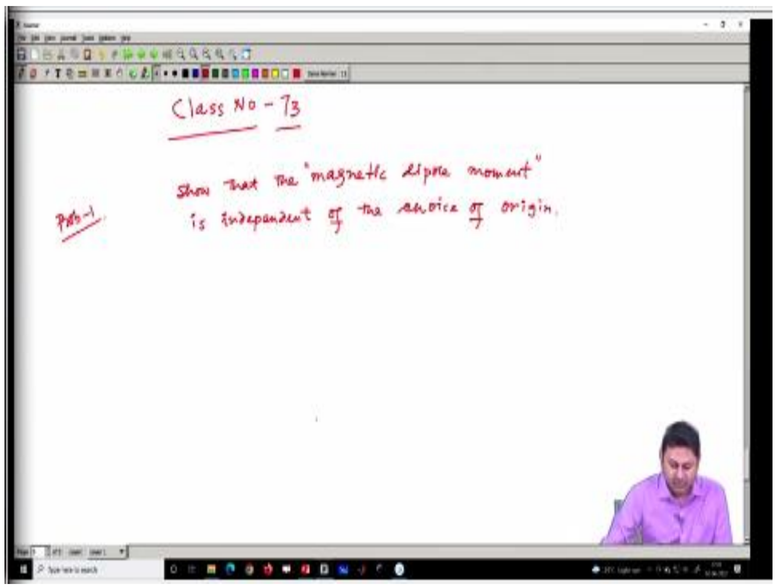

So, this is class number 73, so today we have the first problem, which is interesting. And the first problem is show that the magnetic dipole moment is independent of the choice of origin.

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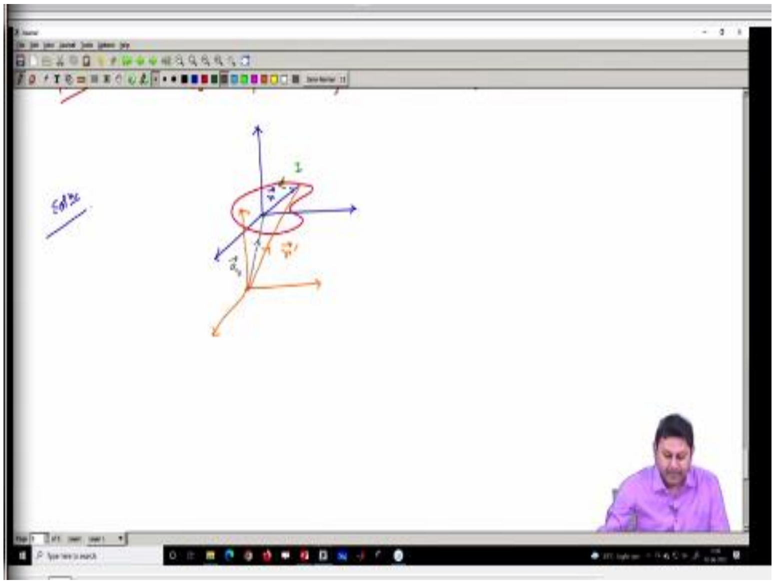

So, solution, so we know suppose this is my coordinate system and if I have a loop like this and where the current is flowing then I can have a magnetic moment current into area. Now what I do that I will shift this coordinate system, so here we have origin, so I shift this coordinate system and if I shift this coordinate system then the question is the magnetic moment will be the same value? And if the problem means saying that show that the magnetic moment is independent of the choice of the origin.

That means if I shift my origin to some other place then still the value of the magnetic moment will be same. So, if I shift, so let me shift this magnetic coordinate system suppose this is my new coordinate system, shifted one. So, if I have a point with the corresponding to the old magnetic system, so this is my $\vec{r}$ and for shifted this is my say $\vec{r}\,'$, this point, this is $\vec{r}\,'$ and this is $\vec{r}$. And I shift the coordinate system from here to here and suppose this vector is a constant vector say, $\vec{a}_0$.

