B.Sc. DEGREE EXAMINATION, APRIL 2018.

III YEAR V SEMESTER

Core Major - Paper IX - MODERN ALGEBRA

Time : 3 Hours Max. Marks : 75

SECTION A – (10 × 2 = 20 marks)

Answer any *TEN* questions

1. Define a group.
2. In a group $G ,$ prove that $\left(a b\right)^{-1}= b ^{-1}a ^{-1}$ for all $a , b \in G$ .
3. Define automorphism.
4. If $∅$ is a homomorphism of $G$ into $\overbar{G}$ , show that $∅ \left( e \right)= \overbar{e}$ , where $e$ and $\overbar{e}$ are the identity elements of $G$ and $\overbar{G}$ respectively.
5. Define a commutative ring.
6. Prove that in a ring R , $a0=0a=0$ for $a \in R.$
7. Define an ideal.
8. Define a maximal ideal.
9. Define relatively prime.
10. Prove that an Euclidean ring possesses a unit element.
11. Define Normal subgroup.
12. Every integral domain can be \_\_\_\_\_\_\_\_\_\_\_ in a field.

SECTION B – (5 × 5 = 25 marks)

Answer any *FIVE* questions

1. Prove that the relation $a ≡b mod H$ is an equivalence relation.
2. Prove that every permutation is the product of its cycles.
3. If R is a ring in which $x^{2}=x$ for all $x \in R ,$ show that
4. $2x=0 $for all$ x \in R$ .
5. $ R $is commutative.

[P.T.O.]

1. If R is a commutative ring with unit element and M is an ideal of R , prove that M is maximal ideal of R if and only if ${R }/{M}$ is a field.
2. Let $R$be a Euclidean ring. Then prove that any two elements $a$ and $b$ in $R$ have a greatest common divisor d. Also prove that $d= λa+ μb$ for some $λ , μ ϵ R.$
3. Prove that N is a normal subgroup of $G$ if and only if $gNg^{-1}=N$ for every $g \in G$ .
4. Let $R$be a Euclidean ring. Suppose that for $a , b , c \in R , {a}/{b}c$ but

$\left( a, b \right)=1$, prove that $^{a}/\_{c}$ .

SECTION C – (3 × 10 = 30 marks)

Answer any *THREE* questions

1. Prove that $HK$ is a subgroup of G if and only if $HK=KH.$
2. If $∅$ is a homomorphism of $G$ onto $\overbar{G}$ with kernel K , prove that ${G}/{K ≈}\overbar{G .}$
3. Prove that a finite integral domain is a field.
4. If $R$ is a commutative ring with unit element whose only ideals are ( 0 ) and

 $R $Itself, prove that $R$ is a field.

1. State and prove Unique factorization theorem.