**B.Sc. DEGREE EXAMINATION, APRIL 2016.**

**III YEAR — VI SEMESTER**

 **Major Paper XIII — LINEAR ALGEBRA**

**Time : 3 hours Max. Marks : 75**

**SECTION A — (10 × 2 = 20 marks)**

**Answer any *TEN* questions.**

1. Define a vector space over a field.
2. Define linear span.
3. Define the dimension of *V* over *F.*
4. Define dual space.
5. Define inner product space.
6. Define orthogonal set of vectors.
7. When is an element in *A(V)* said to be regular?
8. Define characteristic roots.
9. Define *m(T)*.
10. The subspace W of V is invariant under if —————.
11. If V is finite-dimensional over F and if is right invertible, then it is ———.
12. The algebra of all *n* **×** *n*  matrices over *F* denoted as —————.

**SECTION B — (5 × 5 = 25 marks)**

**Answer any *FIVE* questions.**

1. Show that if *V* is a vector space over *F*, then
2. 
3. 
4. 
5. If 
6. If is a basis of *V* over *F* and if in *V* are linearly independent over *F*, then 
7. If is an orthonormal set, then the vectors in  are linearly independent.
8. If *V*  in finite dimensional over *F*, the  is invertible if and only if the constant term of the minimal polynomial for *T* is not zero.
9. If *V* is *n*-dimensional over *F* and if  has all its characteristic roots in *F,* then prove that *T* satisfies a polynomial of degree *n* over *F*.
10. Show that *L(S)* is a subspace of *V*.
11. If is a characteristic root of , then prove that for any polynomial  is a characteristic root of *q(T)*.

**SECTION C — (3 × 10 = 30 marks)**

**Answer any *THREE* questions.**

1. If V is the internal direct sum u1,…., un,then show that V is isomorphic to the external direct sum of u1,…., un.
2. If V is finite – dimensional and if W is an subspace of V, the prove that W is finite – dementional, and dim W ≤ dim V and dim V/W = dim V-dim W.
3. If $u , v \in V $ then show that $\left|\left(u,v\right)\right|\leq \left‖u\right‖\left‖v\right‖.$
4. If $λ ϵ F$ is a characteristic root of $T ϵ A\left(V\right)$ then show that $λ$ is a root of the minimal polynomial of T.
5. If V is n – dimensional over F and if $T ϵ A(V)$ has the matrix m1(T) in the basis and the matrix *m2(T)* is the basis *w1,….,* wn of *V* over *F,* then show that there is an element such that . Also if *S* is the linear transformation of *V* defined by  for *i = 1,2, …,* then prove that C can be taken as 