# M.Sc. DEGREE EXAMINATION,NOVEMBER 2018
## I Year II Semester
## Core Major- Paper IV
## LINEAR ALGEBRA

**Time : 3 Hours** **Max.marks :75**

**Section A** $(10 \times 2 = 20)$ Marks

Answer any **TEN** questions

1. Define an algebraic element of degree n.
2. Define algebraic extension.
3. When is an element $\mathrm{a} \in \mathrm{K}$ is said to be a root of $\mathrm{p(x)} \in \mathrm{f(x)}$ of multiplicity m?
4. Define simple extension.
5. Define fixed field of the group of automorphisms of a filed K.
6. Define the Galois group of $\mathrm{f(x)} \in \mathrm{F[x]}$.
7. Define the index of nilpotence of a linear transformation.
8. Define the invariants of a linear transformation.
9. Define basic Jordan block belonging to $\lambda$.
10. Define rational canonical form of T.
11. Define a transcendental number and give an example.
12. Define characteristic polynomial of T.

**Section B** $(5 \times 5 = 25)$ Marks

Answer any **FIVE** questions

13. If $\mathrm{a},\ \mathrm{b}$ in $\mathrm{K}$ are algebraic over F, the prove that $\mathrm{a} \pm \mathrm{b},\ \mathrm{a.b}$ and a/b (if b≠0 ) are all algebraic over F.
14. If L is an algebraic extension of K and K is an algebraic extension of F, the prove that L is an algebraic extension of F.
15. Show that if $\mathrm{p(x)} \in \mathrm{F(x)}$ of degree n $\geq$ 1 and is irreducible over F then there is an extension E of F such that [E : F ] $=$ n in which $\mathrm{p(x)}$ has a root.
16. Define G(K, F). show that G(K, F) is a subgroup of all automorphisms of K.
17. Show that if $\mathrm{T} \in \mathrm{A}(\mathrm{V})$ is nilpotent the $\alpha_0 + \alpha_1 \mathrm{T} + \ldots + \alpha_{\mathrm{m}} \mathrm{T}^{\mathrm{m}}$ , where the $\alpha_{\mathrm{i}} \in \mathrm{F}$ is invertible if $\alpha_0 \neq 0$.
18. Prove that $\mathrm{A} = \begin{pmatrix} 1 & 1 & 1 \\ -1 & -1 & -1 \\ 1 & 1 & 0 \end{pmatrix}$ is nilpotent and find its invariants.

19. Prove that every linear transformation $T \in A(V)$ satisfies its characteristic polynomial.

**Section C** $(3 \times 10 = 30)$ Marks

Answer any **THREE** questions

20. Prove that the number e is transcendental.
21. If F is of characteristic 0 and if a, b are algebraic over F, show that there exists an element $c \in F(a, b)$ such that F(a, b) = F(c).
22. Show that there is one-to-one correspondence between the sub fields of the splitting field of f(x) and the subgroups of its Galois group.
23. Prove that two nilpotent linear transformations are similar if and only if they have the same invariants.
24. Prove that the elements S and T in $A_F(V)$ are similar in $A_F(V)$ if and only if they have the same elementary divisors.

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15. Show that if $p(x) \in F(x)$ of degree n $\geq$ 1 and is irreducible over F then there is an extension E of F such that [E : F ] $=$ n in which $p(x)$ has a root.
16. Define G(K, F). show that G(K, F) is a subgroup of all automorphisms of K.
17. Show that if $T \in A(V)$ is nilpotent the $\alpha_0 + \alpha_1 T + \ldots + \alpha_m T^m$ , where the $\alpha_i \in F$ is invertible if $\alpha_0 \neq 0$.
18. Prove that $A = \begin{pmatrix} 1 & 1 & 1 \\ -1 & -1 & -1 \\ 1 & 1 & 0 \end{pmatrix}$ is nilpotent and find its invariants.

19. Prove that every linear transformation $T \in A(V)$ satisfies its characteristic polynomial.

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