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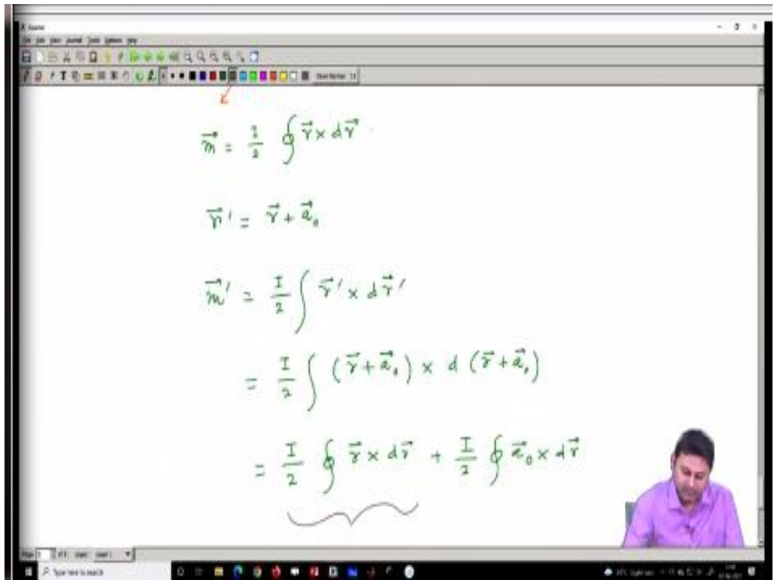

So, according to the definition the magnetic moment $\vec{m}$ is $\frac{I}{2}$ and then $\vec{r} \times d\vec{r}$. Now the origin is shifted to some other point $\vec{a}_0$. And I can now from the new origin I have $\vec{r}\,'$, which is equal to $\vec{r} + \vec{a}_0$. For new origin my magnetic moment is prime and the current is same $\frac{I}{2}$ but here I should write it is $\vec{r}' \times d\vec{r}'$ rather. Now I have a relationship with $\vec{r}$ and $\vec{r}'$, so let us write it this is $\frac{1}{2}$ integral $\vec{r}'$ is $\vec{r} + \vec{a}_0$ and it is cross and it is $d(\vec{r} + \vec{a}_0)$, $\vec{a}_0$ is constant, so $d\vec{a}_0$ will be 0.

So, eventually we will be going to have $\frac{I}{2} \oint \vec{r} \times d\vec{r} + \frac{I}{2} \oint \vec{a}_0 \times d\vec{r}$. Now this quantity is simply my old value $\vec{m}$ for old coordinate system. In old coordinate system that is the value we define this one. Now what is the rest part?

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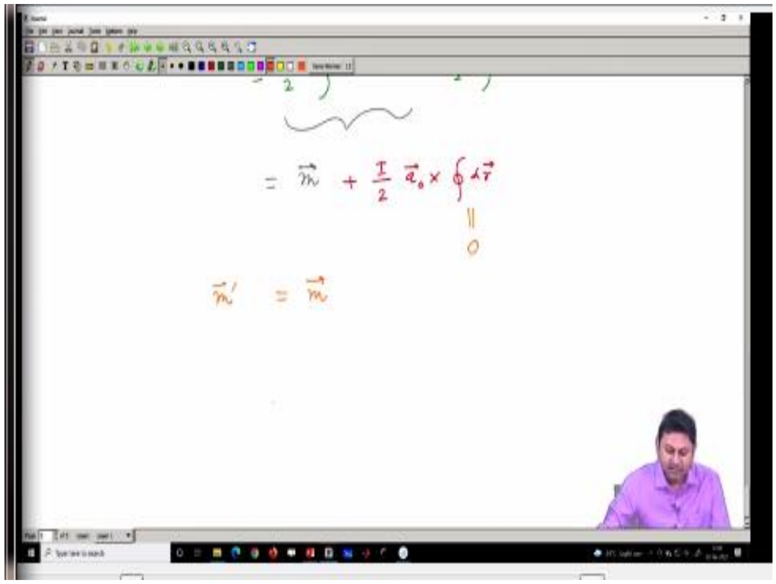

The rest part is interesting because if I add the additional part is this and then we have $\vec{a}_0 \times \oint d\vec{r}$. And now $\oint d\vec{r}$ is 0, so we simply have $\vec{m}' = \vec{m}$ that suggests that if I change my origin from here to some other point and then measure the magnetic moment for this given loop then the magnetic moment should not change, so it is independent of the choice of the origin.

