B.Sc. DEGREE EXAMINATION, APRIL 2018.

I YEAR II SEMESTER

Core Major - Paper III - CLASSICAL ALGEBRA

Time : 3 Hours Max. Marks : 75

SECTION A – (10 × 2 = 20 marks)

Answer any *TEN* questions

1. Write down the expansion of $ \left( 1 -x \right)^{^{-p}/\_{q}}$
2. Show that, $\frac{e+1}{e-1}= \frac{\frac{1}{1! } + \frac{1}{3!} +…}{\frac{1}{2! } + \frac{1}{4! }+….}$
3. Form the rational cubic equation whose two of the roots are 1, 3 -$ \sqrt{-2}$.
4. Solve $x^{3}-12x^{2}+39x-28=0$ whose roots are in Arithmetic Progression.
5. What is receiprocal equation.
6. Show that, the equation $x^{4}-3x^{3}+4x^{2}-2x+1=0$ can be transformed into a reciprocal equation by diminishing the roots by unity.
7. Define Hermitian matrix with an example.
8. State Cayley-Hamilton theorem.
9. Find the number of divisors of 480.
10. State Fermat's theorem.
11. Find the eigen values of $ \left(\begin{matrix}4&-2\\3&3\end{matrix}\right)$
12. When do you say two integers a and b are congruent with respect to modulo m?

SECTION B – (5 × 5 = 25 marks)

Answer any *FIVE* questions

1. Sum the series to infinity: $\frac{1 . 4}{5 . 10}+ \frac{1 . 4 . 7}{5 . 10 . 15}+ \frac{1 . 4 . 7 . 10}{5 . 10 . 15 . 20}+ …$
2. Solve the equation $ 81x^{3}- 18 x^{2} -36 x+8=0$ whose roots are in Harmonic Progression.
3. Remove the fractional coefficients from the equation $x^{3} - \frac{1}{4} x^{2}+ \frac{1}{3} $x - 1 = 0
4. Show that, any real square matrix A may be uniquely written as the sum of symmetric and skew-symmetric matrices.
5. Find the smallest integers with 18 divisors.

[P.T.O.]

1. Show that, $ \frac{5}{1 . 2 . 3}+ \frac{7}{3 . 4 . 5}+ \frac{9}{5 . 6 . 7}+ …$ = 3 log 2 - 1.
2. Find the eigen values and eigen vectors of the matrix $\left(\begin{matrix}2&-2&3\\1&1&1\\1&3&-1\end{matrix}\right)$

SECTION C – (3 × 10 = 30 marks)

Answer any *THREE* questions

1. Show that, $1+ \frac{1+2}{2!}+ \frac{1+2+ 2^{2}}{3!}$ + … ∞ = e ( e - 1 ).
2. Solve the equation, $x^{4} -2 x^{3}+ 4 x^{2}+ 6 x -21=0$ given that two of its roots are equal in magnitude and opposite in sign.
3. Solve the reciprocal equation $x^{6}+ 2 x^{5}+ 2 x^{4} -2 x^{2} -2 x -1=0.$
4. Verify Cayley-Hamilton theorem for A = $\left(\begin{matrix}2&2&0\\2&1&1\\-7&2&-3\end{matrix}\right)$ and hence find its inverse.
5. (i) Show that, 8th power of any number is of the form 17m or 17m ± 1.

(ii) Show that, (18)! + 1 is divisible by 437.