13UPHCT3005/ UPH/CT/3005

B.Sc. DEGREE EXAMINATION,NOVEMBER 2018 II Year III Semester Core Major - Paper V MATHEMATICAL PHYSICS AND STATISTICAL MECHANICS

Time : 3 Hours

Max.marks :60

Section A $(10 \times 1 = 10)$ Marks

Answer any **TEN** questions

- 1. The sum of the eigen values of matrix is equal to the ———– of the matrix.
- 2. What is diagonalisation of a matrix?
- 3. Define beta function.
- 4. Evaluate Γ -1/2.
- 5. Prove that $\sum_{n=0}^{\infty} Pn(x) = \frac{1}{\sqrt{2-2x}}$.
- 6. Find the value of $H_1(x)$ in terms of x.
- 7. What is the phase point?
- 8. What is an ensemble?
- 9. What do you mean by Fermi energy and Fermi temperature?
- 10. What are Bosons?
- 11. How does probability depend upon the number of microstates?
- 12. What is characteristic equation?

Section B $(5 \times 4 = 20)$ Marks

Answer any **FIVE** questions

- 13. State and prove Cayley Hamilton's theorem.
- 14. Evaluate beta function.
- 15. Derive an expression for the generating function for Laguerre's polynomial.
- 16. Give the postulates of statistical mechanics.
- 17. Calculate the Fermi energy of aluminium that the metal has one free electron per atom. Given that Planck's constant (h) = 6.625×10^{-34} JS,Mass of the electron (m)= 9×10^{-31} kg Density of aluminium (ρ) = 7700 kg/m³ and atomic weight of aluminium M = 27.
- 18. Show that $\int_0^1 [35x^2/(32\sqrt{1-x})] dx = 1$ using gamma function.
- 19. Prove that $J_{-n}(x) = (-1)^n J_n(x)$ where n is a positive integer.

Section C $(3 \times 10 = 30)$ Marks

Answer any **THREE** questions

- 20. Determine the eigenvalues and Eigen vectors of the matrix
 - $\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$
- 21. (a) Find the relation between Beta and Gamma function. (b) Evaluate the integral $\int_0^\infty e^{-x^2} dx$.
- 22. Integrate in series Bessel's equation $x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + (x^2 n^2)y = 0$ Where n is not an integer and obtain the general solution of it.
- 23. Find the average energy of a molecule to an ideal gas using Maxwell Boltzmann distribution law.
- 24. Obtain the expression for Fermi-Dirac distribution law. Using it derive expression for the fermi energy of an electron in a metal.

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