**B.Sc. DEGREE EXAMINATION, APRIL 2017.**

**III YEAR — V SEMESTER**

**Major Paper IX — MODERN ALGEBRA**

**Time : 3 hours Max. Marks : 75**

**SECTION A — (10 × 2 = 20 marks)**

**Answer any *TEN* questions.**

1. When we call a group an abdian group?
2. Define normal subgroup.
3. State cayley’s theorem.
4. Define isomorphism.
5. Define zero divisor.
6. Give an example for a integral domain which is not field.
7. If *A* is an ideal of ring *R* with unity and *1 A* then Show that *A=R.*
8. Show that every field is Principal ideal.
9. Define Euclidean ring.
10. When you say that two element are relatively prime?
11. Define cyclic group.
12. Define disjoint cycles.

**SECTION B — (5 × 5 = 25 marks)**

**Answer any *FIVE* questions.**

1. State and prove Cancellation laws in a group.
2. Express the permutation *(135) (5432) (5678)* as a product of disjoint cycles.
3. Show that a field is necessarily an integral domain.
4. If *U* is an ideal of *R* and *1  U* prove *U=R.*
5. If *R* is commutative ring with unity whose ideals are (0) and *R* itself then prove that *R* is a field.
6. Prove that Euclidean ring possesses unit element.
7. If *G* is a finite group of order *η* and *a G* then show that *aη =e.*

**SECTION C — (3 × 10 = 30 marks)**

**Answer any *THREE* questions.**

1. State and prove Lagrange’s theorem.
2. State and prove Cayley’s theorem.
3. Prove that a finite integral domain is a field.
4. Prove that Kernal of a ring homomorphism is an ideal.
5. If *R* is a Eudidean ring show that any elements *a* and *b* in *R* have a greatest common divisor *d* , also prove that *d=λ a+µb* for some λ, *µ R.*

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