M.Sc. DEGREE EXAMINATION,NOVEMBER 2018
I Year II Semester
Core Elective Paper - II
MATHEMATICAL STATISTICS

Time : 3 Hours **Max.marks :75**

Section A $(10 \times 2 = 20)$ Marks

Answer any **TEN** questions

1. Write the moment generating function of chi-square distribution
2. Define the t-distribution.
3. Define point estimation.
4. Define consistent estimate.
5. Define test-function.
6. Define uniformly most powerful test.
7. Define liklihood ratio test.
8. Write the two tailed t-test, when σ^2 is known.
9. Define confidence coeffiecient.
10. Define a pivot of an random variable $\mathrm{T(x,}\theta)$.
11. Define non-central F-distribution with (m, n) degrees of freedom.
12. Write the necessary and sufficient condition for an estimate T to be most efficient.

Section B $(5 \times 5 = 25)$ Marks

Answer any **FIVE** questions

13. Let $\mathrm{X} \sim \mathrm{F(m,n)}$. Prove that for k $>$ 0, integral $EX\hat{}K = (n/m)\hat{}K \ (\Gamma[\mathrm{k} + (\mathrm{m}/2)\ ]\Gamma[(\mathrm{n}/2) - \mathrm{k}])/\Gamma[(\mathrm{m}/2)\Gamma(\mathrm{n}/2)\ ] \quad \mathrm{for\ n} > 2k$.
14. Let $\{\mathrm{T_n}\}$ be a sequence of UMVUE's and T be a statistic with $\mathrm{E}_\theta \mathrm{T}^2 < \infty$ and such that $\mathrm{E}_\theta\{\mathrm{T_n} - \mathrm{T}\}^2 \to 0$ as $\mathrm{n} \to \infty$ for all $\theta\epsilon\Theta$. Prove that T is also the UMVUE.
15. Let T(x) be a maximal invariant with respect to $\mathcal{G}$. Prove that φ is invariant under $\mathcal{G}$ if and only if φ is a function of T.
16. A die is rolled 120 times with the following results

Face	1	2	3	4	5	6
Frequency	20	30	20	25	15	16

Test at 5% level of significance if the die is honest.

17. Let $X_1, X_2, \ldots, X_n$ be independent, identically distributed $N(\mu, \sigma^2)$ random variables. Prove that $\overline{X}$ and $(X_1-\overline{X}, X_2-\overline{X}, \ldots, X_n-\overline{X})$ are independent.
18. Let $X_1, X_2, \ldots, X_n$ be a sample from U(0, θ). Find the shortest length confidence interval for θ at level 1-α based on a sufficient statistic for θ.
19. State and prove Chapman and Robbins inequality.

Section C $(3 \times 10 = 30)$ Marks

Answer any **THREE** questions

20. If $X \sim X^2(n)$, then prove that $\lim_{n\to\infty} p\left\{\sqrt{2X}-\sqrt{2n-1} \le z\right\} = \int_{-\infty}^{t} \frac{1}{\sqrt{2\pi}} e^{-\frac{y^2}{2}} dy$.
21. State and prove Rao-Blackwell theorem.
22. State and prove the Neyman-Pearson fundamental lemma.
23. In a 72 hour period on a long holiday weekend there was a total of 306 fatal automobile accidents. The data are as follows.

No.Of fatal accidents per hour	0 or 1	2	3	4	5	6	7	8 or more
No. of hours	4	10	15	12	12	6	5	7

Test the hypothesis that the number of accidents per hour is a poisson random variable.

24. The following tables gives the yield(pounds per plot) of three varieties of wheat, obtained with four different kinds of fertilizers

	VARIETY OF WHEAT		
FERTILIZER	A	B	C
W	8	3	7
X	10	4	8
Y	6	5	6
Z	8	4	7

Test the hypothesis of equality in the average yields of the three varieties of wheat and the null hypothesis that the four fertilizers are equally effective.

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