B.Sc. DEGREE EXAMINATION, NOVEMBER 2017.

III Year V Semester

Core Major - Paper X - REAL ANALYSIS

Time : 3 Hours Max. Marks : 75

SECTION A – (10 × 2 = 20 marks)

Answer any *TEN* questions

1. Define least upper bound axiom.

2. Define bounded sequence.

3. Define Cauchy sequence.

4. Define conditionally convergent and absolutely convergent.

5. Define metric space.

6. If $lim\_{x\rightarrow a}f\left(x\right)=L$ and $lim\_{x\rightarrow a}g\left(x\right)=M$ then prove that $lim\_{x\rightarrow a}\left[f\left(x\right)+g\left(x\right)\right]=L+M$

7. Define open subset of a metric space.

8. Prove that the set of all irrationals is of second category.

9. State Chain rule in derivatives.

10. State second fundamental theorem of calculus.

11. Prove that the series is divergent.

12. State Comparison test.

SECTION B – (5 × 5 = 25 marks)

Answer any *FIVE* questions

13. Prove that the sequence $\left\{\left(1+\frac{1}{n}\right)^{n}\right\}\_{n=1}^{\infty }$ is convergent.

14. Prove that any bounded sequence of real numbers has a convergent subsequence.

15. Let $f$ be a non decreasing function on the bounded open interval (a, b). If $f$ is bounded above

 on (a, b) then prove that $lim\_{x\rightarrow b-}f\left(x\right)exists.$

16. If $f and g$ are real-valued functions, if $f$ is continuous at $a$, and if $g$ is continuous at $f(a)$

 then prove that $gο f $is continuous at $a$.

17. State and prove Rolle’s theorem.

18. Let G be an open subset of the metric space. Then prove that $G^{'}=M-G$ is closed. If F is a

 closed subset of M then prove that $F^{'}=M-F$ is open.

19. Prove that every convergent sequence is bounded.

SECTION C – (3 × 10 = 30 marks)

Answer any *THREE* questions

20. Prove that the set $\left[0, 1\right]=\left\{x \right|0\leq x\leq 1\}$ is uncountable.

21. If is a sequence of positive numbers such that i) and

 ii) , then show that is convergent.

22. Let $<M, ρ>$ be a metric space and let a be a point in M. Let $f and g$ be real-valued

 functions whose domains are subsets of M. $lim\_{x\rightarrow a}f\left(x\right)=L$ and $lim\_{x\rightarrow a}g\left(x\right)=N$ then

 prove that $lim\_{x\rightarrow a}\frac{f(x)}{g(x)}=\frac{L}{N}$

23. Prove that $f$ is continuous if and only if the inverse image of every open set is open.

24. State and prove fundamental theorem of calculus.

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