**B.Sc. DEGREE EXAMINATION, APRIL 2017.**

**III YEAR — V SEMESTER**

**Major Paper X — REAL ANALYSIS**

**Time : 3 hours Max. marks : 75**

**SECTION A — (10 × 2 = 20 marks)**

**Answer any *TEN* questions.**

1. State least upper bound axiom.
2. Show that the set of all real numbers is uncountable.
3. Define a Cauchy sequence and give an example.
4. If converges to , then show that .
5. Define a metric space.
6. Is every Cauchy sequence in a metric space convergent? Justify.
7. Show that every one point set is open in .
8. Is infinite union of closed sets closed? Justify.
9. If is not of measure zero, if and if is of measure zero, prove that is not of measure zero.
10. State Rolle’s theorem.
11. Define bounded sequence and give an example of a sequence which is bounded but not convergent.
12. Does conditional convergence imply absolute convergence? Justify.

**SECTION B — (5 × 5 = 25 marks)**

**Answer any *FIVE* questions.**

1. Show that every convergent sequence is bounded.
2. Test for convergence: .
3. Let be a non-decreasing function on the bounded open interval . If is bounded above on , then show that exists. Also, if is bounded below on , then show that exists.
4. Show that the real valued function is continuous at if and only if, whenever is a sequence of numbers converging to , the sequence is converges to .
5. State and prove the law of mean.
6. Show that arbitrary intersection of closed sets is closed.
7. If , then show that . Also prove that

[P.T.O.]

**SECTION C — (3 × 10 = 30 marks)**

**Answer any *THREE* questions.**

1. Show that the sequence is convergent.
2. If is a sequence of real numbers such that (a) , and (b) , then show that the alternating series is convergent.
3. Let be a metric space and let be a point in . Let and are real valued functions whose domains are subsets of . If , and , then show that .
4. Show that is of the second category.
5. State and prove the chain rule.

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