So, whatever the origin you want the magnetic moment will remain same. This is interesting problem I thought that I should discuss this problem here in the tutorial. Let us now move to the next problem.

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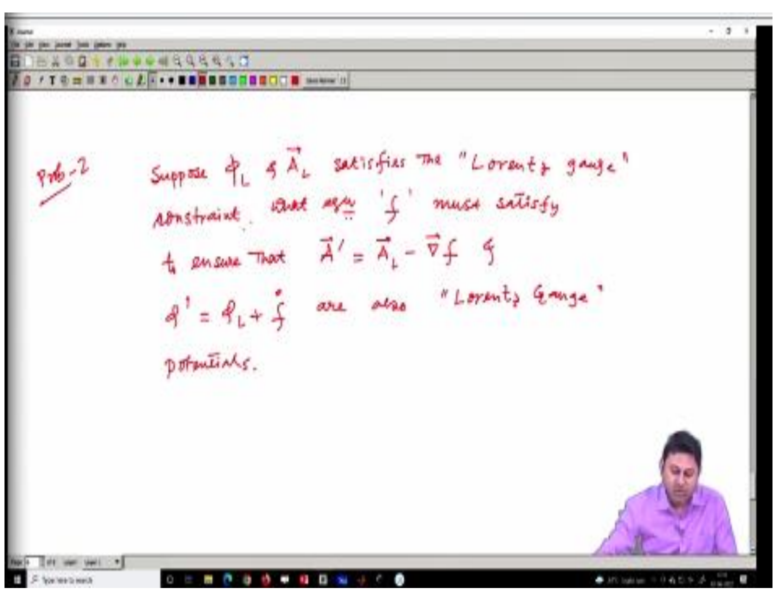

So, that was problem 1. So, I will do the next problem, problem 2 and the problem is saying that suppose ϕ_L and $\vec{r}_L$ satisfies the Lorentz gauge constant. Now the question is what equation 'f' must satisfy to ensure that $\vec{A}' = \vec{A}_L - \vec{\nabla} f$ and $\phi' = \phi_L$ + time derivative of f also Lorentz gauge potentials. So, what is the meaning of this problem? That ϕ_L and $\vec{A}_L$ is a given scalar potential and vector potential that is satisfying the Lorentz gauge constant.

Now I make we know that $\vec{A}$ and ϕ are not unique, so I can make another $\vec{A}$ the vector potential $\vec{A}'$ and another ϕ, ϕ' in this way $\vec{A}_L$ - ϕ an arbitrary scalar function f and demand that $\vec{A}'$ and ϕ' both are satisfying the Lorentz gauge condition, ϕ_L $\vec{A}_L$ also satisfying the Lorentz gauge, $\vec{A}'$ ϕ' also satisfying the Lorentz gauge condition. And if that is the case we need to find out that f, what is f and what equation f must follow?

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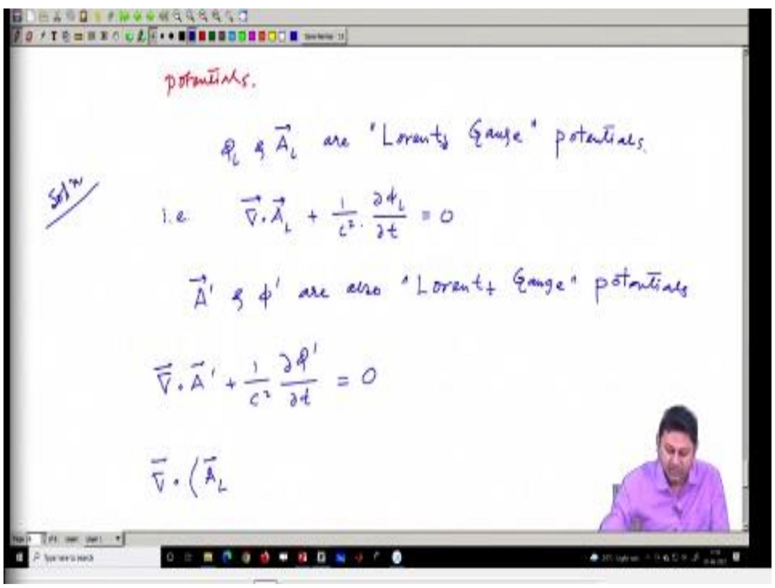

