**B.Sc. DEGREE EXAMINATION, APRIL 2016.**

**III YEAR — V SEMESTER**

**Major Paper IX — MODERN ALGEBRA**

**Time : 3 hours Max. Marks : 75**

**SECTION A — (10 × 2 = 20 marks)**

**Answer any *TEN* questions.**

1. Define the right coset in *G.*
2. Define Normal Subgroup.
3. Define  with usual notation.
4. State Cayley’s theorem.
5. Define a field.
6. State the Pigeon Hole Principle.
7. Define an ideal.
8. When is an ideal said to be maximal ideal?
9. Define Euclidean ring.
10. Define principal ideal ring.
11. State Lagrange’s theorem.
12. Define an abelian group.

**SECTION B — (5 × 5 = 25 marks)**

**Answer any *FIVE* questions.**

1. If *G* is a group, then prove the following
2. The identity element of *G* is unique.
3. For every 
4. For all .
5. If  is a homomorphism of *G* into with kernel *K,* then show that *K* is normal sub group of *G.*
6. Give an example of a ring which is not a field.
7. If *R* is a commutative ring with unit element and *M*  is an ideal of *R* then prove that *M* is a maximal ideal of *R* if and only if *R/M*  is a field.
8. Show that a Euclidean ring possesses a unit element.
9. Prove that *HK* is a subgroup of *G* if and only if *HK=KH.*
10. Show that every permutation is the product of its cycles.

**SECTION C — (3 × 10 = 30 marks)**

**Answer any *THREE* questions.**

1. Prove that the subgroup *N* of *G* is a normal subgroup of *G* if and only if every left coset of *N*  in *G* is a right coset of *N* in *G*.
2. If *G* is a group, then show that *A(G),* the set of automorphisms of *G* is also a group.

[P.T.O.]

1. If *R* is a ring, then for all , prove the following
2. *a 0 = 0a = 0.*
3. *a(-b) = (-a)b = -(ab)*
4. *(-a)(-b) = ab*
5. *(-1) a = -a* where *R* has *a* unit element 1
6. *(-1)(-1) = 1.*

1. If *R* is a commutative ring with unit element whose only ideals are *(0)* and *R* itself then show that *R* is a field.
2. State and prove Unique Factorization theorem.

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