**B.Sc. DEGREE EXAMINATION, APRIL 2017.**

**III YEAR — VI SEMESTER**

**Major Paper XIII — LINEAR ALGEBRA**

**Time : 3 hours Max. Marks : 75**

**SECTION A — (10 × 2 = 20 marks)**

**Answer any *TEN* questions.**

1. Define a Vector space over a field F
2. Prove that R is not a vector space over C.
3. If V is finite-dimensional over F then prove that any two basis of V have the same number of

elements.

1. Prove that any two finite-dimensional vector space over F of the same dimension are

isomorphic.

1. Prove that is a basis for .
2. Define an inner product space on V
3. Let be an orthogonal set of non-zero vectors in an inner product space V. Then prove that S is linearly independent.
4. Let V be a finite dimensional inner product space. Let W be a subspace of V. Then prove that
5. Prove that the element is a characteristic root of if and only if for some in V, prove that
6. If V is finite-dimensional over F and if is invertible, then prove that is a

polynomial expression in T over F.

1. Define matrix of the linear transformation T
2. If is a matrix representation of a linear transformation , where is

a vector space over , with respect to basis , then find T(1,1) and T(1,0).

**SECTION B — (5 × 5 = 25 marks)**

**Answer any *FIVE* questions.**

1. If V is a vector space over F then prove that (i)

(ii) (iii)

(iv) If vthen prove that = 0 implies

1. If are in V then prove that either they are linearly independent or some is a

linear combination of the preceding ones ,

1. If V and W are of dimensions m and n, respectively, over F, then prove that Hom(V,W) is of

dimension mn over F.

1. If then prove that .
2. If V is finite-dimensional over F, then prove that is regular if and only if T maps V

onto V.

1. If is invariant under T, then prove that T induces a linear transformation on V/W,

defined by If T satisfies the polynomial then prove that also satisfies the polynomial.

1. If and if is regular, then prove that

[P.T.O.]

**SECTION C — (3 × 10 = 30 marks)**

**Answer any *THREE* questions.**

1. Let V be a vector space over a field F. Let S,TV then prove that

(a) (b)

(c) L(S) = S if and only if S is a subspace of V.

1. If V is finite-dimensional and if W is a subspace of V then prove that W is finite-dimensional, and dim W
2. Let V be a finite-dimensional inner product space, then prove that V has an orthonormal set as a basis.
3. IF In F are distinct characteristic roots of and if are characteristic vectors of T belonging to , respectively, then prove that are linearly independent over F.
4. IF V is n-dimensional over F and if has the matrix in the basis and the matrix in the basis of V over F, then prove that there is an element such that .