So, let us do the problem then it will be easy to, it is not a very complicated problem, it is a very simple problem as soon as you put the things then you will understand. So, $\vec{A}_L$, so ϕ_L and $\vec{A}_L$ are Lorentz gauge potential, it is already given. That means they will follow this equation, we know for Lorentz gauge if ϕ_L and $\vec{A}_L$ are Lorentz gauge potential, so okay $\vec{\nabla} \cdot \vec{A}_L + \frac{1}{c^2}\frac{\partial \phi_L}{\partial t} = 0$.

$\vec{A}_L$ and ϕ_L both are Lorentz gauge potential, so they must satisfy this equation. Now we are saying that $\vec{A}'$ and ϕ' are also Lorentz gauge potential. That means they should satisfy the same equation, it demands that they will satisfy the same equation. Now if they satisfy the same equation I know what is. So, let me now write my $\vec{A}$ in terms of $\vec{A}_L$, $\vec{A}$' in terms of $\vec{A}_L$ what is given in the equation given the problem that $\vec{A}$' should be $\vec{A}_L - \vec{\nabla}f$.

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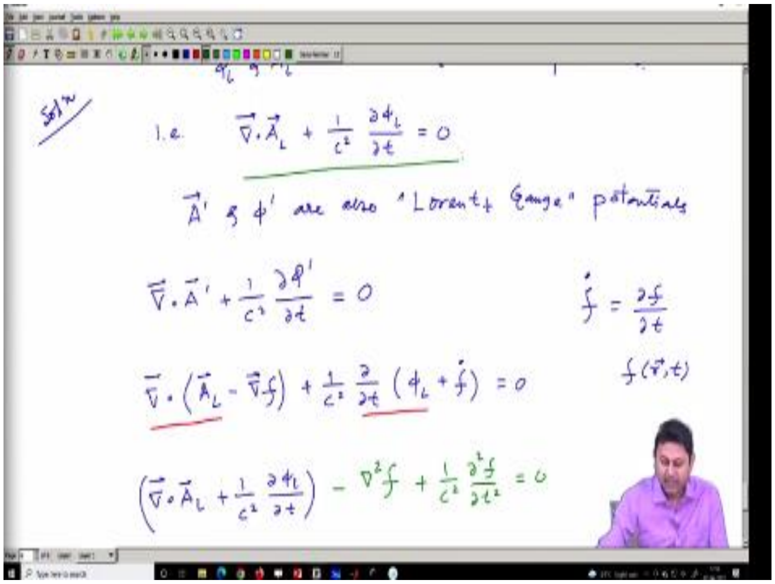

And then + $\frac{1}{c^2}\frac{\partial}{\partial t}$ and ϕ' is $\phi_L + \dot{f}$ that is equal to 0, mind it $\dot{f}$ is $\frac{\partial f}{\partial t}$, f is a function of $\vec{r}$ and t, space and time. So, now we have separate out this portion, this is one part I take this from here, the first term from here rest of the term is $-\nabla^2 f + \frac{1}{c^2}\frac{\partial^2 f}{\partial t^2}$ and that is 0. Now we already know that $\vec{A}_L$ and ϕ_L are satisfying the Lorentz gauge condition. So, this equation is already there and if I put this equation and also this equation is here, so this equation is simply 0.

