**B.Sc. DEGREE EXAMINATION, APRIL 2016.**

**III YEAR — V SEMESTER**

**Major Paper XII — GRAPH THEORY**

**Time : 3 hours Max. Marks : 75**

**SECTION A — (10 × 2 = 20 marks)**

**Answer any *TEN* questions**

1. Define spanning subgraph of a graph and give an example on 5 vertices.
2. Draw all the possible non-isomorphic graphs with 4 vertices.
3. Define Eulerian graph.
4. When do we say that a graph has a Hamiltonian cycle?
5. Define complete bipartite graph.
6. Define independent set of edges.
7. State Euler formula for planar graph.
8. Define sub-division of *e.*
9. Define chromatic number of a graph.
10. Define edge coloring.
11. Prove that if $G$ is a $\left(p, q\right)-graph$ then $\sum\_{v\in V(G)}^{}deg(v)=2q$
12. Define (a) acyclic graph (b) tree.

**SECTION B — (5 × 5 = 25 marks)**

**Answer any *FIVE* questions**

1. Prove that every $\left(p, q\right)-graph $with $q\geq p$ contains a cycle.
2. If $G$ is a Hamiltonian graph, then show that $w(G-S)\leq \left|S\right|$, for any non-empty subset $S$ of $V(G)$.
3. Prove that a graph $G$ is a tree if and only if every two vertices of $G$ are connected by a unique path.
4. State and prove Euler formula for planar graphs.
5. Show that, for any graph $G, χ\left(G\right)\leq ∆\left(G\right)+1$.
6. Show that, a vertex $v $in a connected graph $G$ is a cut vertex if and only if there exists vertices $u and w(\ne v)$ such that every path connecting $u and w contain v$.
7. For a $\left(p, q\right)-graph G, $show that the following statements are equivalent

(a) $G$ is a tree.
(b) $G $is connected and $q=p-1$. (c) $G $is acyclic and $q=p-1$.

**SECTION C — (3 × 10 = 30 marks)**

**Answer any *THREE* questions**

1. Prove that a connected $\left(p, q\right)-graph$ contains a cycle if and only if $q\geq p$.
2. Prove that a non-trivial connected graph is Eulerian if and only if it has no vertex of odd degree.
3. Prove that a $\left(p, q\right)-graph G$ is a bipartite graph if and only if it contains an odd cycle.
4. (a) Define a dual of a graph.
(b). Prove that a graph is planar if and only if it contains no contraction of

 $k\_{5}$ or $k\_{3,3}$.

1. If $G$ is a bipartite graph with $(q(G)\geq 1)$ then show that $χ\_{1}\left(G\right)=∆(G)$.

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