**B.Sc. DEGREE EXAMINATION, APRIL 2016.**

**III YEAR — V SEMESTER**

**Major Paper XII — GRAPH THEORY**

**Time : 3 hours Max. Marks : 75**

**SECTION A — (10 × 2 = 20 marks)**

**Answer any *TEN* questions**

1. Define spanning subgraph of a graph and give an example on 5 vertices.
2. Draw all the possible non-isomorphic graphs with 4 vertices.
3. Define Eulerian graph.
4. When do we say that a graph has a Hamiltonian cycle?
5. Define complete bipartite graph.
6. Define independent set of edges.
7. State Euler formula for planar graph.
8. Define sub-division of *e.*
9. Define chromatic number of a graph.
10. Define edge coloring.
11. Prove that if is a then
12. Define (a) acyclic graph (b) tree.

**SECTION B — (5 × 5 = 25 marks)**

**Answer any *FIVE* questions**

1. Prove that every with contains a cycle.
2. If is a Hamiltonian graph, then show that , for any non-empty subset of .
3. Prove that a graph is a tree if and only if every two vertices of are connected by a unique path.
4. State and prove Euler formula for planar graphs.
5. Show that, for any graph .
6. Show that, a vertex in a connected graph is a cut vertex if and only if there exists vertices such that every path connecting .
7. For a show that the following statements are equivalent

(a) is a tree.  
(b) is connected and . (c) is acyclic and .

**SECTION C — (3 × 10 = 30 marks)**

**Answer any *THREE* questions**

1. Prove that a connected contains a cycle if and only if .
2. Prove that a non-trivial connected graph is Eulerian if and only if it has no vertex of odd degree.
3. Prove that a is a bipartite graph if and only if it contains an odd cycle.
4. (a) Define a dual of a graph.  
   (b). Prove that a graph is planar if and only if it contains no contraction of

or .

1. If is a bipartite graph with then show that .

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