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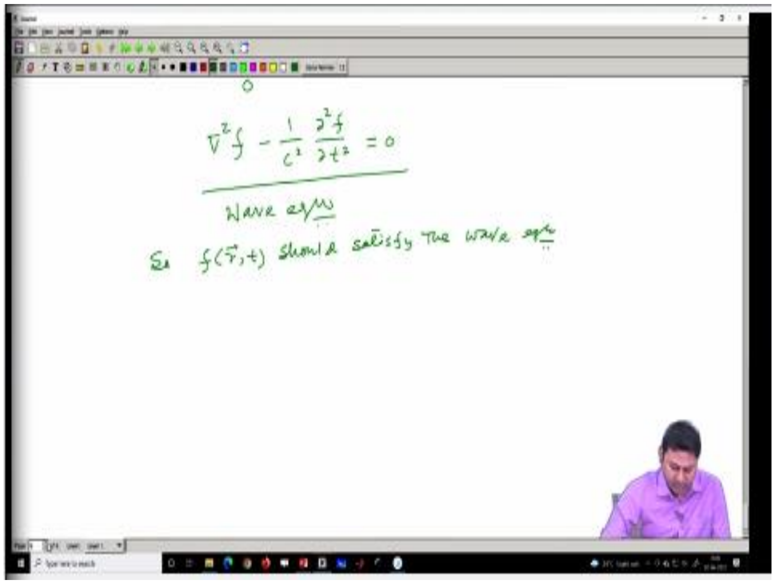

So, the equation the f is simply satisfying is this one, $\nabla^2 f - \frac{1}{c^2}\frac{\partial^2 f}{\partial t^2}$ is equal to 0, this is the wave equation. So, the answer is this f must satisfy the wave equation to ensure that $\vec{A}'$, which is $\vec{A}_L - \vec{\nabla}f$ and ϕ' is ϕ_L + the time derivative of f these 2 will be again going to satisfy the Lorentz gauge condition. So, in order to satisfy the Lorentz gauge condition the arbitrary function should follow or should satisfy the wave equation, so that is the thing. So, f, which is a function of $\vec{r}$ and t should satisfy the wave equation. Now we are going to the final problem, this is the final problem we are having today.

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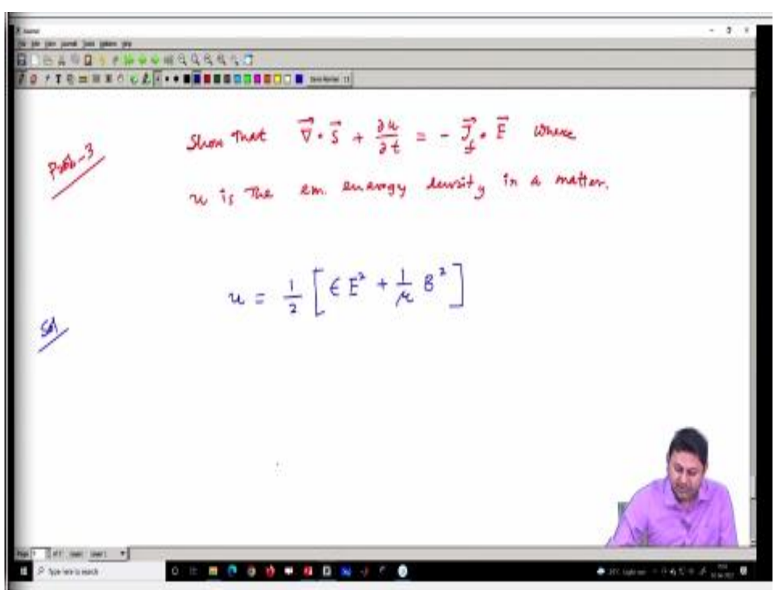

Problem 3, and the problem is show that divergence of the pointing vector $\vec{S}$ + $\frac{\partial u}{\partial t}$ = $-\vec{J}_f \bullet \vec{E}$, where u is the electromagnetic energy density inside a matter. So, we need to just put the value of $\vec{S}$ and u and check that if I calculate we will be going to get this exploiting the Maxwell's equation that is all. So, this is also very straight forward problem. So, let us solve this quickly, solution. So, u is an energy density, so we know that the electromagnetic energy density is $\frac{1}{2}[\epsilon E^2 + \frac{1}{\mu} B^2]$ inside the matter I use ϵ and μ if it is a free medium then I should write ϵ_0 and μ_0. Now what is the time derivative? Because in the equation we have a time derivative of this quantity, so let us first calculate that.

