M.Sc. DEGREE EXAMINATION,NOVEMBER 2018
II Year IV Semester
Core Elective - IV
CALCULUS OF VARIATIONS AND INTEGRAL EQUATIONS

Time : 3 Hours **Max.marks :75**

Section A $(10 \times 2 = 20)$ Marks

Answer any **TEN** questions

1. Define kth order proximity.
2. Find the extremal of the functional v[y(x)] $= \int_0^1 (1 + y''^2)\, dx$, y(0)=0, y'(0)=1,y(1)=1, y'(1)=1.
3. Write the transversality condition for the functional v= $\int_{x_0}^{x_1} F(x, y, z, y', z')dx.$
4. Define an extremal field.
5. Define a degenerate kernel with an example.
6. Define singular integral equations with an example.
7. State any one drawback of using the method of successive approximations.
8. State the Fredholm's second theorem.
9. Give an example for an orthogonal system of functions.
10. Write the Abel integral equation.
11. State the fundamental lemma of calculus of variation.
12. Find the extremal of v= $\int_0^1 \left(y^2 + x^2 y^1\right) dx,$ y(0)=0, y(1) = a.

Section B $(5 \times 5 = 25)$ Marks

Answer any **FIVE** questions

13. Write the ostrogradsky equation for the functional v[z(x,y)] $= \iint_D [(\frac{\partial z}{\partial x})^2 + (\frac{\partial z}{\partial y})^2 + 2z\ f(x, y)]dxdy.$
14. Find the broken-line of extremals of v= $\int_{x_0}^{x_2} {y^1}^2 {(1 - y^1)}^2 dx$
15. Show that the integral equation g(s) = f(s) + $(\frac{1}{\pi}) \int_0^{2\pi} [\sin{(s + t)}\ ]\, g\,(t)\, dt$ possesses infinitely many solutions when f(s)=1.

16. Find the resolvent kernel of g(s) $= 1{+}\lambda \displaystyle\int_0^1 (1-3st)\, g(t)\, dt$ and state the condition for validity.
17. Explain the procedure of obtaining a solution of a symmetric integral equation.
18. Prove that the eigen functions of a symmetric kernel, corresponding to different eigen values are orthogonal.
19. Is the Jacobi condition fulfilled for extremal of the functional v[y(x)] $= \displaystyle\int_0^a \left(y^{1^2} + y^2 + x^2\right) dx$, that passes through the points A(0,0) and B(a,0)?

Section C $(3 \times 10 = 30)$ Marks

Answer any **THREE** questions

20. Derive the differential equation of free vibrations of a string.
21. Find the equation of geodesics on a surface on which the element of length of the curve is of the form $ds^2 = [\phi_1(x) + \phi_2(y)]\ (dx^2 + dy^2)$.
22. Using an approximate method, solve g(s)$= e^s - s - \displaystyle\int_0^1 s\left(e^{st} - 1\right) g(t)\, dt$.
23. Solve the integral equation g(s) $=$ s$+\lambda \displaystyle\int_0^1 \left[st + \sqrt{st}\right] g(t)\, dt$.
24. State and prove Hilbert- Schmidt theorem.

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