B.Sc. DEGREE EXAMINATION, NOVEMBER 2017.

III Year V Semester

Core Major - Paper X - REAL ANALYSIS

Time : 3 Hours Max. Marks : 75

SECTION A – (10 × 2 = 20 marks)

Answer any *TEN* questions

1. Define least upper bound axiom.

2. Define bounded sequence.

3. Define Cauchy sequence.

4. Define conditionally convergent and absolutely convergent.

5. Define metric space.

6. If and then prove that

7. Define open subset of a metric space.

8. Prove that the set of all irrationals is of second category.

9. State Chain rule in derivatives.

10. State second fundamental theorem of calculus.

11. Prove that the series is divergent.

12. State Comparison test.

SECTION B – (5 × 5 = 25 marks)

Answer any *FIVE* questions

13. Prove that the sequence is convergent.

14. Prove that any bounded sequence of real numbers has a convergent subsequence.

15. Let be a non decreasing function on the bounded open interval (a, b). If is bounded above

on (a, b) then prove that

16. If are real-valued functions, if is continuous at , and if is continuous at

then prove that is continuous at .

17. State and prove Rolle’s theorem.

18. Let G be an open subset of the metric space. Then prove that is closed. If F is a

closed subset of M then prove that is open.

19. Prove that every convergent sequence is bounded.

SECTION C – (3 × 10 = 30 marks)

Answer any *THREE* questions

20. Prove that the set is uncountable.

21. If is a sequence of positive numbers such that i) and

ii) , then show that is convergent.

22. Let be a metric space and let a be a point in M. Let be real-valued

functions whose domains are subsets of M. and then

prove that

23. Prove that is continuous if and only if the inverse image of every open set is open.

24. State and prove fundamental theorem of calculus.

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