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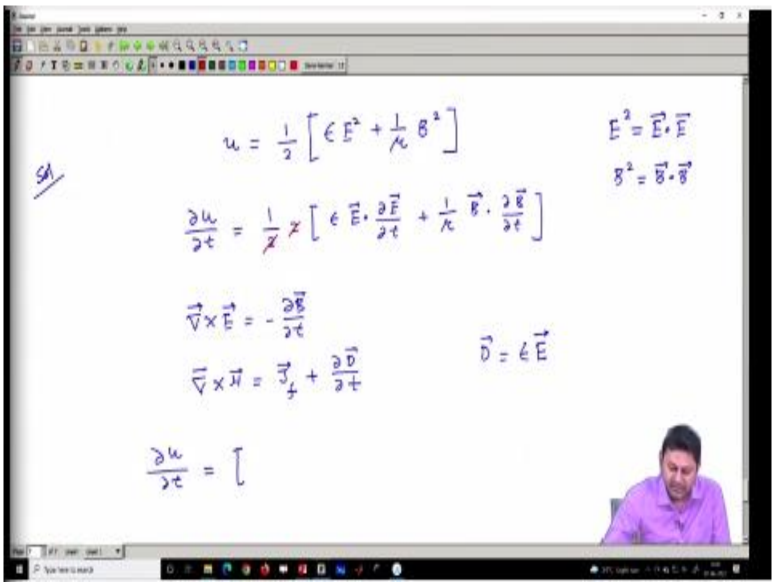

So, the time derivative for u is simply $\frac{1}{2}$ and then 2 will come out and we have ϵ and then $\vec{E} \bullet \frac{\partial \vec{E}}{\partial t}$ + $\frac{1}{\mu} \vec{B} \bullet \frac{\partial \vec{B}}{\partial t}$. So, note that E^2 is $\vec{E} \bullet \vec{E}$ and B^2 is $\vec{B} \bullet \vec{B}$ and we write this. Now inside the matter what we had is $\vec{\nabla} \times \vec{E}$, this is $-\frac{\partial v}{\partial t}$ Faraday's law and $\vec{\nabla} \times \vec{H}$ is $\vec{J}_f$ that is the free current density + $\frac{\partial \vec{D}}{\partial t}$.

So, I can replace these 2 term here, so $\vec{D}$ again in the material what we have is $\vec{E}$, so ϵ multiplied by $\vec{E}$ so $\vec{D} = \epsilon\, \vec{E}$. So, $\frac{\partial u}{\partial t}$ is simply because this $\frac{1}{2}$ and 2 is going to cancel out here, so we are not going to get anything. And then $\vec{E} \bullet \epsilon$ multiplied by $\frac{\partial \vec{E}}{\partial t}$, which is $\frac{\partial \vec{D}}{\partial t}$.

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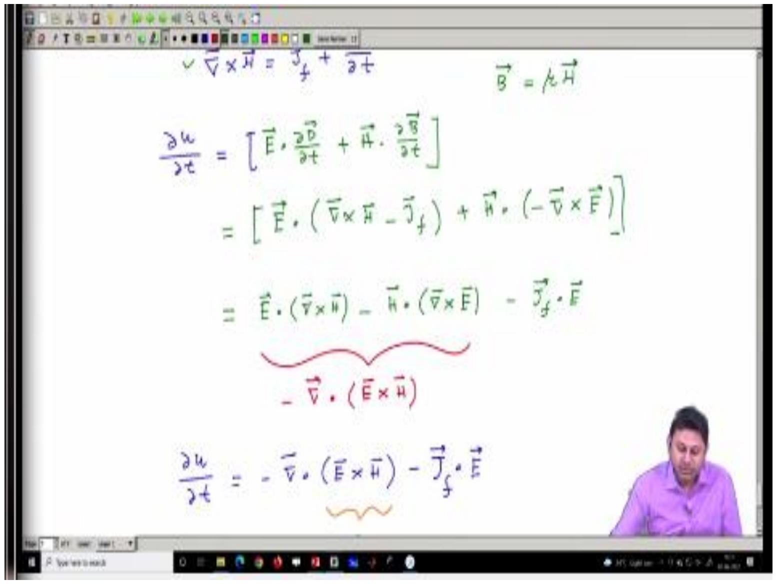

