B.Sc. DEGREE EXAMINATION, APRIL 2018.

III YEAR VI SEMESTER

Core Major - Paper XIII - LINEAR ALGEBRA

Time : 3 Hours Max. Marks : 75

SECTION A – (10 × 2 = 20 marks)

Answer any *TEN* questions

1. Define vector space with an example.

2. Define linear span.

3. Define annihilator of a subspace.

4. If then, prove that .

5. Define an inner product space.

6. If u,vV and then prove that

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7. Prove that the element is a characteristic root of if and only if for some   
 in =

8. Define invertible

9. Define similar transformations.

10. When a subspace is said to be invariant under transformation?

11. Define a characteristic vector.

12. If V is a vector space then prove that

SECTION B – (5 × 5 = 25 marks)

Answer any *FIVE* questions

13. If are in V then prove that either they linearly independent or some is a linear   
 combination of the preceding ones

14. If V and W are of dimensions m and n respectively over F then prove that Hom(V, W) is of   
 dimension mn over F.

15. State and prove Schwarz inequality.

16. If is a characteristic root of , then prove that for any polynomial ,   
 is a characteristic root of .

17. If is n-dimensional over and if has all its characteristic roots in , then prove   
 that satisfies a polynomial of degree over

[P.T.O.]

18. If is a basis of V over F and if in V are linearly independent over   
 F then prove that .

19. If V is finite-dimensional over F then prove that for (i)

(ii) (iii) for S regular in A(V).

SECTION C – (3 × 10 = 30 marks)

Answer any *THREE* questions

20. If T is a homomorphism of U onto V with kernel W, then prove that V is isomorphic to U/W.   
 And also if U is a vector space and W is a subspace of U then prove that there is homomor  
 phism of U onto U/W.

21. If is finite-dimensional and if is a subspace of , then prove that is finite-dimensional, .

22. Let be a finite –dimensional inner product space; then prove that has an orthonormal set   
 as a basis.

23. If is an algebra, with unit element, over , then prove that is isomorphic to a subalgebra of   
 for some vector space over

24. If has all its characteristic roots in then prove that there is basis of in which the   
 matrix of is triangular.