B.Sc. DEGREE EXAMINATION, APRIL 2018.

III YEAR V SEMESTER

Core Major - Paper IX - MODERN ALGEBRA

Time : 3 Hours Max. Marks : 75

SECTION A – (10 × 2 = 20 marks)

Answer any *TEN* questions

1. Define a group.
2. In a group prove that for all .
3. Define automorphism.
4. If is a homomorphism of into , show that , where and are the identity elements of and respectively.
5. Define a commutative ring.
6. Prove that in a ring R , for
7. Define an ideal.
8. Define a maximal ideal.
9. Define relatively prime.
10. Prove that an Euclidean ring possesses a unit element.
11. Define Normal subgroup.
12. Every integral domain can be \_\_\_\_\_\_\_\_\_\_\_ in a field.

SECTION B – (5 × 5 = 25 marks)

Answer any *FIVE* questions

1. Prove that the relation is an equivalence relation.
2. Prove that every permutation is the product of its cycles.
3. If R is a ring in which for all show that
4. for all .
5. is commutative.

[P.T.O.]

1. If R is a commutative ring with unit element and M is an ideal of R , prove that M is maximal ideal of R if and only if is a field.
2. Let be a Euclidean ring. Then prove that any two elements and in have a greatest common divisor d. Also prove that for some
3. Prove that N is a normal subgroup of if and only if for every .
4. Let be a Euclidean ring. Suppose that for but

, prove that .

SECTION C – (3 × 10 = 30 marks)

Answer any *THREE* questions

1. Prove that is a subgroup of G if and only if
2. If is a homomorphism of onto with kernel K , prove that
3. Prove that a finite integral domain is a field.
4. If is a commutative ring with unit element whose only ideals are ( 0 ) and

Itself, prove that is a field.

1. State and prove Unique factorization theorem.