So, I have here $\vec{E} \bullet \frac{\partial \vec{D}}{\partial t}$ plus here we have $\vec{B}$, $\frac{\vec{B}}{\mu}$, so that is $\vec{H}$ because $\vec{B}$ is μ $\vec{H}$, so $\frac{\vec{B}}{\mu}$ is simply $\vec{H}$, so I have $\vec{H} \bullet \frac{\partial \vec{B}}{\partial t}$. Now I will be going to exploit this equation Maxwell's equation this 2 Maxwell's equation here and here what I write. So, these now I write that it is simply $\vec{E} \bullet \frac{\partial \vec{D}}{\partial t}$ I simply write it as $(\vec{\nabla} \times \vec{H} - \vec{J}_f) + \vec{H} \bullet \frac{\partial \vec{B}}{\partial t}$, $\frac{\partial \vec{B}}{\partial t}$ is $-\vec{\nabla} \times \vec{E}$.

So, this thing is simply $\vec{E} \bullet (\vec{\nabla} \times \vec{H})$ - $\vec{H} \bullet (\vec{\nabla} \times \vec{E})$ and then we have - $\vec{J}_f \bullet \vec{E}$. Now this quantity is simply I can write it $-\vec{\nabla} \bullet (\vec{E} \times \vec{H})$ and then in the left-hand side I have $\frac{\partial u}{\partial t} = -\vec{\nabla} \bullet (\vec{E} \times \vec{H})$ and - $\vec{J}_f \bullet \vec{E}$.

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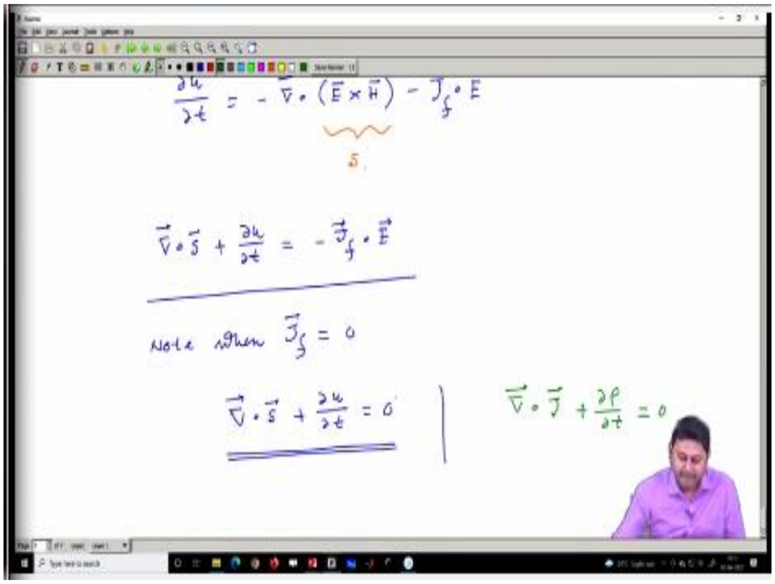

Now $\vec{E} \times \vec{H}$ again is my pointing vector, this is $\vec{S}$. So, I simply have $\vec{\nabla} \bullet \vec{S} + \frac{\partial u}{\partial t} = - \vec{J}_f \bullet \vec{E}$, so that is the expression we wanted to prove. Note that, when there is no free current $\vec{J}_f = 0$ we simply have an expression that the divergence of the pointing vector + the rate of change of energy density = 0, it looks very similar to the continuity equation. The continuity equation if I write side by side.

So, the continuity equation was $\vec{\nabla} \bullet \vec{J} + \frac{\partial \rho}{\partial t} = 0$ that was our continuity equation, it looks very similar to that continuity equation under the condition that there is no free current. Anyway, we need to prove this part including the free current, so we prove it. And in order to prove we just exploit the Maxwell's equation in the matter and we get the results. Well, that is the final problem we wanted to discuss.

So, hope these problems and last couple of classes whatever the problem we had, hope this problem will be useful for you and you will practice more and more problem. And since this is a last course, I believe that you thoroughly enjoyed this course. If you have any doubt and any confusion my email address will be given, so you can contact me directly. So, I will be more than happy to interact with you or if you have an issue in understanding maybe I can fix a meeting with you and discuss face to face. With that note I like to conclude my class here, thank you very much for your attention and best of luck.